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Investigation on magnetron cavity used in rubidium atomic frequency standards

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Abstract: The magnetron cavity was developed for use in the rubidium atomic frequency standards, through which the main characteristics of the magnetron cavity were studied, mainly including the resonant frequency, quality factor, oscillation mode. The results show that the resonant frequency of the magnetron cavity can be attenuated to 6.835 GHz, which is the resonant frequency for rubidium atoms, and Q-factor can be attenuated to 600~1000, the oscillation mode is typical TE_{011} mode which is needed for the rubidium atomic frequency standard, and the cavity has a lower frequency temperature coefficient (32.5~35.0 kHz/ ℃), therefore these derivative magnetron cavities can meet the requirements for rubidium atomic frequency standards well.

Key words: optoelectronics; quantum frequency standard; magnetron cavity; resonant frequency; quality factor

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用于铷原子频标的磁控管腔研究

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摘　要： 研究了一种应用于铷原子频标的磁控管腔，对用于磁控管腔的主要特征和参数计算进行了研究，主要包括磁控管腔的谐振频率、Q 值、微波场模式。研究结果表明磁控管腔的谐振频率可以调谐至 6.835 GHz,Q 值都能够调至 600~1000 之间，其微波场谐振模式是典型的 TE_{011} 模式，频率温度系数较小 (32.5~35.0 kHz/ ℃), 所设计的磁控管腔能够满足铷原子钟物理部分的设计要求。

关键词： 光电子学；量子频标；磁控管腔；谐振频率； Q 值

1　Introduction

Among the various atomic frequency standards, rubidium atomic frequency standard offers the best combination of frequency stability, size, weight, lifetime and cost for many commercial and military applications, especially for the vehicle, marine, airplane and satellite. Up to now, the frequency stability of on-board miniature rubidium atomic frequency standard has already been close to that of miniature cesium frequency standard, but the size of rubidium atomic frequency standard is much less than that of the cesium frequency standard[1,2].

In rubidium atomic frequency standards, the microwave cavity is an essential and important component. The microwave cavity in the rubidium frequency standard is used essentially for two purposes: providing the microwave field interrogating the ^{87}Rb atom in the cell and using as the band-pass filter for the microwave

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frequency multiplier and mixer. There are several microwave cavities adopted in rubidium atomic frequency standards, and the magnetron cavity is the preferential choice for it's high quality factor, smaller volume and nicer magnetic mode[3,4], but the magnetron cavity has a more complicated structure, and it's very difficult to make out the parameters accurate solution by wave equation. And so on when the magnetron cavity is used in rubidium atomic frequency standards, it's resonant frequency, quality factor, oscillation mode need to be studied too.

2 Structure of the magnetron cavity

The structure of the magnetron cavity used in rubidium frequency standard is shown in Fig.1. The outer conductive shield made of copper or aluminum is designed mainly to shield electromagnetic field of the electrodes. It also serves as a supporter of the electrodes, usually four electrodes are used. There is a top cover on the top and a screw thread bottom plate of the cavity. Their function is to adjust the resonant frequency of the cavity by a glass bulb, which serves as a container of rubidium atoms and adjusts the resonant frequency of these cavities.

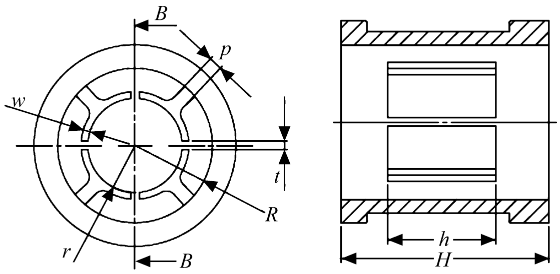

Fig.1 Structure of the magnetron cavity

3 Resonant frequency

The resonant frequency of the cavity depends on the dimensions of the cavity and particular on electrodes. The resonant frequency of the magnetron cavity is[5~8]

$$f = \frac{1}{2\pi r}\sqrt{\left(1+\frac{A_1}{A_2}\right)\frac{Nt}{\mu\varepsilon\pi w}}\sqrt{\frac{1}{1+t/w}}\ , \tag{1}$$

where $A_1 = \pi r^2, A_2 = \pi[R^2-(r+w)^2]$, ε and μ are dielectric constant and magnetic inductivity respectively, and $c = \frac{1}{\sqrt{\mu\varepsilon}}$ is the light velocity in free space, it is about 3×10^8 m/s. $\sqrt{\frac{1}{1+t/w}}$ is the correction coefficient accounting for fringe effect of capacitance between the electrodes, substituting these expressions into expression (1), then we can get the calculation formula

$$f = \frac{c}{2\pi r}\sqrt{\left(1+\frac{A_1}{A_2}\right)\frac{Nt}{\pi w}}\sqrt{\frac{1}{1+t/w}}\ . \tag{2}$$

