Competing Process Hazard Function Models for Player Ratings in Ice Hockey

A.C. Thomas, Samuel L. Ventura, Shane Jensen, Stephen Ma

August 6, 2012

Abstract

Evaluating the overall ability of players in the National Hockey League (NHL) is a difficult task. Existing methods such as the famous "plus/minus" statistic have many shortcomings. Standard linear regression methods work well when player substitutions are relatively uncommon and scoring events are relatively common, such as in basketball, but as neither of these conditions exists for hockey, we use an approach that embraces these characteristics. We model the scoring rate for each team as its own semi-Markov process, with hazard functions for each process that depend on the players on the ice. This method yields offensive and defensive player ability ratings which take into account quality of teammates and opponents, the game situation, and other desired factors, that themselves have a meaningful interpretation in terms of game outcomes. Additionally, since the number of parameters in this model can be quite large, we make use of two different shrinkage methods depending on the question of interest: full Bayesian hierarchical models that partially pool parameters according to player position, and penalized maximum likelihood estimation to select a smaller number of parameters that stand out as being substantially different from average. We demonstrate this on games through five NHL seasons.

1 Introduction

In many situations where a desired outcome depends on the performance of a group, it can be difficult to evaluate the individual contributions of its members. The study of sports provides a number of examples of this over time; the easier decomposition of baseball into what are essentially head-to-head match-ups makes it comparatively easy to tell whether one batter is superior to another, given enough observations.

The study of goal-based team sports – hockey, basketball, soccer, and lacrosse, among others – is considerably more difficult, as the separation of roles is much more difficult to measure with modern game statistics, especially when player efforts do not directly lead to goals. In hockey, the subject of our investigations, player abilities are historically quantified by citing offensive statistics, such as goals and assists, and defensive statistics such as blocked shots and a goaltender's saves. However, these are measured on different combinations of players on the ice, so an overall assessment of ability is not as obvious. Even if we assume that goaltenders have no role in team offense, there is surely a defensive assessment that can be made for other players, which is not as easily captured by these count-based statistical measured.

The first well-known attempt at capturing an overall effect of a player in hockey was the Plus/Minus statistic (+/-). Consider all goals that happen when a particular player is on the ice; the number of goals that were scored for their team minus the number of goals scored by the opposition, whether or not the player was directly involved in the goal-scoring plays.[1] There are several known issues with this measure, mostly related to factors outside the player's control. It does not take into account each player's quality of teammates, quality of opponents, and position; good players on bad teams often have similar +/- statistics as bad players on good teams.

The nature of ice hockey means that scoring events are often quite rare. If we divide a game into many segments when the total number of goals scored is less than ten, the

[1] Goals scored by a team with a man advantage are not typically counted in this measure.

majority of these may be empty of scoring events, requiring a treatment that is considerate of this imbalance; segments of unequal length must also be handled appropriately.

This rarity also contributes to another important consideration – what if the data are insufficient to adequately separate players from each other in their ratings or, worse yet, have little to no predictive value, either for a player's own future performance or for in-game outcomes? Any method we use to generate these ratings should take this into account, either as an integral part of the method or as a post-analysis check.

To manage these factors and generate meaningful player ratings, we propose to measure the abilities of players in ice hockey according to goal-scoring rates when they are on the ice, much as in the plus-minus approach. However, we have two particular features of our approach that adjust for these factors. First, we consider goal-scoring to be the combination of at least two semi-Markov processes, modulated by the players on the ice for each team, so that each player on the ice contributes to both their team offense and team defense. Second, we regularize these estimates to ensure better predictive performance, which may also have the benefit of selecting a subset of players to have non-zero (i.e. non-average) ratings.

Ideally, our method for obtaining meaningful player ratings will have several important properties:

- The ratings will be directly interpretable in terms of game outcomes.
- It can distinguish the offensive and defensive capabilities of each player from each other, allowing for a superior assessment of ability.
- It will control for the quality of a player's team and teammates by factoring their abilities into each observed event.
- It will control for the quality of a player's opposition in the same fashion.
- If required, it can distinguish "average" from those who have exceptional skills at offense or defense, in either form.

We continue by describing previous methods for rating the offensive and defensive skill for players in hockey and other sports in Section 2, as well as describing the data available for this work. In Section 3 we describe our methodological approach to the problem, demonstrating many of its applications in Section 4. We conclude in Section 6 by discussing potential extensions to our approach.

2 Previous Approaches for Player Ratings

2.1 Count-Based Measures: Simple Plus/Minus

The notion of tracking the number of goals scored, both for and against, for each player on the ice is decades old, but its full application took years to reach its current state. In the National Hockey League (NHL), the world's premier professional ice hockey organization, its initial use is said to be pioneered by the Montreal Canadiens hockey club in the 1950s, though only for their own purposes and in secret. The system was popularized by NHL coach Emile Francis during the 1960s, though the existing weaknesses of this approach were obvious even then. The most obvious to address is the effective rarity of goals, with an average of roughly three per team per game. By adding other events that can lead to goals, more information can be attributed to the efforts of players on the ice. These typically include shots on goal, either unweighted or adjusted for the distance from the net, possibly including those that are blocked by the opposing team's skaters or miss the net entirely; these include the Fenwick- and Corsi- weighted Plus/Minus; Macdonald (2012b) lists these and others that have been adapted to the general approach.

Lock and Schuckers (2009); Schuckers et al. (2011) extend this idea by accounting for all events that are recorded in a modern NHL game, including faceoffs, turnovers, and hits, all of which are thought to change the likelihood of the scoring of goals, either due to changes in puck possession or location on the ice. Each of these has an effective "weight" in terms of the expected number of goals scored or prevented because that event did or did not occur;

for example, a team that wins a faceoff near their opponent's goal is more likely to score in the following seconds than they are to be scored upon, and have a higher probability of scoring than if their opponent had won the faceoff instead. For a player in a game, the sum of the weights of events in which they are involved can then inform us about that player's overall contribution to the game.

2.2 Regression-Adjusted Measures

The other most notable weakness of the standard Plus/Minus measure, or any of its derivatives, is coincident play: if two or more players are on the ice together for much of their shared time, it can be difficult to distinguish the abilities of each player from each other when so many of the outcomes to which they contribute are common to both. This problem is common to all goal-based team sports.

To handle this issue in basketball, Rosenbaum (2004) proposed to divide a National Basketball Association (NBA) game into intervals marked by the substitution of players onto the court. From this, he derived a number of independent events, each containing a number of scoring opportunities for each team. The outcome of each event is the difference in points scored between the two teams divided by the time elapsed during the interval; the predictors are indicators of the players on the court for each team – positive for the home team, negative for the away team. Using a linear regression model of these player-predictors on the scoring outcome, each player's associated coefficient represents their contribution to the change in score in favor of their team; this is their "adjusted plus-minus" rating. Ideally, this measure will isolate a player's contribution to their own rating and remove it from others, as the quality of their teammates and their opponents is accounted for.

Ilardi and Barzilai (2008) modify this approach by taking every interval as not one but two events – home scoring and away scoring – and treating them as independent, conditional on the length of the event. Each player on the court appears in each of these two events, as an offensive and defense player respectively, and therefore has a distinct rating for each of these "skills"; the combination of the two can then be taken as the total adjusted player rating.

Each of these procedures was conducted by Macdonald (2011) on NHL data by noting player substitutions from official game logs and using these to construct a table of events.

2.3 Regularization Methods and Variable Selection

One consequence of this modeling approach is the relatively large number of predictors against the number of events we can observe; in one season, there are roughly 400 different players in the NBA, and 1000 different players in the NHL. Because of this, estimates of ability on all players can be imprecise due to a potentially small sample on a subset of these individuals, through large variance or collinearity. One way to adjust for this is to regularize the estimates of each coefficient, producing biased estimates with lower variance. Ridge regression (Hoerl and Kennard, 1970) is used by Sill (2010) for the NBA, and Macdonald (2012a) for the NHL, to account for these difficulties; the degree of regularization was chosen through cross-validation on withheld observations.

Ridge regression is equivalent to specifying a Bayesian model on the ensemble of parameters. In particular, for the linear model, each coefficient has its own independent Gaussian prior $\beta_i \sim N(0, \sigma_\beta^2)$, equivalent to a constraint on the squared sum of the coefficients, where the prior parameter σ_β^2 is either specified beforehand or selected using out-of-sample validation. The ridge method is in fact the simplest form of hierarchical model for this data; there is potential for more flexibility by assigning a hyperprior distribution to this parameter, such as the conjugate form $\sigma_\beta^2 \sim InvGamma(a, b)$, and obtaining a posterior estimate of each of the coefficients and the variance parameter simultaneously. These approaches, plus other Gaussian-derived models such as James-Stein estimation James and Stein (1961), are compared for the case of batting averages in Brown (2008); this type of comparison is equally valid in this case.

Each of these methods of Gaussian regularization produces estimates that are non-zero, but if the point is to distinguish the relative ability of two or more players, it may be that we are far less interested in the comparison between players ranked 499 and 500 than we would be between players ranked 1 and 2. Many of these other players may simply be nuisances in estimating parameters of greater interest. As a result, incorporating a method for variable selection may be useful; the most automatic in this case would be the Lasso (Tibshirani, 1996), in which we obtain both a subset of non-zero parameters as well as estimates for these parameters. In this case, each coefficient has an independent Laplace prior, $\beta_i \sim Laplace(\lambda)$.

2.4 Process Models

The nature of substitution and scoring data from the NBA is vastly different from that of the NHL. In the NBA, there are typically several scoring events for either team per rotation (the equivalent of a "shift" in hockey), and there are relatively few substitutions per game. In the NHL, scoring events are much rarer, on the order of 10 minutes between goals, while players typically only spend about 30-60 seconds on the ice before returning to the bench for a substitution. As we show in Section 2.5, roughly 98% of these intervals have a total of zero goals scored; using this linear regression approach, the event durations will not factor in, and significant information will be lost. Additionally, since the data are clearly non-Gaussian, methods based on Gaussian convergence properties may not be reliable, as the error terms and the prediction terms must be highly dependent to produce the majority-zero data.

The rarity of scoring events relative to the number of observable intervals suggests the use of a Poisson-type process model. Each event represents an observation of the same players on the ice, and any event that does not end in a goal is essentially censored by the change in players. This directly incorporates the observed duration of the event as well as accounting for the relatively sparse number of goals. Simple Poisson models have been used for making strategic decisions in hockey (Morrison, 1976; Beaudoin and Swartz, 2010); these methods can be improved to account for heterogeneity in the scoring rate over time (Thomas, 2007).

Moreover, the game can often be divided into a number of discrete states that give additional information about the game. Hirotsu and Wright (2002) examine soccer as a continuous-time Markov process with 6 states: 2 teams can possess the ball on either half of the field, plus the state of having a goal scored in either net. Thomas (2006) considers a larger state space for hockey with a semi-Markov process instead. Only when a team has possession of the ball/puck in their opponent's territory can they score a goal, so that this underlying state will then directly influence the scoring rate for each team. This method can be applied if data on location and possession is available, but this is not currently available to the public.

We expect that players in the game will similarly affect the scoring rates for each team. The Cox process model (Cox, 1972) decomposes the rate of this process, described by the hazard function $h(t, X) = \lambda(t, X)$, into a time-varying component $\lambda_0(t)$ and a time-independent term for the inclusion of covariates $\lambda_x(X)$. Just as in the linear model case, these models can also be regularized, such as with the Lasso (Tibshirani, 1997).

2.5 Source of Data

Records of many National Hockey League (NHL) games are available to varying levels of detail. For the sake of dividing the game into discrete intervals, we use the interpretation of Rosenbaum (2004) and Macdonald (2011) that an interval should end either when a player substitution is made by either team or when an event occurs (e.g. when a goal is scored). This level of detail is available with ease in game records from the 2007-2008 season until the 2011-2012 season. We select those shifts in which both teams are at full strength – each team has five skaters and one goaltender on the ice – and note the duration of the event in seconds. The outcome is one of three possibilities: the home team scores, the away team scores, or neither team scores and at least one player substitution occurs. As Table 1 shows, over 98% of the observations are non-goal outcomes, which is highly disproportionate compared to the examples in basketball.

For this analysis, we consider only goal-scoring events as those produced by the process.

Seasons: 2007-2012	Away Goal	No Goal (Changes)	Home Goal
Total Events	10,935	1,301,799	11,981
Percent of Total Events	0.83	98.27	0.90

Table 1: A count of the events of each type in the database. A home team advantage is apparent.

We have additional information on shots on goal that did not result in goals, on penalties called that result in man-advantage situations, and on time-outs called (extremely rarely) by coaches. We do not include these at this stage to keep the analysis on events that directly influence the final result of winning or losing the game, since shots on goal only lead to goals a fraction of the time, and the relationship between shots on goal and goals is not as simple as a fixed fraction of events. Any processes that lead to shots must also lead to goals, and to add additional competing processes to the model would add an additional level of complexity that is beyond the scope of this investigation. (See Macdonald (2012b) for how this can be used in a standard regression set-up.)

For each season, we divide the data randomly into two groups – one for in-sample training (all observations from 80% of the games) and one for out-of-sample validation (20%).

3 Model Specification

We model the stochastic nature of the game as a model of two competing processes for the scoring of a goal, censored by player substitutions. Each process has parameters for offensive and defensive characteristics, and these parameters are regularized by partial pooling. We use either penalized maximum likelihood and full hierarchical Bayesian models to infer the parameters of interest.

3.1 Events Obey A Competing Processes Model

There are, at a minimum, two opposing processes in a hockey game: the home team tries to score on the away team, and vice versa. Both of these events are relatively rare compared to the number of observed event intervals, so that it is natural to model these as competing stochastic processes. Predictors that modulate these processes can be the teams in the game, the score of the game, the players on the ice, or some other combination. In particular, each of these predictors has a role in each process, though the magnitude and sign of the effects ought to be different.

