第四章　　平面图形的几何性质

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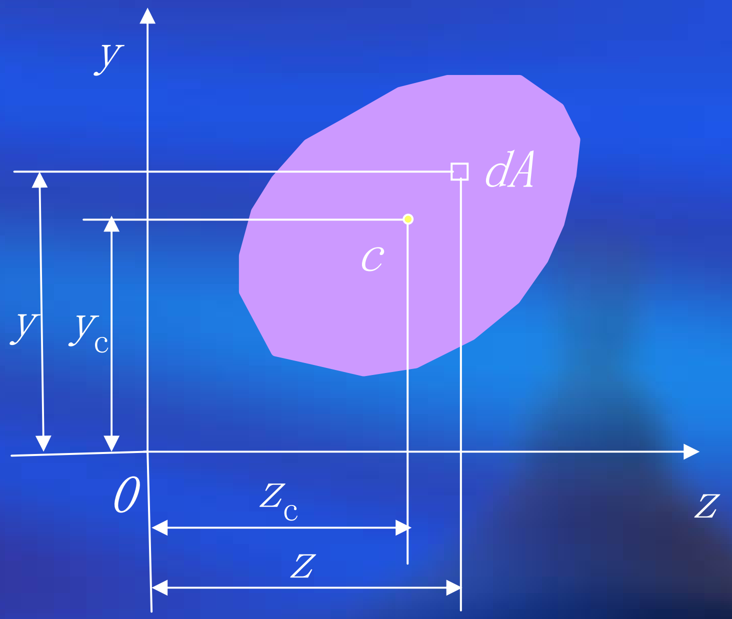
§4.1 静矩和形心

