# I. Introduction

## (I) The Stylized Facts of Economic Growth

(Kaldor, Kuznets, Romer, Lucas, Barro, Mankiw-Romer-Weil, others)

- In the short run, important fluctuations: Output, employment, investment, and consumption vary a lot across booms and recessions.
- In the long run, balanced growth:
  - Output per worker and capital per worker ($Y/L$ and $K/L$) grow at roughly constant, and certainly not vanishing, rates.
  - The capital-to-output ratio ($K/Y$) is nearly constant.
  - The return to capital ($r$) is roughly constant.
  - The wage rate ($w$) grows at the same rate as output.
  - The income shares of labour and capital ($wL/Y$ and $rK/Y$) stay roughly constant.
- Substantial cross-country differences in both income levels and growth rates.
  - GDP *per capita* in 2000:
    Luxembourg: \$44,000 U.S.:\$32,500
    China: \$4,000 Tanzania: \$570
  - Growth rates of GDP (1960 $\sim$ 2000):
    Mean growth rate: 1.8% per annum
    Taiwan: 6% Zambia: -1.8%
- Persistent differences versus conditional convergence.
- Formal education:
  - Highly correlated with high levels of income (obviously two-direction causality).

  - Together with differences in saving rates can "explain" a large fraction of the cross-country differences in output.
  - An important predictor of high growth performance.

- R&D and IT: Most powerful engines of growth (but require high skills at the first place).

- Government polices: Taxation, infrastructure, inflation, law enforcement, property rights and corruption are important determinants of growth performance.

- Democracy: An inverted U-shaped relation; that is, autarchies are bad for growth, and democracies are good, but too much democracy can slow down growth.

- Openness: International trade and financial integration promote growth (but not necessarily if it is between the North and South).

- Inequality: The Kuznets curve, namely an inverted U-shaped relation between income inequality and GDP *per capita* (growth rates as well). That is, income inequality tends to increase with income at low levels of income and to decrease with income at higher levels of income.

- Fertility: High fertility rates correlated with levels of income and low rates of economic growth; and the process of development follows a Malthus curve, meaning that fertility rates initially increase and then fall as the economy develops.

- Financial markets and risk-sharing: Banks, credit, stock markets, social insurance.

- Structural transformation: agriculture → manufacture → services.

- Urbanization: the features of urbanization are as follows: family production → organized production; small villages → big cities; extended domestic trade.

- Other institutional and social factors: colonial history, ethnic heterogeneity, social norms.

**(II) Features of Business Cycles**

- Magnitudes of fluctuations in output ($Y$) and aggregate hours ($H$) are nearly equal: $\sigma_Y \approx \sigma_H$.
- Employment fluctuates almost as much as output and total hours, whereas average weekly hours fluctuate less: $\sigma_Y \approx \sigma_H > \sigma_{AWH}$.
- Consumption of nondurable goods and services fluctuates less than output: $\sigma_C < \sigma_Y$.
- Investment fluctuates more than output: $\sigma_I > \sigma_Y$ and is highly procyclical.
- Productivity is slightly procyclical but varies less than output: $cor(A, Y) > 0, \sigma_A < \sigma_Y$.
- Government expenditures are essentially uncorrelated with output: $cor(G, Y) \approx 0$.
- Import is more correlated with output than export: $cor(IM, Y) > cor(EX, Y)$.
- Net exports are countercyclical.
- Real wage is acyclical.

The stylized facts above should guide us in the modeling of economic growth and business cycles. The theories of macroeconomics we will review in this course seek to explain how all the above factors interrelate with the process of economic growth and business cycles.

**(III) Goals of this course**

- Learn how models work.
- Understand the models and methods used by macroeconomists.
- Think about current and historical events in a disciplined way.
- Acquire research skills.

**(IV) Introduction to Macroeconomic Models**

Why do we use formal models?

- Models identify particular features of reality and study their consequences in isolation.
- Models provide a rigorous way of investigating whether a proposed theory can answer a particular question and whether it generates additional predictions.

Classification of Models

$$\text{number of agents} \begin{cases} \text{the representative agent model} \\ \text{overlapping generations (OLG) model} \\ \text{model with many different agents (heterogeneous agents model)} \end{cases}$$

$$\text{markets clear or not} \begin{cases} \text{Yes: market-clearing models} \\ \text{No: non-market-clearing models} \end{cases}$$

- Market-clearing models are sometimes called Walrasian or Equilibrium models and are associated with *classical economics*.
- Non-market-clearing models are sometimes called disequilibrium models and tend to be identified with *Keynesianism*.

$$\text{role for policy intervention} \begin{cases} \text{Yes: new Keynesian macroeconomics} \\ \text{No: new classical macroeconomics} \end{cases}$$

$$\text{time} \begin{cases} \text{static models: models with one period} \\ \text{dynamic models: models with many periods} \end{cases}$$

Common features of Macroeconomic models

- Current models tend to derive decision rules from optimization problems, and make them compatible with much of microeconomics. (Models with Micro foundations)

  Advantage of using micro foundations: macro models with micro foundations

  - are by construction coherent and explicit;
  - can use standard methods of welfare economics;
  - broadens the source of empirical evidence that can be used to assign numerical values to the models' parameters.