We choose a Cox proportional hazards model for each process, so that the hazard function has separate components for time dependence and predictors, as $h(X,t) = h_0(t)h_1(X)$, where X can represent various factors such as the players and/or team on the ice. For this investigation we begin with $h_0(t) = 1$; more information on the location of the puck at each $t = 0$ may allow us to refine the time-based component in future investigations.

From this, each team's scoring rate is modeled as a log-linear Poisson process. The intercept terms, labeled r^h and r^a, represent the baseline scoring rates for the home and away teams, since as we see in Table 1, the overall scoring rate for the home team is greater than for the away team; in this way, we can explicitly detect a "home-ice advantage". For each predictor indexed by p, let (ω_p, δ_p) be a measure of the offensive and defensive contribution for that predictor, so that a rating of zero corresponds to an "average" contribution; the corresponding indicators are X_p^h and X_p^a. The scoring rates for each process are therefore

- $\lambda^h = \exp(r^h + \sum_p (X_p^h \omega_p + X_p^a \delta_p))$;
- $\lambda^a = \exp(r^a + \sum_p (X_p^a \omega_p + X_p^h \delta_p))$

for this combination. For each instance of this process, T^h and T^a are the times to each event for these processes, and let t be the first time at which any players on the ice are substituted, thereby censoring the scoring process. We assume that the (unmodeled) censoring time is independent of these event times, and that conditional on the predictors, these events are independent of each other. The outcome can then be registered as

$$Y = \begin{cases} 1 & if\ T^h < T^a,\ T^h < t \\ -1 & if\ T^a < T^h,\ T^a < t \\ 0 & otherwise \end{cases}$$

so that $(1, 0, -1)$ represents a home goal, no goal and away goal respectively. Let $T = \min\{t, T_h, T_a\}$ be the observed time of the event.

Because of the independence condition, the likelihood for this event is then the product of the individual likelihoods, noting if either or each of the events was censored. With the survival function form $S(x) = P(T > x)$, we have

$$f(Y|\lambda_h, \lambda_a, T) = f_h(T|\lambda_h)^{\mathbb{I}(Y=1)} S_h(T|\lambda_h)^{\mathbb{I}(Y\neq 1)} \times f_a(T|\lambda_a)^{\mathbb{I}(Y=-1)} S_a(T|\lambda_a)^{\mathbb{I}(Y\neq -1)}.$$

Using this approach, each predictor's offensive parameter coefficient represents the change in the team goal scoring rate with respect to a baseline rate (in particular, if they are replaced by another player of typical ability), and likewise for their defensive parameter and the opposing goal rate.

This method has several advantages for this class of data. Rather than trying to model a single outcome, such as a differential of goals, we can simultaneously calculate both the offensive and the defensive player ability parameters for each player, which are known to be distinct. The parameters we calculate have an immediately meaningful interpretation in terms of game outcomes, since it reflects an increase or decrease in scoring rate. We can assess a player's marginal goal fraction over data in question by comparing the expected number of goals scored and allowed by their team given their ratings against the same data with ratings set to zero.

In addition to the offensive and defensive abilities of each player, we can take into account several other possible influences. We can fit parameters to a whole team to capture their average ability, rather than simply including all the players independently. If we include both teams and players as predictors, this would change the interpretation of a "player effect" to be relative to the performance of one's team. We can also model an effect for the in-game score differential, since many teams may change their offensive and defensive strategies depending on how far ahead or behind they are in the game. This may best be accomplished by selecting a different intercept term depending on the score.

3.2 Regularization of Parameter Estimates

Even though we observe hundreds of thousands of discrete shift intervals in a season, the sheer number of parameters in this model can be excessively large, and many of the player ability measures will be made with only a small number of observations, such as players who appear in only one game; worse yet are those players who are not on the ice for any goal by one team and therefore have a maximum likelihood estimate of minus infinity for each of their parameters. To account for this, we use a hierarchical model to shrink parameter estimates toward a common mean (namely, zero), with the possibility that different positions (center, goaltender, winger and defenseman) have different shrinkage behavior. We have a number of choices for how to carry this out: the choice of prior distribution or penalty term, the degree of hierarchical structure we impose, and whether we choose to minimize a function or integrate over a distribution.

The two standard choices for a prior/penalty distribution are the Gaussian and the Laplace, which penalize the mean squared error and absolute error respectively. We can also consider a third class that joins the two, in the spirit of the Elastic Net method (Zou and Hastie, 2005):

Prior Type	Distribution	PDF
LASSO/L1	Laplace(λ)	$f(x\|\lambda) = \frac{\lambda}{2} \exp(-\lambda\|x\|)$
Ridge/L2	Gaussian$(0, \sigma^2)$	$f(x\|\sigma^2) = \exp(-x^2/(2\sigma^2))/\sqrt{2\pi\sigma^2}$
Elastic Net/L1+L2	Laplace-Gaussian(λ, σ^2)	$f(x\|\lambda, \sigma^2) = \frac{\exp(-\sigma^2\lambda^2/2 - \lambda\|x\| - x^2/(2*\sigma^2))}{\sqrt{8\pi\sigma^2}\Phi(-\sigma\lambda)} 2$

Each of these families gives a different interpretation for the shrinkage behavior of the covariates, both in terms of minimization of a penalty function and in the nature of the partial pooling distribution of each group of players. All of these regularization options act to stabilize parameter estimates against perturbations in the data, both in cases with few observations and in those pairs or multiples with high collinearity.

If we choose the L1 method and set each λ to a constant, then we have a (relatively standard) Lasso implementation, in which the penalized MLE or MAP estimates for the parameter may be exactly zero with non-zero probability. By selecting a smaller subset of non-zero parameters, we would in effect be choosing a sample of players, teams or circumstances whose scoring rates are distinguishable from the average, without having to perform a re-estimation of the effective subset. The number of non-zero terms would depend on the choices of each λ, which need not be identical for every parameter.

Choosing the L2 method and a constant set of σ^2 terms yields a ridge regression-like result, in which the penalized MLE or MAP estimates for each parameter are brought closer but not exactly to zero. We do not have benefit of automatic variable selection in this case, only that of minimizing those estimates with high variance that could bias other parameter estimates.

Compromising with the L1+L2 method allows for some of the benefits of both properties, but may sacrifice the ease of implementation that can be found in the simpler cases. In the case of simple optimization, the L1 and L2 cases are suited to using cross-validation to choose the penalty weights λ and σ^2. If we are considering multiple partially pooled groups, cross-validation may no longer be computationally feasible, since searching the space of possible parameters becomes more difficult the more dimensions we add. A method that can explore the space in a principled fashion may then be preferred.

Since the data are non-Gaussian, and do not have a convenient prior form, neither of the Gaussian or Laplace distributions is in any way conjugate to the parameters in the likelihood, making direct sampling of the full conditional distributions trickier. Each of the scale parameters has a semi-conjugate prior, meaning that we can sample each of these terms from their full conditionals without resorting to Metropolis-type proposals; this advantage disappears with our Elastic Net prior, but this calculation is light enough that a direct draw can be obtained through direct estimation of the full conditional distribution.

3.3 Implementation

We have several computational methods at our disposal to evaluate the suitability of these models, both for their fit to the data and for the questions we wish to answer:

- We use maximization of a penalized likelihood to get rough parameter estimates, with modest levels of L1 and/or L2 shrinkage to handle parameters with minimal information in the data (such as players who are only in one game and were involved in no goals.)

- We can use this as a starting point for Markov Chain Monte Carlo to obtain estimates for the pooled variance/shrinkage parameters. For each MCMC routine, we discard a sufficient number of initial draws as burn-in and thin the chain sufficiently so that the thinned chain has negligible autocorrelation for all parameters and a sufficient number of uncorrelated draws (in each of our cases, a minimum of 500) for use in inference.

- Alternatively, we can simply scan through a series of values for each shrinkage parameter and obtain the penalized maximum likelihood estimator for each, selecting the optimal value through out-of-sample validation. This is easiest when there is only one such parameter to estimate.

In each of these cases, we can judge the performance of each selected model initially using in-sample measures, then confirming goodness of fit by checking against our withheld data subset.

We have two types of problems that we consider: those in which the total distribution of predictors, and their group-level variance terms, is of direct interest, and those in which we are only interested in selecting a subset of predictors. The former case requires simultaneous estimation of a number of shrinkage parameters, and this dimensionality makes a search of the space difficult to accomplish with cross-validated methods, so we use the full Hierarchical Bayesian approach. In the latter case, there is typically only one dimension of interest, as we wish to select from only one relevant subset of predictors, and so here we can use penalized maximum likelihood estimation much more easily.

3.3.1 MCMC Estimation

The full hierarchical model has three levels, from the data, to the predictor coefficients, to their partial pooling prior distributions:

Level 1 Each outcome $(Y|X^h, X^a, \omega, \delta, t)_i$ is distributed as the competing process model. Each predictor block (X_i^h, X_i^a) is stored as a sparse vector, given that there are typically no more than 16 total non-zero terms in each row.

Level 2 Each coefficient pair $(\omega, \delta)_p$ is distributed according to its prior distribution. In the Laplace-Gaussian case, this has four terms corresponding to the group $g(p)$ that has predictor p as a member: the Laplace terms $(\lambda_{\omega,g}, \lambda_{\delta,g})$ and the Gaussian terms $(\sigma^2_{\omega,g}, \sigma^2_{\delta,g})$.

As the intercept terms r^h and r^a effectively correspond to their own (ω, δ) pair and belong to their own group, each acts as their own group mean; weak hyperpriors on their own prior terms act marginally as weak prior distributions.

Level 3 Each Laplace λ term has a weak Gamma conjugate prior; each Gaussian σ^2 term has a weak Inverse Gamma conjugate prior. If the Laplace-Gaussian is used, these priors are no longer conjugate to their respective parameter forms.

We initialize the method by finding the penalized maximum likelihood estimate for all (ω, δ) terms with loose shrinkage parameters. We then use a standard Gibbs sampling routine blocked on each relevant pair of variables:

- Each pair (ω_p, δ_p) is updated using a Metropolis sampler with a bivariate Gaussian proposal distribution. Indexing each observed shift with i, the target distribution is

$$f(\omega_p, \delta_p|Y, X, \sigma_{\omega,g(p)}, \sigma_{\delta,g(p)}, \lambda_{\omega,g(p)}, \lambda_{\delta,g(p)}) \propto f(\omega_p, \delta_p|\sigma_{\omega,g(p)}, \sigma_{\delta,g(p)}, \lambda_{\omega,g(p)}, \lambda_{\delta,g(p)}) \times \prod_{i:p\in(X_i^h, X_i^a)} f(Y|X^h, X^a, \omega, \delta, t)_i.$$

- Each pair $(\lambda_{\omega,g}, \sigma^2_{\omega,g})$ is updated through a pair of univariate grid approximation samplers. The first samples according to the density along the sum of approximate total shrinkage, $1/\sigma_{\omega,g} + \lambda_{\omega,g}/\sqrt{2}$, while keeping the relative fraction of shrinkage $\frac{\lambda_{\omega,g}/\sqrt{2}}{\lambda_{\omega,g}/\sqrt{2}+1/\sigma_{\omega,g}}$ constant;[3] after updating these values, the second samples the relative fraction while keeping the approximate total constant. This is repeated for each pair $(\lambda_{\delta,g}, \sigma^2_{\delta,g})$. (One can always sample directly from the bivariate grid approximation as well, though this is less computationally efficient.)

[3] The $\sqrt{2}$ factor is added to reflect the fact that a Laplace distribution with scale 1 has a variance of 2.

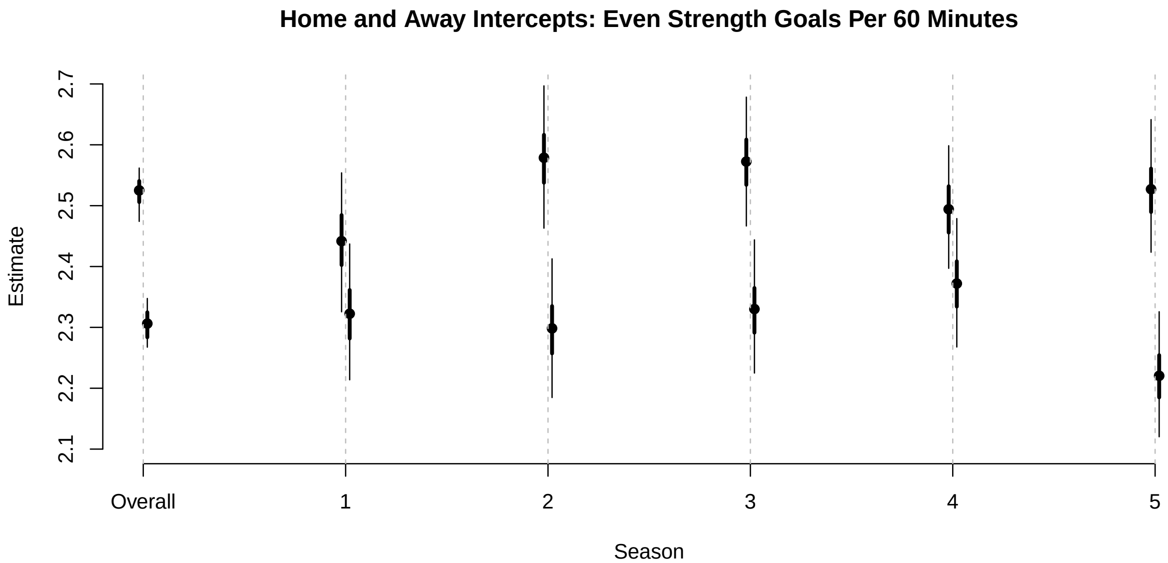


Figure 1: The scoring rates per 60 minutes for generic home and away teams across all seasons and in each individual season. Points are posterior means, thin and thick lines are central 95% and 50% credible intervals. The home team consistently outscores the away team in all five seasons and overall.