1 静矩

$$S_z = \int_A y\,dA$$

$$S_y = \int_A z\,dA$$



2 形心

$$y_c = \frac{\int_A y\,dA}{A} = \frac{S_z}{A} \qquad S_z = y_c A$$

$$z_c = \frac{\int_A z\,dA}{A} = \frac{S_y}{A} \qquad S_y = z_c A$$

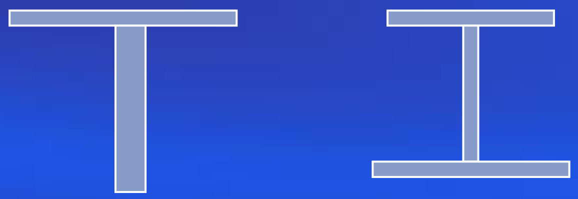
3 组合图形的静矩和形心

$$S_z = \sum y_{ci} A$$

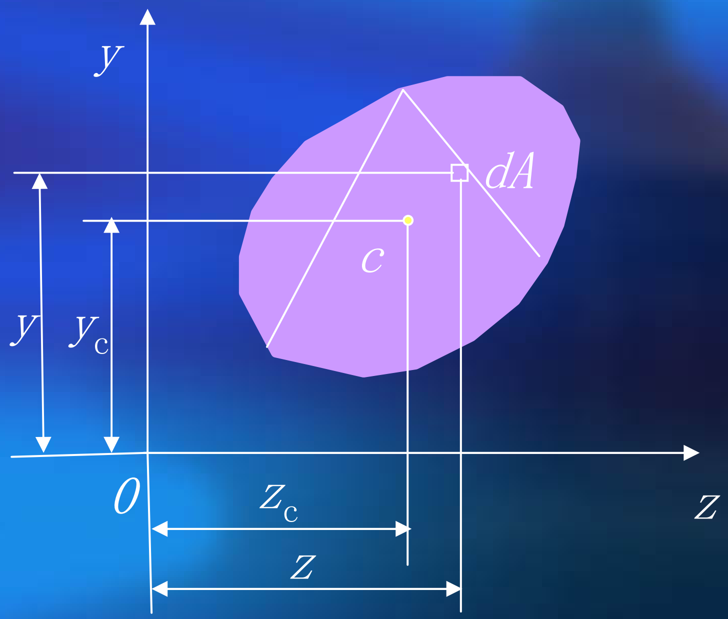
$$S_y = \sum z_{ci} A$$

$$y_c = \frac{\sum y_{ci} A_i}{\sum A_i}$$

$$z_c = \frac{\sum z_{ci} A_i}{\sum A_i}$$



组合图形



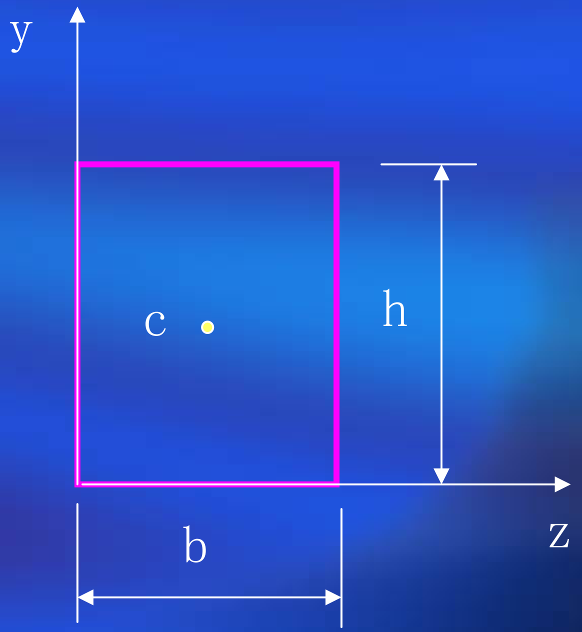