We have designed and manufactured several resonant cavities according to the cavity structure shown in Fig.1 by this method, we could find that the resonant frequency can be attenuated to 6.835 GHz with a glass bulb in them, which is the resonant frequency for the rubidium atoms.

4 Calculation of Q-factor

The Q-factor is related to dimensions of cavity, the material and energy loss of the glass absorption bulb. According to the definition of Q-factor of the cavity, we can obtain

$$Q = \frac{\omega W_0}{P_0}\ , \tag{3}$$

where P_0 is the power loss in the resonating cavity, W_0 is the total energy stored in the resonating cavity, ω is angular frequency. The energy loss in the cavity mainly refers to the loss on the metal surfaces. If the medium loss can be neglected, we can get the approximation formula

$$P_0 = \frac{R_s}{2}\int |\boldsymbol{H}_l|^2 \mathrm{d}s \ , \tag{4}$$

where $\boldsymbol{H}_l$ represents the tangential component of the magnetic density on the metal surfaces in the cavity, S is the total area of metal surface in the cavity, R_s is expressed by

$$R_s = \frac{\delta}{2}\omega\mu_0 \ , \tag{5}$$

where δ is the skin effect depth of electromagnetic field in material. We suppose that the skin effect depths are the same in all the metal surfaces, all R_s are regarded as the same value when calculating the energy loss on metal surfaces.

When calculating the energy loss on the electrodes surfaces, we also deal with the N pieces of electrodes as a cylinder tube approximately. Let S_1 represents S the inner surface area of the outer cavity, S_2 and S_3 the outer and inner surface areas of the cylinder tube respectively, S_4 the inner surface area of top cover and bottom plate of the outer cavity. Let $\boldsymbol{H}_{\text{in}}$ represents S magnetic density in area between the electrodes, let $\boldsymbol{H}_{\text{ex}}$ represents S magnetic density in area between the electrodes and outer shielding, and both them are regarded as uniformly distributed in their respective areas. By using expression (4) we can get

$$P_0 = \frac{R_s}{2}\left(\int_{S_1}|\boldsymbol{H}_{\text{ex}}|^2\mathrm{d}s + \int_{S_2}|\boldsymbol{H}_{\text{ex}}|^2\mathrm{d}s + \int_{S_3}|\boldsymbol{H}_{\text{ex}}|^2\mathrm{d}s + \int_{S_4}|\boldsymbol{H}_{\text{ex}}|^2\mathrm{d}s\right) . \tag{6}$$

Substituting expression (5) into expression(6) and having $S_4 = 2\pi R^2$, we can get

$$P_0 = \pi R_s l H_0^2\left[r + \left(r + R + \frac{R^2}{h}\right)\left(\frac{A_1}{A_2}\right)^2\right] . \tag{7}$$

By using expressions, Q can be described as

$$Q = \frac{r\omega_0\mu_0}{2R_s}\frac{1+\dfrac{A_1}{A_2}}{1+\left(1+\dfrac{R}{r}+\dfrac{R^2}{hr}\right)\left(\dfrac{A_1}{A_2}\right)^2} .$$

Then substituting expression (13) into this one, we can get the final expression

$$Q = \frac{r}{\delta}\frac{1+\dfrac{A_1}{A_2}}{1+\left(1+\dfrac{R}{r}+\dfrac{R^2}{lr}\right)\left(\dfrac{A_1}{A_2}\right)^2} . \tag{8}$$

Substituting the shape dimensions of the cavity into expressions (8), and the material is copper, then δ could be considered as $\delta = 1.4 \times 10^{-6}\text{m}$[7,8], the Q-factors of the empty cavities are achieved.