4 Applications Where MCMC Is Optimal: Group Variability and Complete Abilities

4.1 Measuring Home-Ice Advantage

The simplest version of this process model has only two coefficients, the intercepts for the home team and away team processes:

$$\lambda^h = \exp(r^h); \qquad \lambda^a = \exp(r^a).$$

Figure 1 shows estimates for the scoring rates obtained by results of MCMC, by taking $\exp(r^h)$ and $\exp(r^a)$, the per-second rates, and multiplying up to a full 60-minute game.

It is clear that the home team has a consistent advantage. Whether or not the effective home scoring rate is actually identical in each of the five seasons, they are so close as to be

	Insample			Outsample		
Season	L1	L2	L1+L2	L1	L2	L1+L2
2007-2008	57701.00	57703.41	**57691.29**	14088.33	14087.59	**14084.88**
2008-2009	59996.23	59966.41	**59961.80**	15070.79	15064.35	**15063.55**
2009-2010	61414.61	61358.52	**61347.06**	15709.86	15704.05	**15702.43**
2010-2011	62551.85	62521.21	**62515.34**	15540.85	15537.76	**15536.86**
2011-2012	62397.71	62391.72	**62377.40**	15983.34	15982.60	**15981.68**

Table 2: DIC for the Laplace, Gaussian and Laplace-Gaussian pooling priors for the model with teams as explanatory variables.

indistinguishable from each other; this is similar for the away scoring rate. The year-to-year variability in home and away mean rates is consistent with a common goal-scoring rate across all five seasons; simulations verify that the change in estimated means is consistent with the spread in estimation based on the generation of a season's worth (1230 games) of goals for each team from the Poisson model.

4.2 Overall Team Performance, Per Season

Because each of the 30 teams in the data is present in roughly one fifteenth of the total events, we do not expect the degree of sparsity as when we model the impact of individual players. This does not mean, however, that the model cannot benefit from partial pooling on team parameters, both to reduce the effective dimensionality of the model and to improve predictive accuracy. This model is then specified as

$$\lambda^h = \exp(r^h + \omega_{home} + \delta_{away}); \qquad \lambda^a = \exp(r^a + \omega_{away} + \delta_{home})$$

with partial pooling under one of our chosen schemes; in general, this is of the form

$$\omega_{team} \sim \mathrm{Laplace-Gaussian}(\lambda_{team}, \sigma^2_{team})$$

where the shrinkage behavior depends on the prior specification for $(\lambda_{team}, \sigma^2_{team})$.

We estimate these parameters within each season using MCMC for each of the three

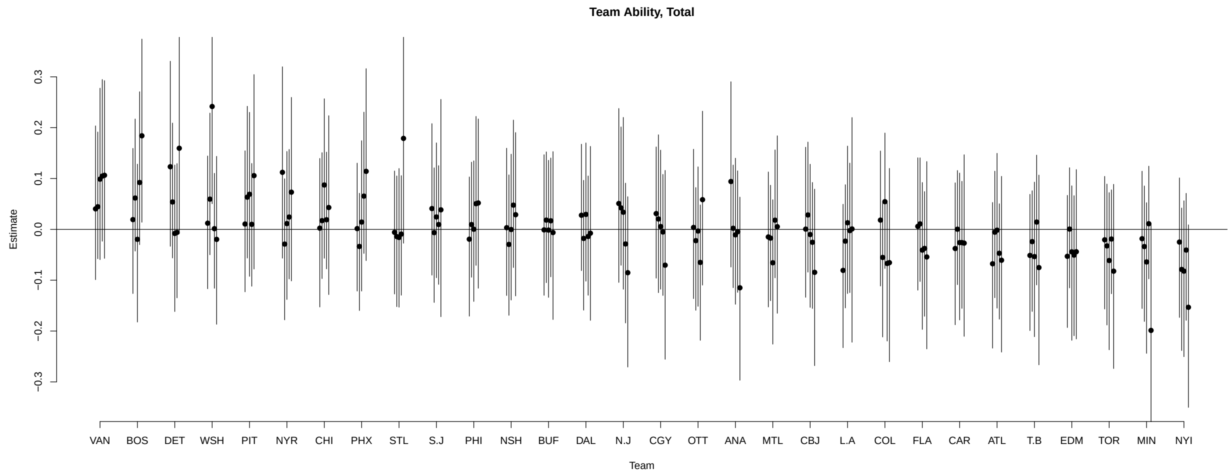


Figure 2: Total team ability estimates for each team in the NHL, grouped by team for each season; order is by overall team rating. Points are posterior means, lines are central 95% credible intervals. A rating of 0.1 corresponds to a differential of roughly 0.3 goals per game scored or prevented.

submodels for pooling, with the home and away intercepts pooled from all seasons as specified from the previous section. For each shrinkage mode, two variance components are estimated, for total offensive and defensive ability respectively. For the Laplace-Gaussian prior form, there are then four total parameters, rather than two, and this mode has the lowest Deviance Information Criterion for all five seasons, both in- and out-sample, as shown in Table 2. From this point on, we share results of our methods using only the full Laplace-Gaussian prior.

Figure 2 shows the posterior distributions for each team's net ability, or their offensive ability minus their defensive liability, $\omega_i - \delta_i$, within each season, using the Laplace-Gaussian prior. As expected, these track well with the number of goals scored and allowed by each team during these seasons, since the correlation of parameters across teams is minimal; teams play each other no more than eight times per season out of a total of 82 games. There are also several significant deviations for some teams for one season compared to the rest, such as St. Louis in 2012 (very positive) and Minnesota in 2012 (very negative), that are not statistically distinguishable from their other performances but still illuminating nonetheless.

4.3 Distribution of Player Abilities, Across All Seasons

The estimation procedure for team effects is relatively straightforward, given the relative balance of the design matrix. Once we consider individual players, many more questions arise since the design matrix can be far more unbalanced; for example, a player's defensive rating may be trickier to estimate because they share the majority of their shifts with a single goaltender. Arguably, it gets worse if both players are *great* players, since they may both be retained by a single team for much of their careers.

There is some relaxation of this when dealing with data from multiple seasons, as the more players change teams, the more the players in the league will mix. We therefore model player abilities with all five seasons together, which we refer to as the "grand model". The model is specified with the following terms:

- Overall home and away effects.
- Offensive and defensive parameters for all skaters (centers, wingers and defensemen).
- Defensive parameters only for goaltenders.
- Laplace-Gaussian pooling for each type of ability and each position parameter (center, left wing, right wing, defenseman, goaltender).

We do not include team effects at this stage specifically due to the fact that we are trying to compare players across teams, and their collinearity with goaltenders would be needlessly complicating. We are still resigned to the degree of confounding in defensive estimates, since the goaltender not only plays a large role, but is not typically replaced throughout the game, often only relieved during a poor outing. We use the standard MCMC set-up to estimate parameters.

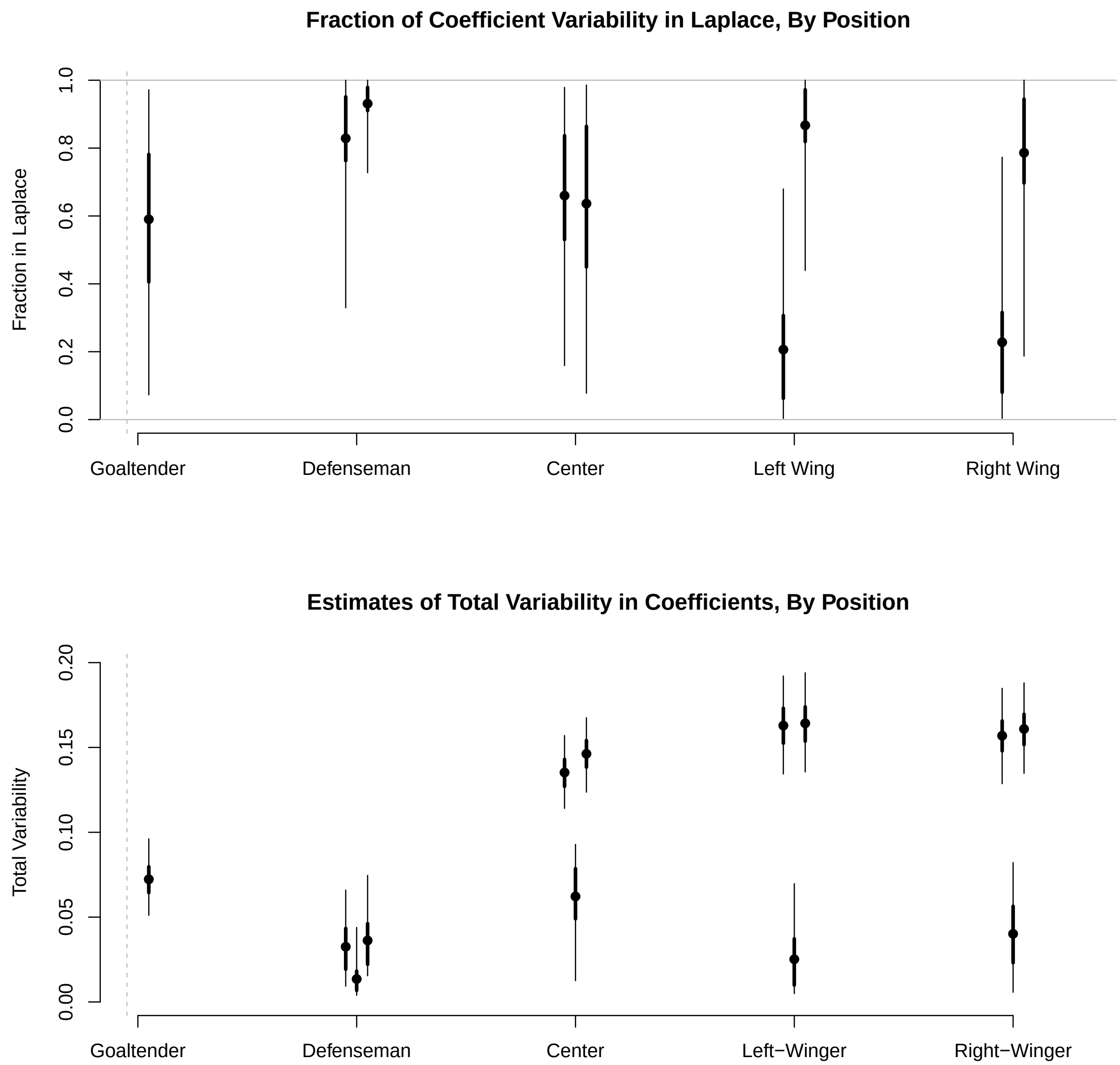


Figure 3: Variability properties of coefficient estimates by position. Thick and thin lines represent 50% and 95% credible intervals; points represent estimated means. **Top**, the approximate fraction of the variability that can be attributed to the Laplace component of the Laplace-Gaussian error distribution, for each position and for offense and defense. For most positions there is a strong tendency towards the Laplacian distribution, with heavier tails and more outliers; this is less pronounced for winger offence. **Bottom**, the offensive, defensive and total variability by position. Goaltenders have only defensive variability, which is considerably more variable than defense for any skating position. Offensive players (centers and wingers) have more variability in offense, and every skating position has minimal variability in defence.

4.3.1 Overall Variability of Rating By Position

Figure 3 shows the variability of player abilities at each position according to their respective Laplace-Gaussian distributions. The first graph shows us an approximate proportion of the fraction of variability best explained by the Laplace term, as an indicator of the degree to which a distribution of players has heavier tails; the higher this is, the higher the number of "extreme" players. The second graph shows the total variability of player abilities as the standard deviation of player estimates at each iteration of the MCMC. Several matters are apparent:

- There is considerable variability in offensive ability for forwards (centers and wingers) but far less for defensemen. This is consistent with the notion that defensemen have less impact on offensive output during even-strength situations.

- For all positions other than goaltender, defensive variability is far smaller than it is for offense. Two explanations are immediate. First, it may be that the collinearity between skaters and goaltenders is causing our estimates of goaltender ability to be more variable than they are in reality, and less variable for the skaters. Second, since the total defensive burden is shared by six players (five skaters plus one goaltender) rather than the five for offense, and the bulk of defensive skill is taken up by the goaltender, the total amount of "defensive skill" available to be shared by skaters is considerably smaller, and therefore there is less total variability between players.

- How valuable is an individual position to a team? A typical starting goaltender plays about 60 full games a season for their team, while first-line offensive and defensive players will have the equivalent of roughly 30 and 35 full games respectively. On average, a good goaltender is worth roughly what a good offensive player is to a team's total output with respect to "average" players, while a good defensive player appears to be worth considerably less.

- The center position has, on the whole, more effect on defensive performance than a defenseman does, and wingers seem to have roughly equal defensive variability as the defense position has total variability. This would seem to confirm the case that when forwards have control of the puck, particularly in their offensive zone, they deny the likelihood of their opponents being able to score. As we show soon, this does not mean that a player with a high ω rating must therefore have a high δ rating.

From these overall results, we move on to describe the individual performances of players over the five-season period, as organized by position. Table 3 lists the top three players in each position group under the grand model; we provide a more complete list of players at each position in the supporting material, including several of the worst players at each position.

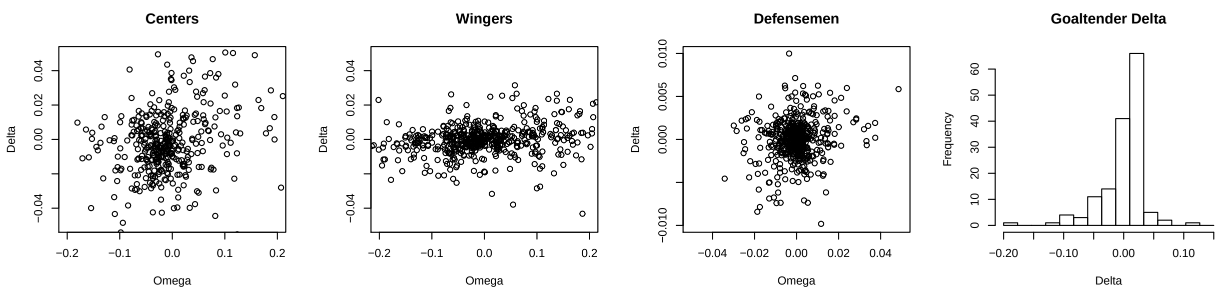


Figure 4: Scatterplots of player ability estimates by position. There is little if any correlation between a player's estimates of offensive and defensive ability.