已知：矩形截面$b\times h$

求：　S_z和 S_y

解：

$$S_z = y_c A = \frac{bh^2}{2}$$

$$S_y = z_c A = \frac{hb^2}{2}$$



已知： 图示图形

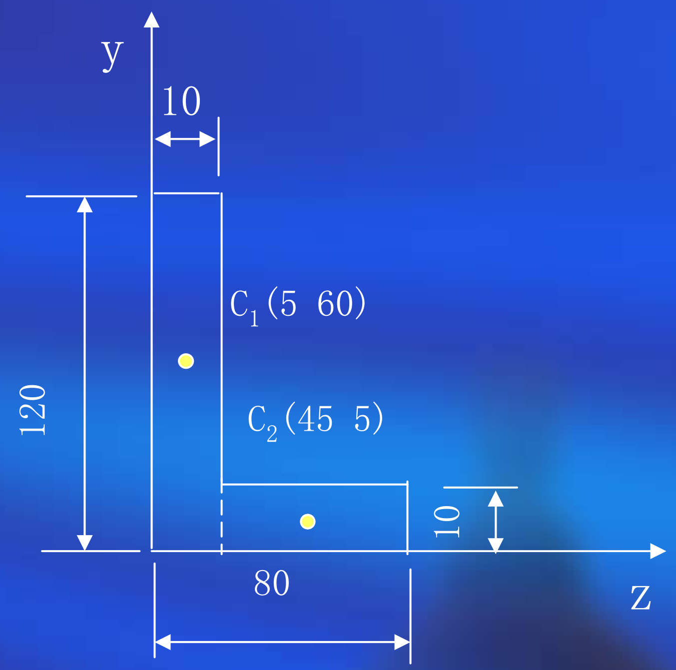
求： z_c和y_c

解：

$$z_c = \frac{A_1 z_1 + A_2 z_2}{A_1 + A_2}$$

$$= \frac{120\times10\times5 + 70\times10\times45}{120\times10 + 70\times10}$$

$$= 19.7mm$$



$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

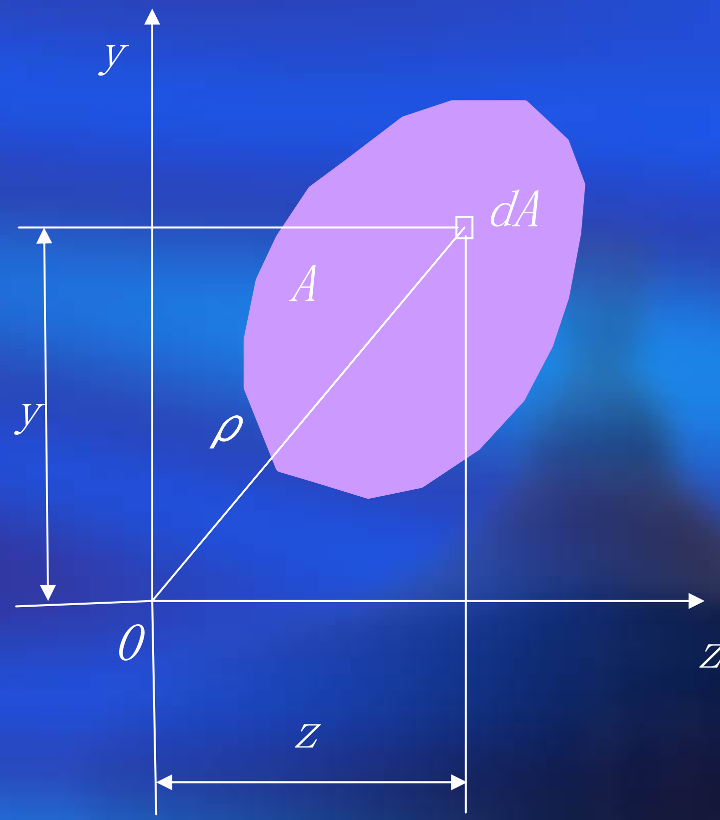
$$= \frac{120\times10\times60 + 70\times10\times5}{120\times10 + 70\times10} = 39.7mm$$