5 Oscillation mode

With the values of electrical and dielectric parameters and the shapes dimension of the cavity, we have obtained the distribution of microwave magnetic field in the cavity and the cavity response through a high frequency three-dimensional numerical simulation and experiments. Fig.2 shows the simulation results using the HFSS soft of microwave magnetic field in the cavity. It can be seen that the distribution of the microwave magnetic field in the cavity is the typical TE_{011} mode[8,9], whose magnetic field intensity is stronger in the middle than in the margin. The microwave signal is injected into the cavity by a microwave coupling loop, and the direction of the microwave magnetic field is almost parallel to the direction of cavity axis, which meets the requirement of the system[10,11].

The magnetic field structure is the TE_{011} mode, so the Q-factor is high, according to the test results, the Q-factors of the empty cavities simulated by the HFSS soft are in the scope of 1500~3200 for different empty cavities. These results validate the formulas used to evaluate parameters of magnetron cavities, and we also offer another table for evaluated results of magnetron cavities shown in Table 1. The results in Table 1 show that with the closer resonant frequencies, the bigger volume of cavities, the higher Q-factor is achieved, which offers a wider Q-factor range for rubidium frequency standards to select.

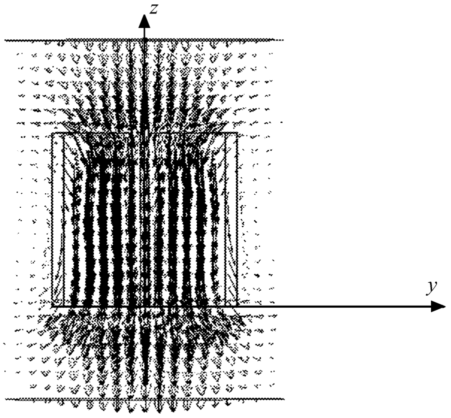

Fig.2 Distribution of microwave magnetic field

Table 1 Geometric dimensions, calculation results for the magnetron cavities

t(mm)	r(mm)	w(mm)	R(mm)	h(mm)	Evaluated f (GHz)	Evaluated Q
1.8	8.0	1.0	13.0	16	7.05	3197
1.8	8.0	1.0	12.5	16	7.35	2839
1.8	8.0	1.0	12.0	16	7.57	2438
1.8	8.0	1.0	11.5	16	8.05	1994
1.8	8.0	1.0	11.0	16	8.60	1625

6 Test results

We have designed and manufactured several resonant cavities according to the cavity structure shown in Fig.1 by this method, their geometric dimensions (in millimeters) is listed in Table 2. Substituting these parameters into expressions (2) and (8), and the material is copper, then considering that $\delta = 1.4 \times 10^{-6}\text{m}$[4,5], the resonant frequencies and Q-factors of the empty cavities are achieved. The Network Analyzer was used to test the parameters for these magnetron cavities. The test results of these cavities are also shown in Table 2.

Table 2 Geometric dimensions, calculation and test results for the magnetron cavities

t(mm)	r(mm)	w(mm)	R(mm)	h(mm)	Evaluated f (GHz)	Evaluated Q	Test result f (GHz)	Test result Q
1.8	8.0	1.0	13.0	16	7.05	3197	7.04	3032
2.0	8.0	1.0	13.0	16	7.22	3197	7.23	3036
2.2	8.0	1.0	13.0	16	7.33	3197	7.35	3095
2.4	8.0	1.0	13.0	16	7.43	3197	7.48	3053
2.6	8.0	1.0	13.0	16	7.54	3197	7.58	3105

By comparing the measured values with the calculated values, we find that the resonant frequencies tally well with the calculated values, and the Q-factors are all smaller than the calculated values because of roughness of the cavities inner surfaces. The microwave cavity in the rubidium frequency standard is used essentially for two purposes: providing the microwave field interrogating the $^{87}\mathrm{R_b}$ atom in the cell and using as the band-pass filter for the microwave frequency multiplier and mixer. Fig.3 is the test result for the cavities with a glass bulb. From the figure we could find that most of the cavities resonant frequency listed in Table 2 can also be attenuated to 6.835 GHz with a glass bulb in them, which is the resonant frequency for the rubidium atoms[11,12], and Q-factor can be attenuated to 600~1000, which can meet the requirement for the rubidium atomic frequency standard[13,14] too.