Which Players Made The Greatest Total Difference?

Since the ratings represent multipliers to the default scoring rate, we can quickly estimate the total contribution of a player over the observation period as the difference in expected goals, scored and allowed by any average team, relative to an average player,

$$G_{net} = [(\exp(r_{base} + \omega_p) - \exp(r_{base})) - (\exp(r_{base} - \delta_p) - \exp(r_{base}))] \times T_{total,p}.$$

Player	Total Rating	SE(Rating)	Player	Total Rating	SE(Rating)
	Center			**Winger**	
Pavel Datsyuk	0.463	0.105	Alexander Semin	0.321	0.0744
Sidney Crosby	0.388	0.116	Alex Ovechkin	0.318	0.0805
Henrik Sedin	0.355	0.133	Marian Gaborik	0.308	0.0875
	Goaltender			**Defense**	
Henrik Lundqvist	0.186	0.0546	Zdeno Chara	0.077	0.0739
Tim Thomas	0.12	0.0581	Mark Streit	0.0427	0.0626
Jonathan Quick	0.102	0.0594	Jaroslav Spacek	0.0373	0.0528

Table 3: Top players at each position, by overall rating, over five NHL seasons (2007-2012).

A mean intercept parameter $r_{base} = -7.3$ corresponds to roughly 2.4 goals per 60 minutes. Table 4 lists the top 20 total goal producers and preventers over the five season period. Four goaltenders make the top 20 list; despite the fact that defensemen typically log more ice time than forwards, no defencemen make the top 20. We can adjust these ratings to reflect teammates and opponents by using the expected goals in each shift given all other player ratings, to handle nonlinearity in the rate relationship.

5 Applications of Variable Selection

5.1 "Most Valuable Player" Awards, Per Team, Per Season

The term Most Valuable Player has many interpretations throughout the sports world. One that appeals to us is the notion that a player is most valuable to their team if their team's performance suffers the most compared to a "replacement" player in their stead. In the context of this model, we propose that each player should be judged with respect to the rest of their team. Since selecting an exceptional player can be treated as a special case of variable selection, we propose the following scheme to pick exceptional players on each team:

- Take a model with teams and individual players as predictors. (Omit goaltenders for this ranking.)

Rank	Player	Position	Total Time	+Scored	+Prevented	Net Goals
1	Henrik Lundqvist	G	928100	0.00	127.80	127.80
2	Pavel Datsyuk	C	320200	103.70	15.93	119.60
3	Henrik Sedin	C	350300	100.60	0.10	100.70
4	Alex Ovechkin	L	373500	94.81	-0.18	94.63
5	Sidney Crosby	C	240400	98.37	-13.32	85.06
6	Alexander Semin	L	271100	69.88	-0.40	69.48
7	Evgeni Malkin	C	309400	81.26	-12.63	68.63
8	Marian Gaborik	R	276700	67.61	-0.22	67.39
9	Loui Eriksson	L	330900	66.44	-0.45	65.99
10	Jarome Iginla	R	393100	73.61	-8.72	64.89
11	Tim Thomas	G	732300	0.00	63.31	63.31
12	Joe Thornton	C	360000	55.96	6.92	62.88
13	Ilya Kovalchuk	L	376500	72.66	-13.66	59.01
14	Martin Brodeur	G	814100	0.00	58.64	58.64
15	Roberto Luongo	G	799600	0.00	57.10	57.10
16	Jonathan Toews	C	306800	57.22	-0.17	57.05
17	Martin St. Louis	R	384400	68.73	-12.16	56.56
18	Jason Spezza	C	318400	69.67	-13.21	56.46
19	Patrick Sharp	R	300200	54.65	-1.00	53.65
20	Henrik Zetterberg	L	339700	52.07	-0.80	51.27
	...					
81	Zdeno Chara	D	436700	21.640	1.83200	23.470
	...					

Table 4: The top 20 even-strength players in the NHL over 5 seasons (2007-2012) according to the net number of goals scored or prevented, assuming a baseline scoring rate of roughly 2.4 goals per team per 60 minutes. At position 81, Zdeno Chara is the highest-ranked defenseman in this time period.

- Fix the estimates for team ability and the grand means to be those obtained in Section 4.2. This is to ensure that all subsequent player ratings obtained will roughly sum to zero, since all ratings are relative to their team rating for each of offense and defense.

- Using a single shrinkage penalty for player ratings, choose an appropriate penalty size. Here we choose a Lasso penalty of $\lambda = 8$ as it produces the highest likelihood for the out-of-sample data in three of five seasons; in the other two, the optimal penalty was such that no player had a non-zero relative rating. In each case, the fit to out-of-sample data was virtually identical for penalties greater than 5.

- For each team, select players with the highest and lowest offensive, defensive and overall ratings. Place them in the appropriate MVP and LVP tables.

- If necessary, steadily decrease the penalty, filling in empty cells in the MVP and LVP table as new players emerge. Stop when all cells in the table are filled. (This occurs for between 2 and 5 teams in 30 at most.)

One demonstration of the method is shown in Figure 5, for the 2011-2012 season, and Table 5 lists the top 10 MVPs and bottom 10 LVPs for that year; a full list of named MVPs and LVPs, for offense, defense and overall, is given in the supplementary material. Most of the results are consistent with expectations, though we can spot some interesting trends. First, quite often, the most valuable player for offense will be the least valuable player for defense, such as Joffrey Lupul with the 2011-2012 Maple Leafs, or vice versa. In many ways this is not surprising; since the best players have the most ice time, they would be more likely to have ratings that are not shrunk completely to zero on that basis alone, and because these ratings tend to not be correlated (see Figure 4 for ratings in the five-season grand model) it is not unexpected that this rating will sometimes be negative.

Second, some of the more surprising Least Valuable Players are centers who specialize in taking faceoffs, often at critical times, such as David Steckel of the Washington Capitals in 2009-2010 and again with the Toronto Maple Leafs in 2011-2012. These players are often

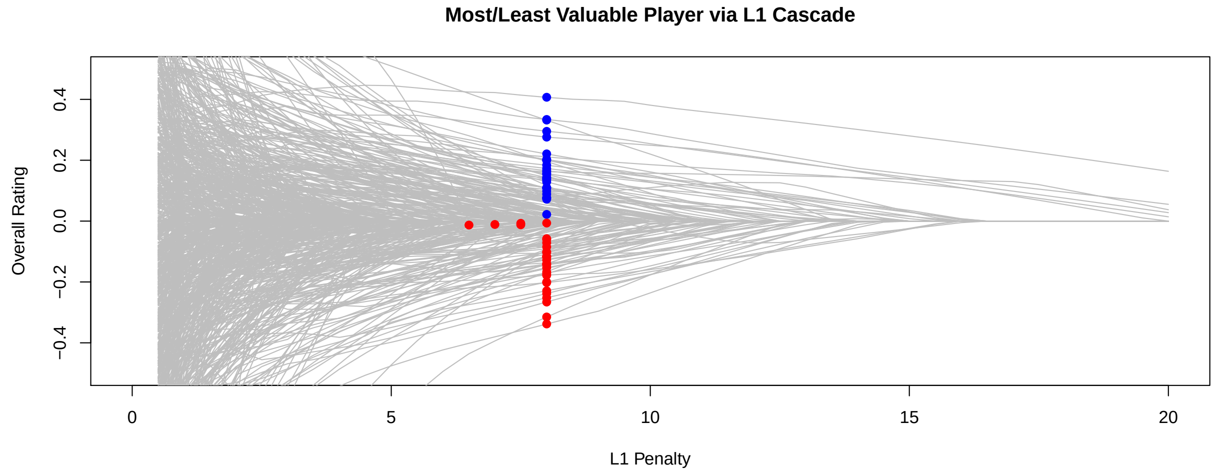


Figure 5: The Lasso Cascade method for picking team Most/Least Valuable Players for the 2011-2012 season. Team-level effects are fixed, and player effects are subjected to a steadily decreasing penalty beginning with $\lambda = 8$ as chosen by out-of-sample validation. Points indicate where MVPs (in blue) and LVPs (in red) are first declared for overall ability.

brought into the game specifically to take faceoffs, often in their team's defensive zone, before switching off for another player at their next opportunity. Because they are given fewer opportunities to score goals, merely to help prevent them, their offensive ratings will suffer accordingly; their defensive ratings can be insignificant by comparison. Taking puck location into account has been the subject of previous research (Thomas, 2006) and its role in this model will be the subject of a future investigation.

5.2 Identifying Exceptional Player Pair Interactions

By taking advantage of variable selection methods as a part of the modeling process, we allow for the possibility of including a substantially large number of alternate predictors to any of our models. One compelling inclusion is interaction effects; in this context, this would allow us to see whether two players have additional "chemistry" that yields a higher or lower total in their offensive or defensive abilities. If this is the case, we must see whether there are any corresponding changes to the individual player abilities as well.

Team	MVP	Rel. Rating	Team	LVP	Rel. Rating
EDM	Jordan Eberle	0.407	N.J	Ryan Carter	-0.338
T.B	Steven Stamkos	0.334	NYI	Nino Niederreiter	-0.315
PIT	Sidney Crosby	0.332	DET	Tomas Holmstrom	-0.266
NYI	John Tavares	0.295	BOS	Shawn Thornton	-0.252
FLA	Stephen Weiss	0.276	CHI	Michael Frolik	-0.238
PHX	Adrian Aucoin	0.221	MTL	Alexei Emelin	-0.229
OTT	Marcus Foligno	0.203	T.B	Dominic Moore	-0.202
WSH	Alexander Semin	0.200	WSH	Michael Knuble	-0.200
STL	David Perron	0.200	BUF	Robyn Regehr	-0.178
DAL	Jamie Benn	0.184	CGY	Tim Jackman	-0.173

Table 5: The top 10 MVPs and bottom 10 LVPs for the 2011-2012 season, calculated as the rating of a player relative to their team's average and selected by the Lasso method.

Since the MCMC procedure gets considerably slower with the addition of a large number of predictors and coefficients, and since our particular interest is in variable selection, we use the Lasso method of penalized maximum likelihood to select a number of non-zero coefficients for the new group. The procedure is similar to those used in previous analyses:

- Begin with the specification of the grand model in Section 4.3. Use the mean value of each σ_g and λ_g as Laplace-Gaussian penalty terms that we will keep fixed for the individual player effects, to allow for and moderate adjustments due to the pair terms.

- Select a subset of player pairs from the database. For this analysis, we took the top 1000 pairs of players in terms of the number of shifts they played together over the five-year period. We use the condition that both players played forward positions or both players played defense, since these groups tend to co-ordinate their play amongst themselves. Add these pairs as predictors to the model.

- Estimate the model for a series of Lasso penalty values, labelled λ_{pair}, on the player-pair terms, in order from strictest to loosest for computational ease. (Maintain the previously obtained penalty values for player effects.)

- If the goal is to increase predictive accuracy, choose the penalty term that minimizes

Rank	Player 1		Player 2		Team	Time (s)	Rating
1	Brad Boyes	R	Jay McClement	C	STL	35466	0.393
2	Matt Carle	D	Andrej Meszaros	D	PHI	41011	0.314
3	Patrice Bergeron	C	Brad Marchand	C	BOS	85678	0.31
4	Jussi Jokinen	L	Jeff Skinner	C	CAR	46196	0.287
5	Kris Letang	D	Paul Martin	D	PIT	40034	0.275
217	Zach Bogosian	D	John Oduya	D	ATL/WPG	57215	-0.235
218	David Booth	L	Michael Santorelli	C	FLA	34158	-0.241
219	Alex Frolov	L	Anze Kopitar	C	LA	45982	-0.269
220	Sidney Crosby	C	Evgeni Malkin	C	PIT	69217	-0.283
221	Ilya Kovalchuk	L	Todd White	C	ATL	70421	-0.545

Table 6: The top and bottom five player-pair interactions over 5 NHL seasons. These effects represent the additional total rate beyond the abilities of the players themselves.

out-of-sample error. If the goal is to select a fixed number of significant partnerships, choose the penalty term that yields that count.

In this case, we find that the penalty $\lambda_{pair} = 8.5$ minimizes the out-of-sample likelihood for these events. Of the 2000 possible parameters to select from (1000 each for ω and δ), this routine selects 247 non-zero parameters for player pairs for 221 unique player pairs.

Table 6 shows the top and bottom five player pair ratings from the analysis; a more complete list is available in the supporting material. Of particular note is the most extreme case, the pairing of Ilya Kovalchuk of Todd White, whose mutual rating is so low that they effectively wiped out their positive total individual ratings during their time together. Both recorded very high-scoring seasons when they played together, but this accolade effectively masks their mutual liability on defense. The next-lowest pair of Sidney Crosby and Evgeni Malkin is similar; their presence together does not increase their (considerable) offensive prowess beyond their individual levels, but does lead to a substantial increase in the rate of goals scored against their team while they are both on the ice.

Interestingly, the pair of Henrik and Daniel Sedin, twin brothers who play most of their even-strength shifts together, does not appear in the selected group. Indeed, the most total ice time in the top/bottom five is the 135th-most coincident pair of Patrice Bergeron and

Brad Marchand from Boston. This suggests that the levels of shrinkage are appropriate for obtaining a reasonable subset of player pairs that have reasonable deviations.

As a final check, the positions of players in the grand rating table are mostly unchanged, so that the original player ratings are reasonably robust to these new variables. Worth noting is that the top two positions in the grand ratings reverse; Sidney Crosby now has the highest player rating over Pavel Datsyuk, due to the removal of the poorer outcomes when he plays with Evgeni Malkin, as opposed to other potential linemates.