§4.2 惯性矩和惯性半径

1 惯性矩

$$I_z = \int_A y^2 \, dA$$

$$I_y = \int_A z^2 \, dA$$



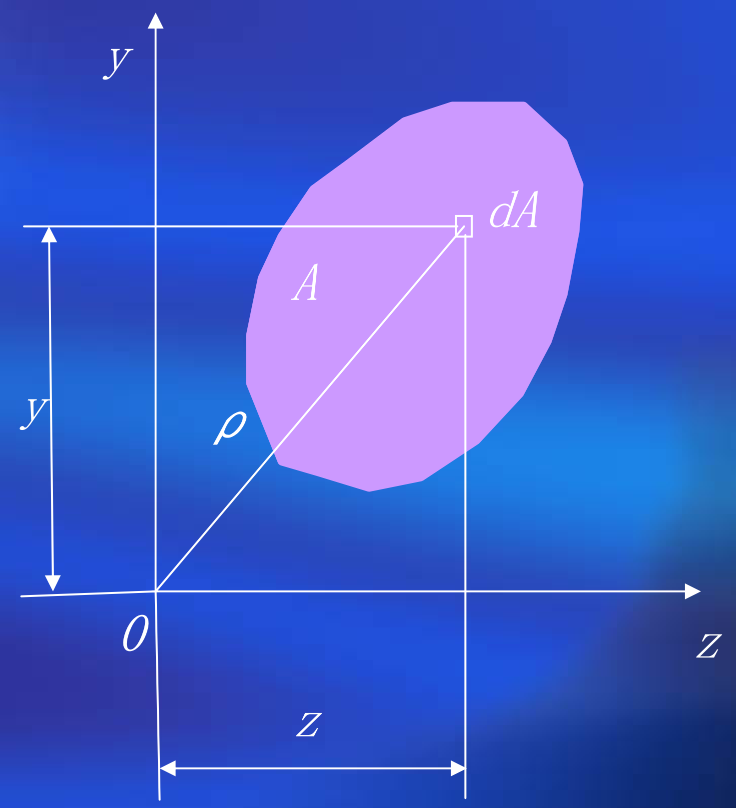
2 惯性半径

$$i_z = \sqrt{\frac{I_z}{A}}$$

$$i_y = \sqrt{\frac{I_y}{A}}$$

3 极惯性矩

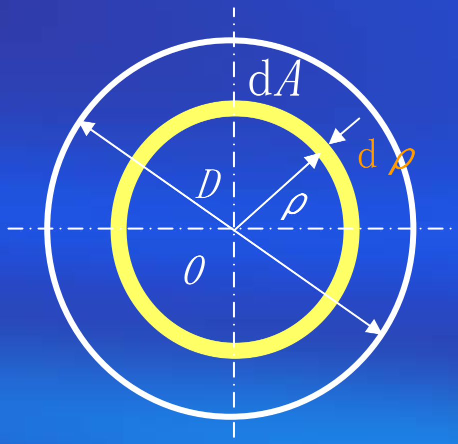
$$I_p = \int_A \rho^2 dA$$



1 圆

$$I_P = \int_A \rho^2 dA$$

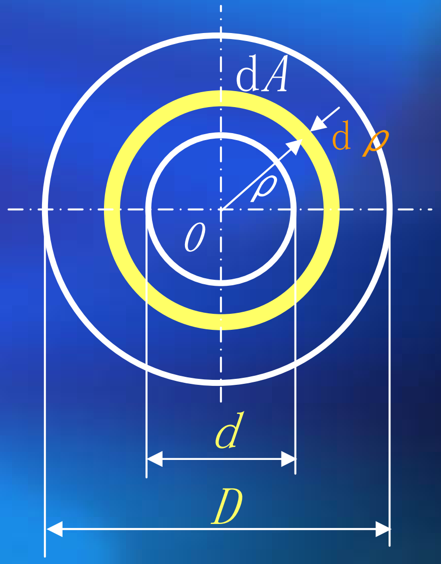
$$= \int_0^{\frac{D}{2}} \rho^2 2\pi\rho \, d\rho = \frac{\pi D^4}{32}$$



2 空心圆

$$I_P = \int_A \rho^2 dA$$

$$= \int_{\frac{d}{2}}^{\frac{D}{2}} \rho^2 2\pi\rho d\rho = \frac{\pi D^4}{32}(1-\alpha^4)$$