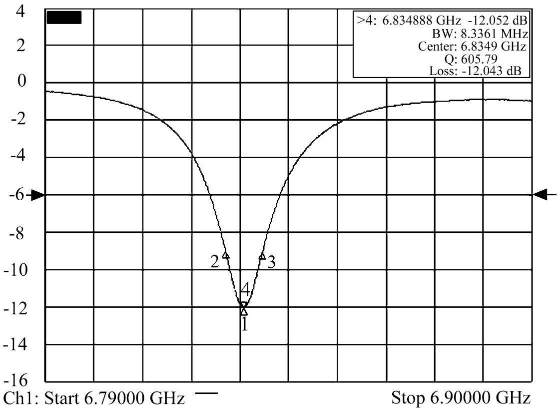

Fig.3 The resonant frequency and quality factor test results of magnetron cavity

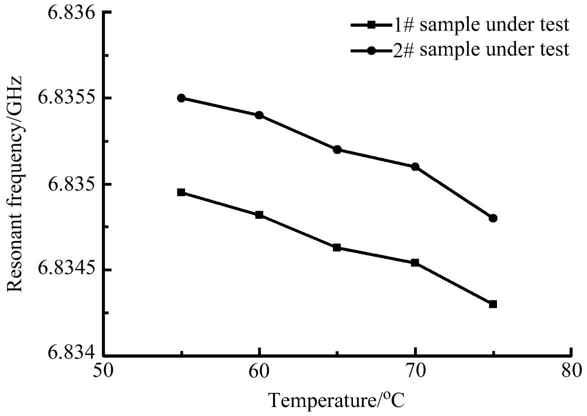

Fig.4 The temperature-frequency coefficients of magnetron cavities

7 Temperature-frequency coefficient and rubidium atomic frequency standard

The temperature-frequency coefficients of two magnetron cavities were also tested. In the temperature scope of 55°C~75°C, which is the working temperature region of the cavity, both frequencies of the magnetron cavities are linearly decreaseing with the temperature ascend as shown in Fig.4. The temperature-frequency coefficients of the two cavities were also obtained. One of the cavities temperature-frequency coefficient is 32.5 kHz/°C, and the other is 35.0 kHz/°C. These results also show that temperature-frequency coefficients is more smaller than $\mathrm{TE_{111}}$ mode cylindrical cavity whose temperature-frequency coefficient is over 200 kHz/°C[12,13]. The lower temperature-frequency coefficient has a smaller pulling effect for the rubidium atomic frequency standard[1].

There is a kind of rubidium atomic frequency standard used in space has been developed based on these magnetron cavities. As the cavity-cell assembly of the magnetron cavity was link to rubidium atomic frequency standard system, the closed loop was realized. After modulation and improvement of the cavity-cell assembly, the 1s stability of rubidium atomic frequency standard based on the magnetron cavity can exceed 3×10^{-12}, as shown in Fig.5.

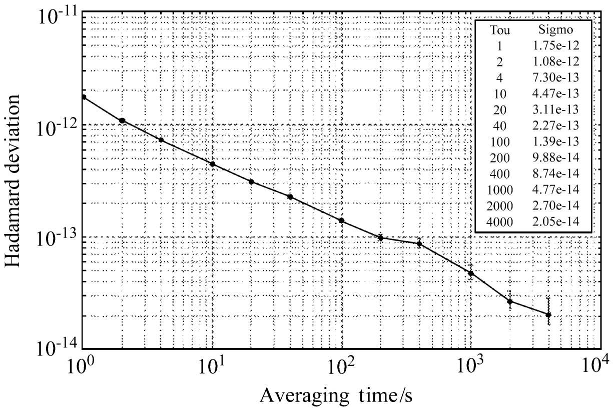

Fig.5 Test result of short term stability for atomic frequency standard

8 Conclusion

In conclusion, based on the theoretical calculations, a kind of magnetron cavity for space rubidium atomic frequency standard is developed. The test results show that the resonant frequency and Q-factor can be accommodated to meet the requirements of the rubidium atomic frequency standard, the microwave magnetic field in the cavity is the typical TE_{011} mode, and the cavity has a lower temperature coefficient. There is a kind of rubidium atomic frequency standard used in space has been developed based on these magnetron cavities, 1s stability of rubidium atomic frequency standard clocks based on the magnetron cavity can be better that 3×10^{-12}.

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