6 Discussion and Extensions

We have presented a model-based method for assessing player ability in ice hockey by treating the game as a competing stochastic process. Given the sheer number of predictors, and the relatively weak explanatory power of each, we use shrinkage methods to improve our estimation of model parameters. We also allow for the possibility of expanding the model specification from a simple flat hazard model to a more general Cox proportional hazards semi-Markov process, to account for other phenomena. Here we address potential ways to better extend the model as a useful interpretation of the game.

One obvious issue is that the methods for estimating parameters in this model are considerably slower than simple regression, whether we use Monte Carlo methods or functional maximization, especially when more parameters or data points are added. If this method is to ever see conventional and public use, the computation must either be considerably faster, or a new method of estimation must be used. Because this is a highly non-standard likelihood function, it is a complicated matter to improve parameter estimates in a general way. Sequential updating may prove to be the easiest method to improve both methods, particularly with regard to particle filtering for the Hierarchical Bayesian methods.

As a practical matter, there are several factors that can be explored immediately. Many have to do with the use of the time-dependent component of the Cox model, which we have kept as constant and unit-valued to this point.

Knowing Location Affects The Short-Term Scoring Rate

A game of hockey begins with a face-off at center ice, immediately after which neither team is very likely to score in the next few seconds. A distribution for the goal hazard after faceoff was proposed by Thomas (2007), which begins at 0 for both teams and rises to a plateau with an exponential decay. If a team has the puck in their offensive zone, they are more likely to score a goal in the immediate future than the mean rate, and their opponents far less likely.

One approach is to include known puck possession and location terms as covariates in a general model; Macdonald (2012b) in particular uses the zone in which the play starts as a mean-altering covariate. In our case, the natural point to include this is in the time-varying component to the Cox model, by choosing a relative hazard that starts at a rate given the state of play and returns to the overall mean.

One benign side effect of this is that "garbage goals" – those scored after a longer scrum in an offensive zone, taking advantage of continued pressure rather than pure skill – would be down-weighted, since we would expect a goal to be much more likely in that scenario.

Including More Events As Outcomes

Since a goal is preceded by a shot on goal in the vast majority of cases, one method to improve the modelling framework is to consider shots to be a non-censored terminating state of a model instead of a goal. Since this would lead to a roughly ten-fold count in the number of uncensored events, it would represent a great increase in the precision of estimates, especially if there was no individual variability on what fraction of shots on goal became goals. But this is certainly not the case, since there is significant variety on the fraction of shots that become goals (let alone shots on net) depending on the player; a defenseman's slap shot is considerably less likely than a forward's wrist shot. How we can include this feature in this

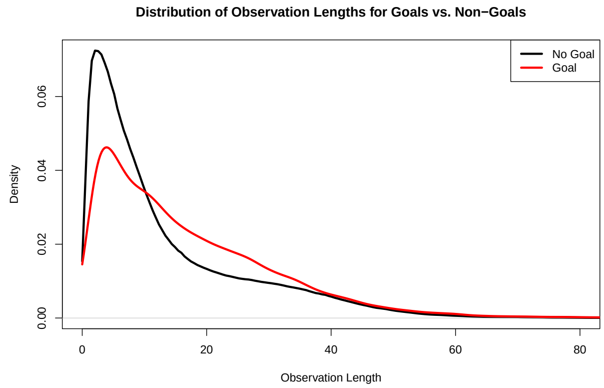


Figure 6: The lengths of shifts, conditioned on whether or not a goal was scored to terminate the observation. Shifts that end in goals are slightly longer.

model framework is an open problem, but may include information on the success rate of shots based on location and type as a post-processing step.

Censoring May Be Slightly Informative

Shift lengths are either obtained directly or censored by player changes. One assumptions we make is that the censoring mechanism is roughly exogenous, and does not depend on of influence the state of the game in progress. While this assumption is clearly incorrect, the distributions of shift time are quite similar, as shown in Figure 6. Two immediate reasons for this are clear. First, a goal is often scored following a longer scrum in the offensive zone, during which players have no opportunity to change off. Second, the changing process can be sequential; three players change, then shortly after, the other two change off, leading to a bias in short shifts. We expect that this factor can be accounted for, either through modeling or stratification, once we take puck possession and location into account.

Does The Power Play Look Like The Process Model?

When a team has a man-advantage over their opponents, the game tends to look very differently than a smooth stochastic process: the team on the power play sets up shop in their offensive zone, plays keep-away from their opponents and maneuvers to make a shot on goal. The short-handed team's prime goal in this period is not to score, but to remove the danger by clearing the puck from their own zone. (Scoring a short-handed goal is often seen as a bonus rather than the main objective while killing a penalty.)

To extend this model to the power-play situation, we would need to account for this in a principled manner. It may be sufficient to simply change the baseline scoring rates, or to replace the penalized player with an indicator for the power play state, but this is subject to a future investigation and not at all obvious given the apparent differences in game play.

Acknowledgements

We thank Brian Macdonald for introducing us to the shift-based method of dividing the game into components and for fruitful discussions.

References

Beaudoin, D. and Swartz, T. B. (2010). Strategies for Pulling the Goalie in Hockey. *The American Statistician*, **64**.

Brown, L. D. (2008). In-season Prediction of Batting Averages – A Field Test of Empirical Bayes and Bayes Methodologies. *The Annals of Applied Statistics*, **2** 113152.

Cox, D. (1972). Regression Models and Life-Tables (with discussion). *Journal of the Royal Statistical Society, Series B*, **34** 187–220.

Hirotsu, N. and Wright, M. (2002). Using a Markov Process Model of an Association Football Match to Determine the Optimal Timing of Substitution and Tactical Decisions. *Journal of the Operational Research Society*, **53**.

Hoerl, A. E. and Kennard, R. W. (1970). Ridge regression: Biased estimation for nonorthogonal problems. *Technometrics*, **12** 5567.

Ilardi, S. and Barzilai, A. (2008). Adjusted Plus-Minus Ratings: New and Improved for 2007-2008.
URL `http://www.82games.com/ilardi2.htm`.

James, W. and Stein, C. (1961). Estimation with quadratic loss. *Proc. 4th Berkeley Symp. Probab. Statist*, **1** 367–379.

Lock, D. and Schuckers, M. (2009). Beyond +/-: A Rating System to Compare NHL Players.

Macdonald, B. (2011). A Regression-based Adjusted Plus-Minus Statistic for NHL Players. *Journal of Quantitative Analysis in Sports*, **7**.

Macdonald, B. (2012a). Adjusted Plus-Minus for NHL Players using Ridge Regression.
URL `http://arxiv.org/abs/1201.0317v1`.

Macdonald, B. (2012b). An Expected Goals Model for Evaluating NHL Teams and Players. In *MIT Sloan Sports Analytics Conference 2012*.

Morrison, D. (1976). On the Optimal Time to Pull the Goalie: A Poisson Model Applied to a Common Strategy Used in Ice Hockey. In *TIMS Studies in Management Science*, vol. 4.

Rosenbaum, D. T. (2004). Measuring How NBA Players Help Their Teams Win.
URL `http://www.82games.com/comm30.htm`.

Schuckers, M. E., Lock, D. F., Wells, C., Knickerbocker, C. J. and Lock, R. H. (2011). National Hockey League Skater Ratings Based upon All On-Ice Events: An Adjusted Minus/Plus Probability (AMPP) Approach.

Sill, J. (2010). Improved NBA Adjusted +/- Using Regularization and Out-of-Sample Testing. *MIT Sloan Sports Analytics Conference.*

THOMAS, A. (2006). The Impact of Puck Possession and Location on Ice Hockey Strategy. *Journal for Quantitative Analysis in Sports*, **2**.

THOMAS, A. (2007). Inter-Arrival Times of Goals in Ice Hockey. *Journal of Quantitative Analysis in Sports*, **3**.

TIBSHIRANI, R. (1996). Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society, Series B (Methodology)*, **58** 267288.

TIBSHIRANI, R. (1997). The Lasso Method for Variable Selection in the Cox Model. *Statistics in Medicine*, **16** 385–395.

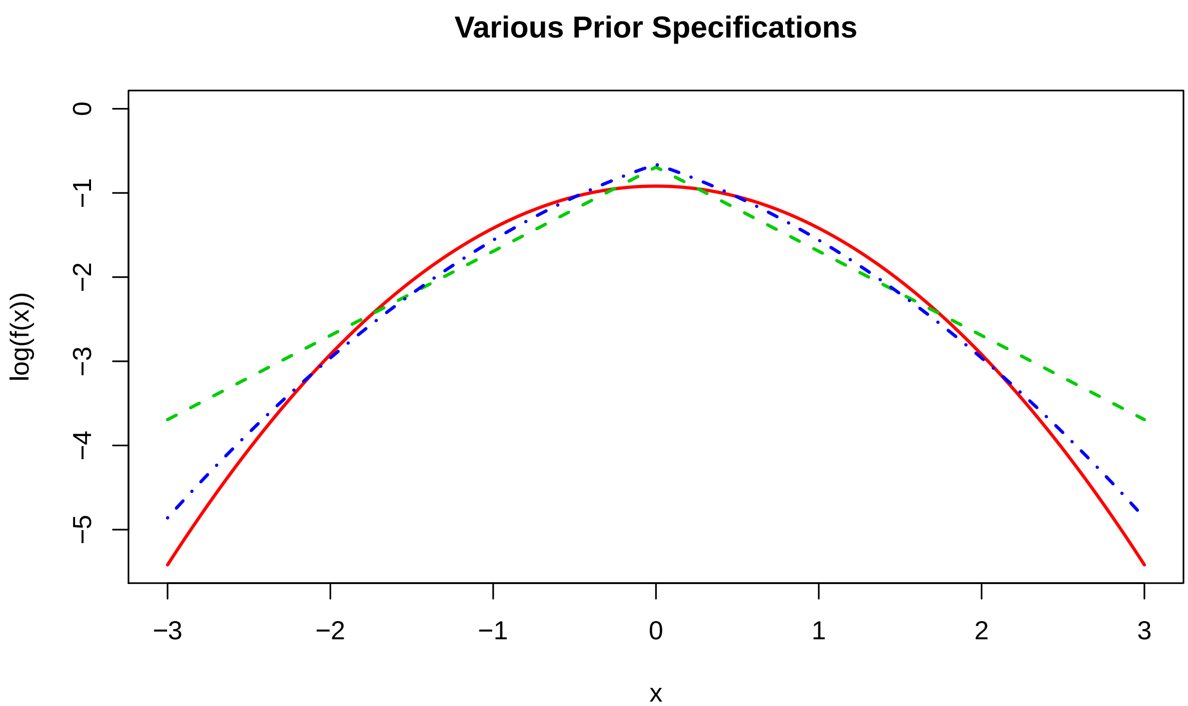
ZOU, H. and HASTIE, T. (2005). Regularization and variable selection via the elastic net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **67** 301–320.

Supplementary Material: Improving NHL Player Ability Ratings with Hazard Function Models for Goal Scoring and Prevention

A.C. Thomas, Samuel L. Ventura, Shane Jensen, Stephen Ma

A Prior Distribution Characteristics

The three prior families we consider (L1, L2, L1+L2) have slightly different properties, as shown in this figure:



Each line represents the log-probability density for three families for the prior/penalty distribution on player parameters. Red/solid is the Gaussian, which is smoothly varying with light tails; green/dash is the Laplace, which is sharp at zero, with exponential tails; and blue/dot-dash is the Laplace-Gaussian, which is also sharp with exponential tails, and has one additional parameter to compromise between the other two. All three distributions have unit variance.

B Player Abilities Across All Five Seasons

Goaltenders:

Rank	Player	$\omega_i - \delta_i$	$\Delta(\omega_i - \delta_i)$	Time (s)	ω_i	δ_i
1	HENRIK.LUNDQVIST	0.186	0.0546	928000	0	-0.186
2	TIM.THOMAS	0.12	0.0581	732000	0	-0.12
3	JONATHAN.QUICK	0.102	0.0594	662000	0	-0.102
4	MARTIN.BRODEUR	0.101	0.0571	814000	0	-0.101
5	ROBERTO.LUONGO	0.1	0.0567	8e+05	0	-0.1
6	PEKKA.RINNE	0.0917	0.0556	678000	0	-0.0917
7	CORY.SCHNEIDER	0.0753	0.0761	169000	0	-0.0753
8	DOMINIK.HASEK	0.0715	0.0815	102000	0	-0.0715
9	ANTTI.NIEMI	0.0602	0.0587	469000	0	-0.0602
10	ILJA.BRYZGALOV	0.0592	0.0484	859000	0	-0.0592
11	SEMYON.VARLAMOV	0.056	0.0635	308000	0	-0.056
12	MIKE.SMITH	0.0509	0.0517	539000	0	-0.0509
13	KARI.LEHTONEN	0.0502	0.0473	617000	0	-0.0502
14	EVGENI.NABOKOV	0.0469	0.0496	674000	0	-0.0469
15	ERIK.ERSBERG	0.0455	0.069	125000	0	-0.0455
145	J-SEBASTIEN.AUBIN	-0.052	0.078	36900	0	0.052
146	JUSTIN.POGGE	-0.0552	0.0801	17400	0	0.0552
147	PATRICK.LALIME	-0.0637	0.0626	188000	0	0.0637
148	HANNU.TOIVONEN	-0.0665	0.0791	52500	0	0.0665
149	ANDREW.RAYCROFT	-0.106	0.071	231000	0	0.106

Wingers:

Rank	Player	$\omega_i - \delta_i$	$\Delta(\omega_i - \delta_i)$	Time (s)	ω_i	δ_i
1	ALEXANDER.SEMIN	0.321	0.0744	271000	0.323	0.00221
2	ALEX.OVECHKIN	0.318	0.0805	374000	0.319	0.000724
3	MARIAN.GABORIK	0.308	0.0875	277000	0.309	0.00118
4	LOUI.ERIKSSON	0.258	0.0814	331000	0.26	0.00202
5	ALEXANDER.RADULOV	0.249	0.127	71600	0.265	0.0161
6	PATRICK.SHARP	0.234	0.0833	3e+05	0.239	0.00494
7	ALEX.TANGUAY	0.232	0.0795	278000	0.235	0.00253
8	RADIM.VRBATA	0.23	0.109	249000	0.187	-0.0432
9	JAKUB.VORACEK	0.227	0.0905	230000	0.239	0.012
10	BOBBY.RYAN	0.221	0.0928	284000	0.232	0.0114
11	THOMAS.VANEK	0.22	0.0915	290000	0.241	0.0203
12	JAROME.IGINLA	0.211	0.0898	393000	0.245	0.0334
13	ZACH.PARISE	0.206	0.0848	295000	0.202	-0.0043
14	HENRIK.ZETTERBERG	0.201	0.0831	340000	0.205	0.00349
15	SCOTT.HARTNELL	0.199	0.0851	322000	0.205	0.00604
510	COLTON.ORR	-0.253	0.139	114000	-0.27	-0.0178
511	RYAN.HOLLWEG	-0.267	0.165	44400	-0.268	-0.00148
512	STEPHANE.VEILLEUX	-0.267	0.114	174000	-0.266	0.00117
513	NINO.NIEDERREITER	-0.281	0.171	38800	-0.26	0.0211
514	RAITIS.IVANANS	-0.292	0.151	80100	-0.297	-0.00487

Centers:

Rank	Player	$\omega_i - \delta_i$	$\Delta(\omega_i - \delta_i)$	Time (s)	ω_i	δ_i
1	PAVEL.DATSYUK	0.463	0.105	320000	0.392	-0.0711
2	SIDNEY.CROSBY	0.388	0.116	240000	0.474	0.0855
3	HENRIK.SEDIN	0.355	0.133	350000	0.354	-0.000419
4	PATRICE.BERGERON	0.28	0.112	239000	0.256	-0.0246
5	EVGENI.MALKIN	0.266	0.0907	309000	0.328	0.0623
6	JONATHAN.TOEWS	0.243	0.0978	307000	0.244	0.000816
7	JOE.THORNTON	0.235	0.0974	360000	0.207	-0.028
8	JASON.SPEZZA	0.217	0.108	318000	0.281	0.0633
9	NATHAN.HORTON	0.207	0.103	286000	0.259	0.0521
10	MATS.SUNDIN	0.195	0.132	91200	0.195	-2.97e-05
11	JORDAN.EBERLE	0.185	0.134	124000	0.21	0.0252
12	STEPHEN.WEISS	0.181	0.0966	328000	0.194	0.013
13	JEFF.CARTER	0.179	0.0857	307000	0.186	0.00652
14	ALEXANDER.STEEN	0.179	0.0994	267000	0.124	-0.0551
15	MARC.SAVARD	0.175	0.114	181000	0.178	0.00354
416	NICK.SPALING	-0.156	0.131	126000	-0.152	0.0039
417	COLTON.GILLIES	-0.16	0.154	67000	-0.171	-0.011
418	TOM.PYATT	-0.17	0.136	107000	-0.164	0.00574
419	RADEK.BONK	-0.19	0.141	106000	-0.181	0.00978
420	ROD.PELLEY	-0.305	0.177	121000	-0.334	-0.0288

Defensemen:

Rank	Player	$\omega_i - \delta_i$	$\Delta(\omega_i - \delta_i)$	Time (s)	ω_i	δ_i
1	ZDENO.CHARA	0.077	0.0739	437000	0.0708	-0.00619
2	MARK.STREIT	0.0427	0.0626	303000	0.0485	0.00585
3	JAROSLAV.SPACEK	0.0373	0.0528	297000	0.0374	0.000195
4	MIKE.GREEN	0.0355	0.0527	315000	0.0374	0.00192
5	MATT.CARLE	0.0341	0.0454	379000	0.0334	-0.000666
6	DAN.HAMHUIS	0.0335	0.0462	393000	0.0333	-0.000192
7	IAN.WHITE	0.0325	0.0471	395000	0.0341	0.00157
8	TOM.GILBERT	0.0309	0.0452	387000	0.0321	0.00119
9	FILIP.KUBA	0.0261	0.0461	331000	0.0284	0.00231
10	LUBOMIR.VISNOVSKY	0.026	0.0433	362000	0.0284	0.00245
11	KRIS.LETANG	0.025	0.0443	326000	0.027	0.002
12	BRENT.BURNS	0.022	0.0427	353000	0.0212	-0.000733
13	NICKLAS.LIDSTROM	0.0215	0.0388	391000	0.0117	-0.00982
14	ALEX.GOLIGOSKI	0.0206	0.0429	253000	0.0238	0.00326
15	KENT.HUSKINS	0.0202	0.0452	225000	0.014	-0.00616
504	NICLAS.WALLIN	-0.0266	0.048	225000	-0.0242	0.00235
505	GARNET.EXELBY	-0.0283	0.0499	151000	-0.0261	0.00218
506	LUCA.SBISA	-0.0295	0.0518	167000	-0.0285	0.000981
507	ANTON.VOLCHENKOV	-0.0296	0.0559	303000	-0.0342	-0.00456
508	FRANCOIS.BEAUCHEMIN	-0.0315	0.045	373000	-0.0299	0.00161

C Team MVP/LVP By Season

2007-2008:

Team	MVP Offense	MVP Defense	MVP Total
ANA	RYAN.GETZLAF	KENT.HUSKINS	RYAN.GETZLAF
ATL	ILYA.KOVALCHUK	BRYAN.LITTLE	ILYA.KOVALCHUK
BOS	ZDENO.CHARA	AARON.WARD	ZDENO.CHARA
BUF	JAROSLAV.SPACEK	ALES.KOTALIK	JAROSLAV.SPACEK
CAR	BRET.HEDICAN	GLEN.WESLEY	BRET.HEDICAN
CBJ	JAN.HEJDA	JAN.HEJDA	JAN.HEJDA
CGY	JAROME.IGINLA	ADRIAN.AUCOIN	JAROME.IGINLA
CHI	JONATHAN.TOEWS	DUNCAN.KEITH	JONATHAN.TOEWS
COL	PAUL.STASTNY	KURT.SAUER	PAUL.STASTNY
DAL	BRENDEN.MORROW	STEVE.OTT	BRENDEN.MORROW
DET	PAVEL.DATSYUK	NICKLAS.LIDSTROM	PAVEL.DATSYUK
EDM	JONI.PITKANEN	MATT.GREENE	MATT.GREENE
FLA	NATHAN.HORTON	JASSEN.CULLIMORE	NATHAN.HORTON
L.A	ALEX.FROLOV	KEVIN.DALLMAN	KEVIN.DALLMAN
MIN	MARIAN.GABORIK	JAMES.SHEPPARD	MARIAN.GABORIK
MTL	ANDREI.KASTSITSYN	FRANCIS.BOUILLON	ANDREI.KASTSITSYN
N.J	DAVID.ODUYA	DAVID.ODUYA	DAVID.ODUYA
NSH	JASON.ARNOTT	JERRED.SMITHSON	JASON.ARNOTT
NYI	MIKE.COMRIE	JOSEF.VASICEK	JOSEF.VASICEK
NYR	SEAN.AVERY	MAREK.MALIK	SEAN.AVERY
OTT	DANY.HEATLEY	CHRIS.PHILLIPS	DANY.HEATLEY
PHI	BRAYDON.COBURN	BRAYDON.COBURN	BRAYDON.COBURN
PHX	SHANE.DOAN	ZBYNEK.MICHALEK	SHANE.DOAN
PIT	EVGENI.MALKIN	JORDAN.STAAL	EVGENI.MALKIN
S.J	JOE.THORNTON	JONATHAN.CHEECHOO	JOE.THORNTON
STL	KEITH.TKACHUK	DAVID.PERRON	DAVID.PERRON
T.B	VINCENT.LECAVALIER	MICHEL.OUELLET	MICHEL.OUELLET
TOR	MATS.SUNDIN	BRYAN.MCCABE	MATS.SUNDIN
VAN	MARKUS.NASLUND	SAMI.SALO	MARKUS.NASLUND
WSH	ALEX.OVECHKIN	BOYD.GORDON	ALEX.OVECHKIN

Team	LVP Offense	LVP Defense	LVP Total
ANA	TRAVIS.MOEN	FRANCOIS.BEAUCHEMIN	TRAVIS.MOEN
ATL	STEVE.MCCARTHY	ILYA.KOVALCHUK	STEVE.MCCARTHY
BOS	SHANE.HNIDY	PHIL.KESSEL	SHANE.HNIDY
BUF	NOLAN.PRATT	THOMAS.VANEK	NOLAN.PRATT
CAR	NICLAS.WALLIN	ERIC.STAAL	NICLAS.WALLIN
CBJ	DAVID.VYBORNY	RICK.NASH	RICK.NASH
CGY	STEPHANE.YELLE	ANDERS.ERIKSSON	STEPHANE.YELLE
CHI	CRAIG.ADAMS	CRAIG.ADAMS	CRAIG.ADAMS
COL	KARLIS.SKRASTINS	RYAN.SMYTH	KARLIS.SKRASTINS
DAL	MATTIAS.NORSTROM	STEPHANE.ROBIDAS	STEPHANE.ROBIDAS
DET	DALLAS.DRAKE	ANDREAS.LILJA	DALLAS.DRAKE
EDM	JARRET.STOLL	SAM.GAGNER	JARRET.STOLL
FLA	BRANISLAV.MEZEI	OLLI.JOKINEN	BRANISLAV.MEZEI
L.A	MICHAL.HANDZUS	PATRICK.OSULLIVAN	MICHAL.HANDZUS
MIN	BRIAN.ROLSTON	SEAN.HILL	SEAN.HILL
MTL	JOSH.GORGES	ANDREI.MARKOV	JOSH.GORGES
N.J	SERGEI.BRYLIN	VITALY.VISHNEVSKI	SERGEI.BRYLIN
NSH	JERRED.SMITHSON	ALEXANDER.RADULOV	JERRED.SMITHSON
NYI	RADEK.MARTINEK	MIKE.COMRIE	MIKE.COMRIE
NYR	RYAN.HOLLWEG	CHRIS.DRURY	RYAN.HOLLWEG
OTT	ANTON.VOLCHENKOV	WADE.REDDEN	ANTON.VOLCHENKOV
PHI	SAMI.KAPANEN	DANIEL.BRIERE	SAMI.KAPANEN
PHX	MICHAEL.YORK	SHANE.DOAN	MICHAEL.YORK
PIT	ADAM.HALL	RYAN.WHITNEY	RYAN.WHITNEY
S.J	MARC-EDOUARD.VLASIC	SANDIS.OZOLINSH	SANDIS.OZOLINSH
STL	RYAN.JOHNSON	PAUL.KARIYA	RYAN.JOHNSON
T.B	NICK.TARNASKY	DAN.BOYLE	NICK.TARNASKY
TOR	TOMAS.KABERLE	JIRI.TLUSTY	TOMAS.KABERLE
VAN	SAMI.SALO	AARON.MILLER	BYRON.RITCHIE
WSH	SHAONE.MORRISONN	MICHAEL.NYLANDER	MICHAEL.NYLANDER

2008-2009:

Team	MVP Offense	MVP Defense	MVP Total
ANA	COREY.PERRY	BRETT.FESTERLING	COREY.PERRY
ATL	ZACH.BOGOSIAN	MARTY.REASONER	ZACH.BOGOSIAN
BOS	MARC.SAVARD	DAVID.KREJCI	MARC.SAVARD
BUF	THOMAS.VANEK	ADAM.MAIR	THOMAS.VANEK
CAR	ERIC.STAAL	PATRICK.EAVES	ERIC.STAAL
CBJ	JAKUB.VORACEK	CHRISTIAN.BACKMAN	JAKUB.VORACEK
CGY	MATTHEW.LOMBARDI	CORY.SARICH	CORY.SARICH
CHI	ANDREW.LADD	DUSTIN.BYFUGLIEN	ANDREW.LADD
COL	RUSLAN.SALEI	IAN.LAPERRIERE	IAN.LAPERRIERE
DAL	LOUI.ERIKSSON	STEPHANE.ROBIDAS	LOUI.ERIKSSON
DET	PAVEL.DATSYUK	BRETT.LEBDA	PAVEL.DATSYUK
EDM	DENIS.GREBESHKOV	LUBOMIR.VISNOVSKY	DENIS.GREBESHKOV
FLA	STEPHEN.WEISS	KARLIS.SKRASTINS	STEPHEN.WEISS
L.A	KYLE.QUINCEY	SEAN.ODONNELL	SEAN.ODONNELL
MIN	ANDREW.BRUNETTE	ANTTI.MIETTINEN	ANDREW.BRUNETTE
MTL	JOSH.GORGES	MAXIM.LAPIERRE	JOSH.GORGES
N.J	ZACH.PARISE	MIKE.MOTTAU	ZACH.PARISE
NSH	JASON.ARNOTT	ANTTI.PIHLSTROM	JASON.ARNOTT
NYI	MARK.STREIT	SEAN.BERGENHEIM	MARK.STREIT
NYR	NIKOLAI.ZHERDEV	RYAN.CALLAHAN	NIKOLAI.ZHERDEV
OTT	DANIEL.ALFREDSSON	FILIP.KUBA	DANIEL.ALFREDSSON
PHI	JEFF.CARTER	CLAUDE.GIROUX	JEFF.CARTER
PHX	STEVE.REINPRECHT	KEN.KLEE	STEVE.REINPRECHT
PIT	EVGENI.MALKIN	ROB.SCUDERI	EVGENI.MALKIN
S.J	DEVIN.SETOGUCHI	MICHAEL.GRIER	MICHAEL.GRIER
STL	PATRIK.BERGLUND	PATRIK.BERGLUND	PATRIK.BERGLUND
T.B	MARTIN.ST LOUIS	ADAM.HALL	MARTIN.ST LOUIS
TOR	ALEXEI.PONIKAROVSKY	IAN.WHITE	ALEXEI.PONIKAROVSKY
VAN	HENRIK.SEDIN	DANIEL.SEDIN	HENRIK.SEDIN
WSH	ALEX.OVECHKIN	MILAN.JURCINA	ALEX.OVECHKIN