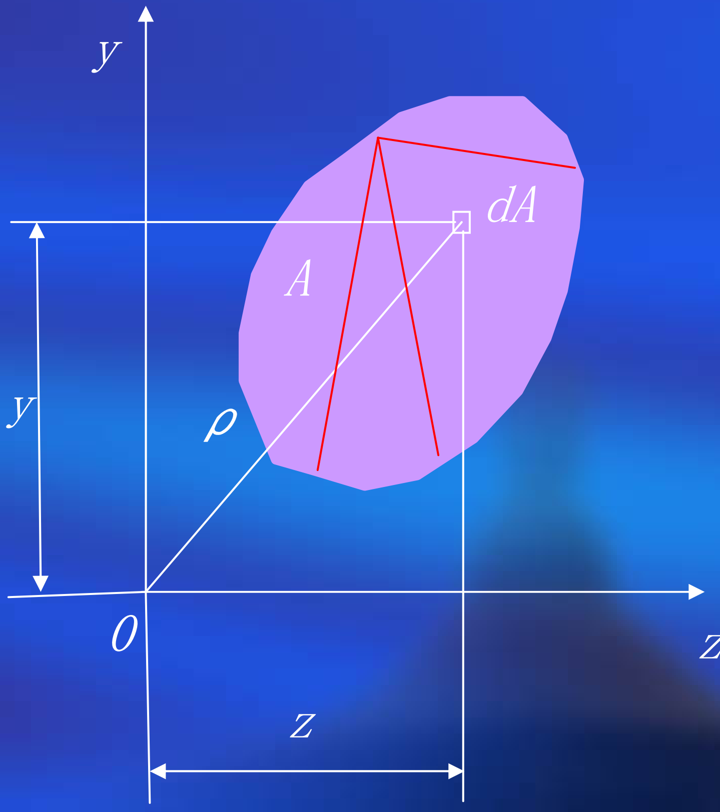


$$\alpha = \frac{d}{D}$$

4 惯性矩与极惯性积的关系

$$I_p = \int_A \rho^2 dA = \int_A (y^2 + z^2) dA$$

$$= \int_A y^2 dA + \int_A z^2 dA = I_z + I_y$$



5 组合图形的惯性矩

$$I_z = \sum I_{z_i}$$

$$I_y = \sum I_{y_i}$$

已知： 矩形 $b \times h$

求： I_y和I_z

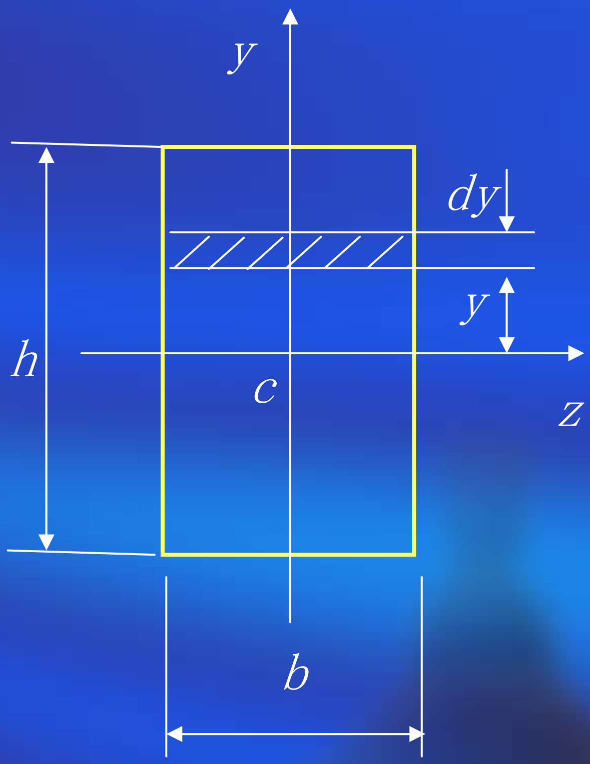
解：

$$I_z = \int_A y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 b dy$$

$$I_z = \frac{bh^3}{12}$$

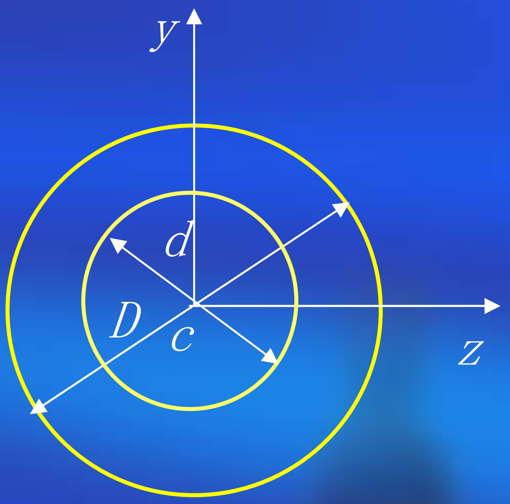
$$I_y = \int_A z^2 dA = \int_{-\frac{b}{2}}^{\frac{b}{2}} z^2 h dz$$

$$I_y = \frac{hb^3}{12}$$



已知：实心圆截面直径D，空心圆截面直径D、d.