Team	LVP Offense	LVP Defense	LVP Total
ANA	TRAVIS.MOEN	ROB.NIEDERMAYER	TRAVIS.MOEN
ATL	GARNET.EXELBY	ILYA.KOVALCHUK	GARNET.EXELBY
BOS	SHAWN.THORNTON	VLADIMIR.SOBOTKA	SHAWN.THORNTON
BUF	JOCHEN.HECHT	THOMAS.VANEK	JOCHEN.HECHT
CAR	ROD.BRINDAMOUR	JOE.CORVO	ROD.BRINDAMOUR
CBJ	ANDREW.MURRAY	KRISTIAN.HUSELIUS	KRISTIAN.HUSELIUS
CGY	ERIC.NYSTROM	DION.PHANEUF	ERIC.NYSTROM
CHI	BEN.EAGER	BRIAN.CAMPBELL	BRIAN.CAMPBELL
COL	DARCY.TUCKER	JORDAN.LEOPOLD	JORDAN.LEOPOLD
DAL	JOEL.LUNDQVIST	MATT.NISKANEN	JOEL.LUNDQVIST
DET	KRIS.DRAPER	DAN.CLEARY	KRIS.DRAPER
EDM	JASON.STRUDWICK	LIAM.REDDOX	JASON.STRUDWICK
FLA	JAY.BOUWMEESTER	NATHAN.HORTON	JAY.BOUWMEESTER
L.A	RAITIS.IVANANS	ANZE.KOPITAR	RAITIS.IVANANS
MIN	STEPHANE.VEILLEUX	MARTIN.SKOULA	STEPHANE.VEILLEUX
MTL	ALEX.KOVALEV	ANDREI.KASTSITSYN	ALEX.KOVALEV
N.J	JAY.PANDOLFO	COLIN.WHITE	JAY.PANDOLFO
NSH	ANTTI.PIHLSTROM	JASON.ARNOTT	RADEK.BONK
NYI	THOMAS.POCK	FREDDY.MEYER	THOMAS.POCK
NYR	COLTON.ORR	MARC.STAAL	COLTON.ORR
OTT	ANTON.VOLCHENKOV	JASON.SPEZZA	ANTON.VOLCHENKOV
PHI	ANDREAS.NODL	ARRON.ASHAM	ANDREAS.NODL
PHX	DAN.CARCILLO	DAVID.HALE	DAVID.HALE
PIT	MARK.EATON	KRIS.LETANG	MARK.EATON
S.J	JODY.SHELLEY	DEVIN.SETOGUCHI	DEVIN.SETOGUCHI
STL	MIKE.WEAVER	BARRET.JACKMAN	BARRET.JACKMAN
T.B	ADAM.HALL	MARK.RECCHI	MARK.RECCHI
TOR	JOHN.MITCHELL	NIK.ANTROPOV	JOHN.MITCHELL
VAN	MASON.RAYMOND	KEVIN.BIEKSA	MASON.RAYMOND
WSH	MILAN.JURCINA	TOMAS.FLEISCHMANN	DONALD.BRASHEAR

Team	MVP Offense	MVP Defense	MVP Total
ANA	RYAN.GETZLAF	GEORGE.PARROS	GEORGE.PARROS
ATL	ILYA.KOVALCHUK	CHRIS.THORBURN	ILYA.KOVALCHUK
BOS	ZDENO.CHARA	MARCO.STURM	ZDENO.CHARA
BUF	JOCHEN.HECHT	TONI.LYDMAN	TONI.LYDMAN
CAR	ERIC.STAAL	BRETT.CARSON	ERIC.STAAL
CBJ	JAKUB.VORACEK	NATHAN.PAETSCH	NATHAN.PAETSCH
CGY	RENE.BOURQUE	CURTIS.GLENCROSS	CURTIS.GLENCROSS
CHI	PATRICK.SHARP	PATRICK.SHARP	PATRICK.SHARP
COL	CHRIS.STEWART	DARCY.TUCKER	WOJTEK.WOLSKI
DAL	BRAD.RICHARDS	MARK.FISTRIC	MARK.FISTRIC
DET	HENRIK.ZETTERBERG	DREW.MILLER	DREW.MILLER
EDM	DUSTIN.PENNER	TOM.GILBERT	DUSTIN.PENNER
FLA	NATHAN.HORTON	DENNIS.SEIDENBERG	NATHAN.HORTON
L.A	WAYNE.SIMMONDS	DREW.DOUGHTY	WAYNE.SIMMONDS
MIN	KIM.JOHNSSON	ERIC.BELANGER	KIM.JOHNSSON
MTL	BRIAN.GIONTA	JOSH.GORGES	BRIAN.GIONTA
N.J	ZACH.PARISE	MARK.FRASER	ZACH.PARISE
NSH	DAN.HAMHUIS	FRANCIS.BOUILLON	FRANCIS.BOUILLON
NYI	MARK.STREIT	ANDREW.MACDONALD	MARK.STREIT
NYR	MARIAN.GABORIK	MARC.STAAL	MARIAN.GABORIK
OTT	ALEXANDRE.PICARD	MARCUS.FOLIGNO	MARCUS.FOLIGNO
PHI	MATT.CARLE	DAN.CARCILLO	MATT.CARLE
PHX	ZBYNEK.MICHALEK	TAYLOR.PYATT	TAYLOR.PYATT
PIT	SIDNEY.CROSBY	JORDAN.STAAL	SIDNEY.CROSBY
S.J	PATRICK.MARLEAU	MANNY.MALHOTRA	MANNY.MALHOTRA
STL	ALEXANDER.STEEN	BRAD.WINCHESTER	BRAD.WINCHESTER
T.B	STEVEN.STAMKOS	STEPHANE.VEILLEUX	VICTOR.HEDMAN
TOR	IAN.WHITE	COLTON.ORR	IAN.WHITE
VAN	DANIEL.SEDIN	KYLE.WELLWOOD	DANIEL.SEDIN
WSH	ALEX.OVECHKIN	JEFF.SCHULTZ	ALEX.OVECHKIN

Team	LVP Offense	LVP Defense	LVP Total
ANA	BRAD.MARCHAND	COREY.PERRY	BRAD.MARCHAND
ATL	ZACH.BOGOSIAN	MAXIM.AFINOGENOV	MAXIM.AFINOGENOV
BOS	STEVE.BEGIN	BLAKE.WHEELER	BLAKE.WHEELER
BUF	CRAIG.RIVET	CLARKE.MACARTHUR	CRAIG.RIVET
CAR	ROD.BRINDAMOUR	ROD.BRINDAMOUR	ROD.BRINDAMOUR
CBJ	JARED.BOLL	R.J..UMBERGER	JARED.BOLL
CGY	JAY.BOUWMEESTER	RENE.BOURQUE	JAY.BOUWMEESTER
CHI	DUSTIN.BYFUGLIEN	ANDREW.LADD	DUSTIN.BYFUGLIEN
COL	DARCY.TUCKER	CHRIS.STEWART	DARCY.TUCKER
DAL	MATT.NISKANEN	BRAD.RICHARDS	MATT.NISKANEN
DET	KIRK.MALTBY	JONATHAN.ERICSSON	KIRK.MALTBY
EDM	TAYLOR.CHORNEY	PATRICK.OSULLIVAN	PATRICK.OSULLIVAN
FLA	DENNIS.SEIDENBERG	NATHAN.HORTON	KEITH.BALLARD
L.A	RAITIS.IVANANS	JACK.JOHNSON	JACK.JOHNSON
MIN	CAL.CLUTTERBUCK	MARTIN.HAVLAT	CAL.CLUTTERBUCK
MTL	MAXIM.LAPIERRE	PAUL.MARA	MAXIM.LAPIERRE
N.J	NICKLAS.BERGFORS	MIKE.MOTTAU	NICKLAS.BERGFORS
NSH	FRANCIS.BOUILLON	MARTIN.ERAT	MARTIN.ERAT
NYI	NATE.THOMPSON	KYLE.OKPOSO	NATE.THOMPSON
NYR	CHRIS.HIGGINS	MICHAEL.DEL ZOTTO	CHRIS.HIGGINS
OTT	RYAN.SHANNON	ALEX.KOVALEV	ALEX.KOVALEV
PHI	RYAN.PARENT	OSKARS.BARTULIS	OSKARS.BARTULIS
PHX	LAURI.KORPIKOSKI	ED.JOVANOVSKI	LAURI.KORPIKOSKI
PIT	RUSLAN.FEDOTENKO	EVGENI.MALKIN	RUSLAN.FEDOTENKO
S.J	JED.ORTMEYER	DAN.BOYLE	JED.ORTMEYER
STL	B.J..CROMBEEN	ERIC.BREWER	ERIC.BREWER
T.B	STEPHANE.VEILLEUX	STEVEN.STAMKOS	STEPHANE.VEILLEUX
TOR	FRANCOIS.BEAUCHEMIN	MATT.STAJAN	MATT.STAJAN
VAN	KYLE.WELLWOOD	RYAN.KESLER	RYAN.KESLER
WSH	DAVID.STECKEL	SHAONE.MORRISONN	DAVID.STECKEL

2010-2011:

Team	MVP Offense	MVP Defense	MVP Total
ANA	BOBBY.RYAN	GEORGE.PARROS	BOBBY.RYAN
ATL	DUSTIN.BYFUGLIEN	DUSTIN.BYFUGLIEN	DUSTIN.BYFUGLIEN
BOS	NATHAN.HORTON	JOHNNY.BOYCHUK	NATHAN.HORTON
BUF	DREW.STAFFORD	PAUL.GAUSTAD	PAUL.GAUSTAD
CAR	JEFF.SKINNER	BRANDON.SUTTER	BRANDON.SUTTER
CBJ	RICK.NASH	NIKITA.FILATOV	RICK.NASH
CGY	ALEX.TANGUAY	CORY.SARICH	JAROME.IGINLA
CHI	JONATHAN.TOEWS	BRIAN.CAMPBELL	BRIAN.CAMPBELL
COL	MATT.DUCHENE	DANIEL.WINNIK	DANIEL.WINNIK
DAL	BRAD.RICHARDS	JEFF.WOYWITKA	BRAD.RICHARDS
DET	BRIAN.RAFALSKI	TOMAS.HOLMSTROM	BRIAN.RAFALSKI
EDM	ALES.HEMSKY	THEO.PECKHAM	ALES.HEMSKY
FLA	CORY.STILLMAN	MIKE.WEAVER	MIKE.WEAVER
L.A	DREW.DOUGHTY	ALEC.MARTINEZ	DREW.DOUGHTY
MIN	BRENT.BURNS	GREG.ZANON	BRENT.BURNS
MTL	TOMAS.PLEKANEC	ROMAN.HAMRLIK	TOMAS.PLEKANEC
N.J	PATRIK.ELIAS	MARK.FAYNE	PATRIK.ELIAS
NSH	SIARHEI.KASTSITSYN	DAVID.LEGWAND	DAVID.LEGWAND
NYI	PA.PARENTEAU	ANDREW.MACDONALD	ANDREW.MACDONALD
NYR	RYAN.MCDONAGH	MICHAEL.SAUER	MICHAEL.SAUER
OTT	JASON.SPEZZA	RYAN.SHANNON	JASON.SPEZZA
PHI	JEFF.CARTER	ANDREAS.NODL	JEFF.CARTER
PHX	LAURI.KORPIKOSKI	SAMI.LEPISTO	SAMI.LEPISTO
PIT	SIDNEY.CROSBY	CHRIS.CONNER	SIDNEY.CROSBY
S.J	DEVIN.SETOGUCHI	KYLE.WELLWOOD	KYLE.WELLWOOD
STL	DAVID.BACKES	DAVID.BACKES	DAVID.BACKES
T.B	STEVEN.STAMKOS	BRETT.CLARK	STEVEN.STAMKOS
TOR	MIKHAIL.GRABOVSKI	FREDRIK.SJOSTROM	MIKHAIL.GRABOVSKI
VAN	HENRIK.SEDIN	KEVIN.BIEKSA	HENRIK.SEDIN
WSH	ALEXANDER.SEMIN	JOHN.CARLSON	ALEXANDER.SEMIN

Team	LVP Offense	LVP Defense	LVP Total
ANA	CAM.FOWLER	SAKU.KOIVU	CAM.FOWLER
ATL	JOHN.ODUYA	ANDREW.LADD	JOHN.ODUYA
BOS	BRAD.MARCHAND	MICHAEL.RYDER	BRAD.MARCHAND
BUF	SHAONE.MORRISONN	CRAIG.RIVET	CRAIG.RIVET
CAR	CHAD.LAROSE	ERIC.STAAL	CHAD.LAROSE
CBJ	NATHAN.PAETSCH	DERICK.BRASSARD	NATHAN.PAETSCH
CGY	RENE.BOURQUE	ALEX.TANGUAY	ALEX.TANGUAY
CHI	FERNANDO.PISANI	TOMAS.KOPECKY	TOMAS.KOPECKY
COL	RYAN.OBYRNE	KEVIN.PORTER	KEVIN.PORTER
DAL	TOM.WANDELL	BRENDEN.MORROW	TOM.WANDELL
DET	RUSLAN.SALEI	TODD.BERTUZZI	RUSLAN.SALEI
EDM	JASON.STRUDWICK	SAM.GAGNER	SAM.GAGNER
FLA	MIKE.WEAVER	DAVID.BOOTH	DAVID.BOOTH
L.A	TREVOR.LEWIS	JACK.JOHNSON	JACK.JOHNSON
MIN	ERIC.NYSTROM	MARTIN.HAVLAT	ERIC.NYSTROM
MTL	MAXIM.LAPIERRE	P.K..SUBBAN	MAXIM.LAPIERRE
N.J	DAVID.CLARKSON	ILYA.KOVALCHUK	DAVID.CLARKSON
NSH	NICK.SPALING	FRANCIS.BOUILLON	NICK.SPALING
NYI	BRUNO.GERVAIS	MILAN.JURCINA	BRUNO.GERVAIS
NYR	BRANDON.PRUST	MICHAEL.DEL ZOTTO	MICHAEL.DEL ZOTTO
OTT	ZACK.SMITH	BOBBY.BUTLER	BOBBY.BUTLER
PHI	DAN.CARCILLO	DARROLL.POWE	DAN.CARCILLO
PHX	DEREK.MORRIS	KYLE.TURRIS	DEREK.MORRIS
PIT	MAXIME.TALBOT	EVGENI.MALKIN	EVGENI.MALKIN
S.J	JAMIE.MCGINN	PATRICK.MARLEAU	PATRICK.MARLEAU
STL	B.J..CROMBEEN	JAY.MCCLEMENT	B.J..CROMBEEN
T.B	ADAM.HALL	DOMINIC.MOORE	ADAM.HALL
TOR	FREDRIK.SJOSTROM	TYLER.BOZAK	TYLER.BOZAK
VAN	TANNER.GLASS	MASON.RAYMOND	TANNER.GLASS
WSH	JEFF.SCHULTZ	JASON.CHIMERA	JASON.CHIMERA