求：I_y和I_z。



解：

1 实心圆

$$I_p=\int_A \rho^2 dA = I_y + I_z = 2I_y = 2I_z$$

$$I_y = I_z = \frac{\pi D^4}{64}$$

2 空心圆

$$I_y = I_z = \frac{\pi D^4 (1-\alpha^4)}{64}$$

§4.3 惯性积

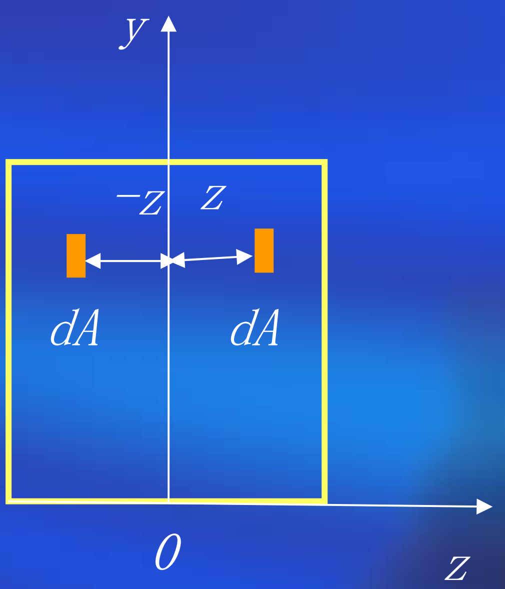
$$I_{yz} = \int_A yz dA$$

1 y、z之一为图形对称轴则 $I_{yz}=0$；

2 惯性积为零的一对座标轴称为惯性主轴；

3 通过形心的主轴称为形心主轴或形心惯性主轴；

4 形心主轴与杆件轴线所确定的平面称为形心主惯性平面。



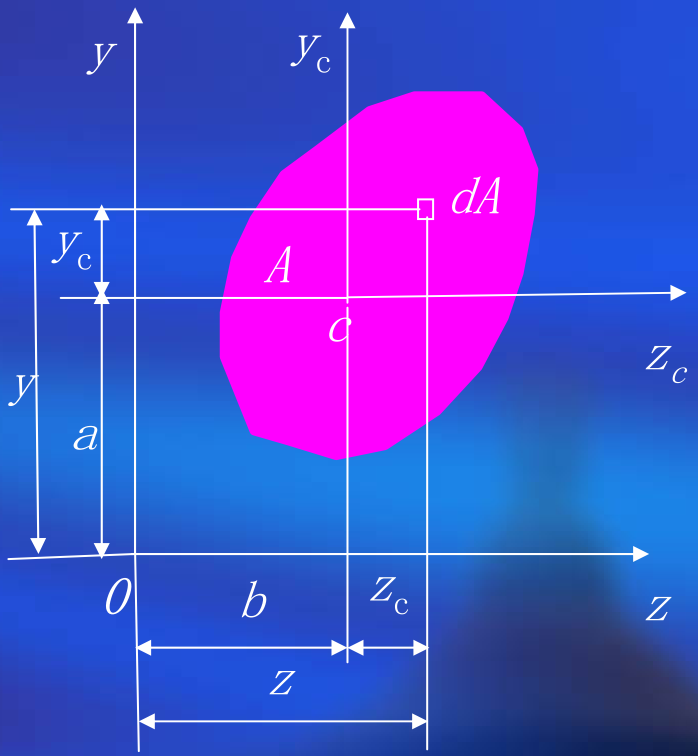
§4.4 平行移轴公式

图形对形心轴的惯性矩和惯性积为：

$$I_{z_c} = \int_A y_c^2 dA$$

$$I_{y_c} = \int_A z_c^2 dA$$

$$I_{y_c z_c} = \int_A y_c z_c dA$$



图形对平行于形心轴y、z轴的惯性矩和惯性积为：

$$I_z = \int_A y^2 dA \qquad I_{yz} = \int_A yz dA \qquad I_y = \int_A z^2 dA$$

$$z = z_c + b \qquad y = y_c + a$$

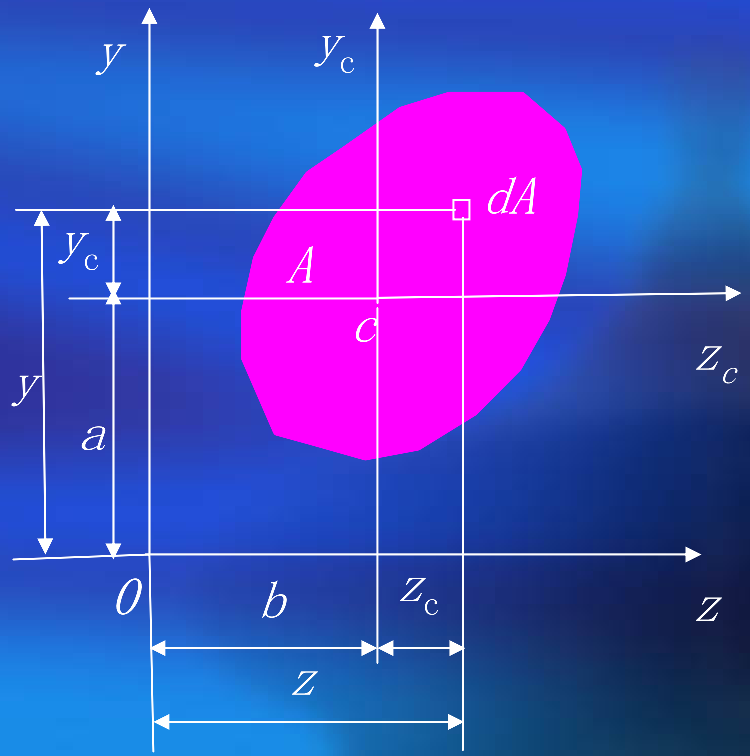