2011-2012:

Team	MVP Offense	MVP Defense	MVP Total
ANA	SAKU.KOIVU	SHELDON.BROOKBANK	SAKU.KOIVU
BOS	TYLER.SEGUIN	ADAM.MCQUAID	TYLER.SEGUIN
BUF	THOMAS.VANEK	ROBYN.REGEHR	THOMAS.VANEK
CAR	JAMIE.MCBAIN	PATRICK.DWYER	TIM.GLEASON
CBJ	VACLAV.PROSPAL	NATHAN.PAETSCH	VACLAV.PROSPAL
CGY	OLLI.JOKINEN	ROMAN.HORAK	ROMAN.HORAK
CHI	PATRICK.SHARP	NIKLAS.HJALMARSSON	PATRICK.SHARP
COL	GABRIEL.LANDESKOG	GABRIEL.LANDESKOG	GABRIEL.LANDESKOG
DAL	JAMIE.BENN	MARK.FISTRIC	JAMIE.BENN
DET	HENRIK.ZETTERBERG	TODD.BERTUZZI	IAN.WHITE
EDM	JORDAN.EBERLE	BEN.EAGER	JORDAN.EBERLE
FLA	STEPHEN.WEISS	JASON.GARRISON	STEPHEN.WEISS
L.A	JUSTIN.WILLIAMS	WILLIE.MITCHELL	JUSTIN.WILLIAMS
MIN	DANY.HEATLEY	NICK.SCHULTZ	NICK.SCHULTZ
MTL	ERIK.COLE	JOSH.GORGES	ERIK.COLE
N.J	PETR.SYKORA	JACOB.JOSEFSON	JACOB.JOSEFSON
NSH	MIKE.FISHER	BLAKE.GEOFFRION	BLAKE.GEOFFRION
NYI	JOHN.TAVARES	MATT.MARTIN	JOHN.TAVARES
NYR	MARIAN.GABORIK	RYAN.MCDONAGH	MARIAN.GABORIK
OTT	MARCUS.FOLIGNO	KYLE.TURRIS	MARCUS.FOLIGNO
PHI	SCOTT.HARTNELL	SEAN.COUTURIER	SCOTT.HARTNELL
PHX	RAY.WHITNEY	ADRIAN.AUCOIN	ADRIAN.AUCOIN
PIT	SIDNEY.CROSBY	DERYK.ENGELLAND	SIDNEY.CROSBY
S.J	PATRICK.MARLEAU	DOUGLAS.MURRAY	PATRICK.MARLEAU
STL	DAVID.PERRON	VLADIMIR.SOBOTKA	DAVID.PERRON
T.B	STEVEN.STAMKOS	BRETT.CONNOLLY	STEVEN.STAMKOS
TOR	JOFFREY.LUPUL	CARL.GUNNARSSON	JOFFREY.LUPUL
VAN	HENRIK.SEDIN	CHRISTOPHER.TANEV	HENRIK.SEDIN
WPG	BLAKE.WHEELER	JOHN.ODUYA	BLAKE.WHEELER
WSH	ALEXANDER.SEMIN	KARL.ALZNER	ALEXANDER.SEMIN
Team	LVP Offense	LVP Defense	LVP Total
ANA	JASON.BLAKE	CAM.FOWLER	JASON.BLAKE
BOS	SHAWN.THORNTON	DAVID.KREJCI	SHAWN.THORNTON
BUF	ROBYN.REGEHR	JASON.POMINVILLE	ROBYN.REGEHR
CAR	PATRICK.DWYER	ERIC.STAAL	ERIC.STAAL
CBJ	JOHN.MOORE	JEFF.CARTER	JOHN.MOORE
CGY	BLAKE.COMEAU	OLLI.JOKINEN	TIM.JACKMAN
CHI	MICHAEL.FROLIK	ANDREW.BRUNETTE	MICHAEL.FROLIK
COL	JAN.HEJDA	KYLE.QUINCEY	JAN.HEJDA
DAL	VERNON.FIDDLER	ADAM.PARDY	VERNON.FIDDLER
DET	TOMAS.HOLMSTROM	HENRIK.ZETTERBERG	TOMAS.HOLMSTROM
EDM	MAGNUS.PAAJARVI	JORDAN.EBERLE	MAGNUS.PAAJARVI
FLA	BRIAN.CAMPBELL	TOMAS.KOPECKY	BRIAN.CAMPBELL
L.A	TREVOR.LEWIS	ANZE.KOPITAR	TREVOR.LEWIS
MIN	NICK.SCHULTZ	DEVIN.SETOGUCHI	DEVIN.SETOGUCHI
MTL	MATHIEU.DARCHE	ALEXEI.EMELIN	ALEXEI.EMELIN
N.J	RYAN.CARTER	ILYA.KOVALCHUK	RYAN.CARTER
NSH	KEVIN.KLEIN	FRANCIS.BOUILLON	KEVIN.KLEIN
NYI	NINO.NIEDERREITER	MILAN.JURCINA	NINO.NIEDERREITER
NYR	MARC.STAAL	BRAD.RICHARDS	BRAD.RICHARDS
OTT	JARED.COWEN	STEPHANE.DA COSTA	JARED.COWEN
PHI	MATT.CARLE	HARRISON.ZOLNIERCZYK	MATT.CARLE
PHX	DEREK.MORRIS	DEREK.MORRIS	DEREK.MORRIS
PIT	JOE.VITALE	STEVE.SULLIVAN	JOE.VITALE
S.J	DOUGLAS.MURRAY	JAMIE.MCGINN	JAMIE.MCGINN
STL	SCOTT.NICHOL	ANDY.MCDONALD	SCOTT.NICHOL
T.B	DOMINIC.MOORE	STEVEN.STAMKOS	DOMINIC.MOORE
TOR	DAVID.STECKEL	JOFFREY.LUPUL	DAVID.STECKEL
VAN	DALE.WEISE	ALEXANDER.EDLER	DALE.WEISE
WPG	TANNER.GLASS	DUSTIN.BYFUGLIEN	TANNER.GLASS
WSH	MICHAEL.KNUBLE	TROY.BROUWER	MICHAEL.KNUBLE

D Exceptional Player Pairs, Overall

These ratings represent the total increase or decrease in team scoring rates if these two players play together, rather than separately. (We do not claim, for example, that the net impact of playing Crosby and Malkin is extremely negative, since their individual abilities are each positive; merely that their partnership leads to worse results than when these players play separately.)

Rank	Player 1		Player 2		Total Time (s)	Rating
1	BRAD.BOYES	R	JAY.MCCLEMENT	C	35466	0.393
2	MATT.CARLE	D	ANDREJ.MESZAROS	D	41011	0.314
3	PATRICE.BERGERON	C	BRAD.MARCHAND	C	85678	0.31
4	JUSSI.JOKINEN	L	JEFF.SKINNER	C	46196	0.287
5	KRIS.LETANG	D	PAUL.MARTIN	D	40034	0.275
6	MICHAL.HANDZUS	C	WAYNE.SIMMONDS	R	96815	0.265
7	TOM.GILBERT	D	RYAN.WHITNEY	D	32378	0.247
8	BARRET.JACKMAN	D	KEVIN.SHATTENKIRK	D	59537	0.24
9	JASON.BLAKE	L	DOMINIC.MOORE	C	36983	0.231
10	PASCAL.DUPUIS	L	JORDAN.STAAL	C	48978	0.225
11	KEITH.BALLARD	D	NICHOLAS.BOYNTON	D	51300	0.212
12	ALEX.FROLOV	L	PATRICK.OSULLIVAN	C	31117	0.211
13	VALTTERI.FILPPULA	C	JIRI.HUDLER	C	115005	0.198
14	MATT.CARLE	D	CHRIS.PRONGER	D	112174	0.197
15	MILAN.HEJDUK	R	WOJTEK.WOLSKI	L	46147	0.191
16	KEVIN.BIEKSA	D	DAN.HAMHUIS	D	102515	0.191
17	MARTIN.HANZAL	C	RADIM.VRBATA	R	157171	0.188
18	KARL.ALZNER	D	JOHN.CARLSON	D	120008	0.187
19	ALES.HEMSKY	R	DUSTIN.PENNER	R	90640	0.186
20	MATT.GREENE	D	SEAN.ODONNELL	D	42201	0.183
21	VERNON.FIDDLER	C	LEE.STEMPNIAK	R	42605	0.181
22	CRAIG.CONROY	C	CURTIS.GLENCROSS	L	44530	0.178
23	BRIAN.CAMPBELL	D	JASON.GARRISON	D	62456	0.177
24	NICKLAS.LIDSTROM	D	BRIAN.RAFALSKI	D	173306	0.177
25	ZACH.PARISE	L	TRAVIS.ZAJAC	C	150049	0.176
26	MATT.GREENE	D	ALEC.MARTINEZ	D	57029	0.175
27	TROY.BROUWER	R	PATRICK.SHARP	R	38506	0.174
28	JIRI.HUDLER	C	HENRIK.ZETTERBERG	L	63952	0.172
29	BRETT.CLARK	D	VICTOR.HEDMAN	D	53748	0.169
30	SHANE.DOAN	R	STEVE.REINPRECHT	C	51277	0.164
31	TONI.LYDMAN	D	LUBOMIR.VISNOVSKY	D	88426	0.161
32	JOHNNY.BOYCHUK	D	ZDENO.CHARA	D	92395	0.155
33	DREW.DOUGHTY	D	ROB.SCUDERI	D	125662	0.154
34	ALEX.BURROWS	L	HENRIK.SEDIN	C	173247	0.15
35	PATRICK.DWYER	R	BRANDON.SUTTER	C	80414	0.148
36	PATRIK.ELIAS	L	BRIAN.ROLSTON	R	57073	0.147
37	PAUL.MARTIN	D	DAVID.ODUYA	D	85861	0.147
38	BRUNO.GERVAIS	D	MARK.STREIT	D	66853	0.138
39	JOSH.GORGES	D	P.K..SUBBAN	D	67710	0.138
40	ZDENO.CHARA	D	DENNIS.WIDEMAN	D	82733	0.138
41	MATT.DUCHENE	C	MILAN.HEJDUK	R	87232	0.133
42	FILIP.KUBA	D	PAUL.RANGER	D	46711	0.131
43	VERNON.FIDDLER	C	TAYLOR.PYATT	L	54765	0.124
44	CARLO.COLAIACOVO	D	ALEX.PIETRANGELO	D	73311	0.123
45	MILAN.JURCINA	D	JEFF.SCHULTZ	D	31131	0.12
46	BRAD.MARCHAND	C	GEORGE.PARROS	R	32479	0.119
47	WILLIE.MITCHELL	D	SLAVA.VOYNOV	D	37854	0.116
48	RYAN.CALLAHAN	R	CHRIS.DRURY	C	72179	0.116
49	ROB.BLAKE	D	MARC-EDOUARD.VLASIC	D	97836	0.109
50	TRAVIS.HAMONIC	D	ANDREW.MACDONALD	D	107992	0.101
201	MICHAEL.CAMMALLERI	L	ANZE.KOPITAR	C	29798	-0.123
202	BRAD.BOYES	R	KEITH.TKACHUK	C	68156	-0.125
203	MILAN.HEJDUK	R	RYAN.SMYTH	L	74613	-0.125
204	DERICK.BRASSARD	C	RICK.NASH	L	78314	-0.125
205	DAVID.BACKES	R	ANDY.MCDONALD	C	84713	-0.128
206	MILAN.MICHALEK	L	JASON.SPEZZA	C	104458	-0.131
207	VYACHESLAV.KOZLOV	L	TODD.WHITE	C	54517	-0.132
208	FRANCOIS.BEAUCHEMIN	D	CAM.FOWLER	D	74655	-0.138
209	JAROME.IGINLA	R	OLLI.JOKINEN	C	113358	-0.145
210	TYLER.BOZAK	C	PHIL.KESSEL	R	131533	-0.148
211	MICHAEL.DEL ZOTTO	D	DAN.GIRARDI	D	54129	-0.157
212	MICHAEL.CAMMALLERI	L	JAROME.IGINLA	R	52981	-0.177
213	NIK.ANTROPOV	C	EVANDER.KANE	L	39117	-0.197
214	PAUL.STASTNY	C	CHRIS.STEWART	R	65308	-0.197
215	MARTIN.ST LOUIS	R	STEVEN.STAMKOS	C	173577	-0.206
216	KYLE.OKPOSO	R	JOHN.TAVARES	C	60880	-0.211
217	ZACH.BOGOSIAN	D	JOHN.ODUYA	D	57215	-0.235
218	DAVID.BOOTH	L	MICHAEL.SANTORELLI	C	34158	-0.241
219	ALEX.FROLOV	L	ANZE.KOPITAR	C	45982	-0.269
220	SIDNEY.CROSBY	C	EVGENI.MALKIN	C	69217	-0.283
221	ILYA.KOVALCHUK	L	TODD.WHITE	C	70421	-0.545