$$I_z = \int_A y^2 dA = \int_A (y_c + a)^2 dA$$

$$= \int_A y_c^2 dA + 2a\int_a y_c dA + a^2 \int_A dA$$

$$I_z = I_{z_c} + a^2 A$$

$$I_y = I_{y_c} + b^2 A$$

$$I_{yz} = I_{y_c z_c} + abA$$



已知：T形截面。

求：I_{zc}

解：形心　c (0　y_c)

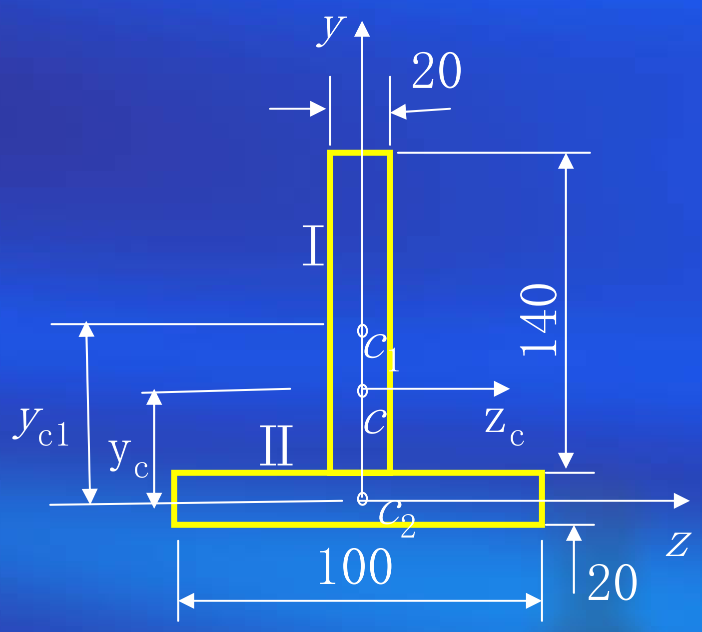
$$y_c=\frac{A_1y_1+A_2y_2}{A_1+A_2}=\frac{A_1y_1+A_2\times 0}{A_1+A_2}$$

$$=\frac{0.14\times 0.2\times 0.8+0}{0.14\times 0.02+0.1\times 0.02}=0.0467m$$

$$I_{z_c}^{\mathrm{I}}=\frac{0.02\times 0.14^3}{12}+(0.08-0.0467)^2\times 0.02\times 0.14=7.69\times 10^{-6}m^4$$

$$I_{z_c}^{\mathrm{II}}=\frac{0.1\times 0.02^3}{12}+0.0467^2\times 0.1\times 0.02=4.43\times 10^{-6}m^4$$

$$I_{z_c}=I_{z_c}^{\mathrm{I}}+I_{z_c}^{II}=12.12\times 10^{-6}m^4$$



- 作业
- 4.2
- 4.7
- 4.9