- Most macro models are of the general equilibrium variety, so that all interdependencies are taken into account.
- More and more empirical evidence is used. To some extent, Macroeconomics is the empirical application of dynamic, stochastic, general equilibrium (DSGE) models.

What is a model?

A model consists of endogenous/explained/dependent variables and exogenous/forcing/ independent variables. These variables are related through a set of equations including:

- accounting identities such as those in the national accounts,
- behavioral relationships (e.g. the consumption function) describing agents' strategies or rules of behavior,
- institutional rules such as tax schedules,
- technological constraints such as production functions.

What does it mean to solve a model?

Solving a model simply means solving this set of equations, figuring out how the endogenous variables depend on the exogenous variables.

Examples:

Example 1: A simple static and deterministic model

(i.e. a model containing no random elements or uncertainty)

$$\begin{aligned} y &= c + i \\ c &= \beta y \end{aligned}$$

exogenous variable: i, endogenous variables: y, c.

Sometimes the model is called “structural form”.

$$\begin{aligned} y &= \frac{i}{1-\beta} \\ c &= \frac{\beta i}{1-\beta} \end{aligned}$$

Sometimes the solution is called “reduced/closed form”.

Comparative statics:

to see how a change in an exogenous variable affects the endogenous variables. E.g.:

$$\frac{dy}{di} = \frac{1}{1-\beta} \qquad \text{(multiplier)}$$

Sensitivity analysis:

to check the sensitivity of a model's predictions to a change in parameter values. E.g.:

$$\frac{d(dy/di)}{d\beta} = \frac{1}{(1-\beta)^2} > 0 \qquad (\beta \uparrow \Rightarrow \text{multiplier} \uparrow)$$

Example 2: A nonlinear model

$$\begin{aligned} y &= f(c, i) \\ c &= g(y) \end{aligned}$$

where $f$ and $g$ are some nonlinear functions.

$$\begin{aligned} dy &= \frac{\partial f(c,i)}{\partial c}dc + \frac{\partial f(c,i)}{\partial i}di = f_c dc + f_i di \\ dc &= \frac{\partial g(y)}{\partial y}dy = g_y dy \\ \Rightarrow \qquad \frac{dy}{di} &= \frac{f_i}{1 - f_c g_y}. \end{aligned}$$

Example 3: Dynamic Stochastic Model

Dynamic: there are many periods and all the periods are linked together, we consider the effects of current period on the future.
Stochastic: some or all of the exogenous variables are *random variables* (variables with shock or error term).

$$\begin{aligned} y &= c + i \\ c &= \beta y \end{aligned}$$

now assume $i$ is a random variable with mean 0 and variance $\sigma^2$, that is, $E[i] = 0, var(i) = \sigma^2$.

$$y = \frac{1}{1-\beta}i, \quad c = \frac{\beta}{1-\beta}i \qquad \text{(both are random variables)}$$

$$\begin{aligned} E[y] &= E\left[\frac{1}{1-\beta}i\right] = \frac{1}{1-\beta}E[i] = 0 \\ var(y) &= var\left(\frac{1}{1-\beta}i\right) = \frac{1}{(1-\beta)^2}var(i) = \frac{\sigma^2}{(1-\beta)^2} \\ E[c] &= E\left[\frac{\beta}{1-\beta}i\right] = \frac{\beta}{1-\beta}E[i] = 0 \\ var(c) &= var\left(\frac{\beta}{1-\beta}i\right) = \left(\frac{\beta}{1-\beta}\right)^2 var(i) = \frac{\beta^2\sigma^2}{(1-\beta)^2} \end{aligned}$$

the above solutions are the predictions of our model.

Then how do we test the model? Econometric methods.

**(V) Some Useful Tools**

- *Log approximation.* If $x$ is a small number, then

$$\ln(1+x) \approx x.$$

- *Growth rates.* In continuous time, the growth rate of a variable $x$ is

$$\dot{x} \equiv \frac{dx/dt}{x} = \frac{d\ln x}{dt}.$$

  The growth rate of a ratio is the difference between the two underlying growth rates:

$$\dot{\left(\frac{x}{y}\right)} = \dot{x} - \dot{y}$$

  The growth rate of a product is the sum:

$$\dot{(xy)} = \dot{x} + \dot{y}.$$

  In discrete time, the growth rate is:

$$\frac{x_t - x_{t-1}}{x_{t-1}}.$$

  Sometimes $x_t/x_{t-1}$ is called the gross growth rate.

  For small growth rates, we can approximate the growth rate as

$$\frac{x_t - x_{t-1}}{x_{t-1}} \approx \ln x_t - \ln x_{t-1} = \Delta \ln x_t,$$

  where $\Delta$ denotes a difference.

- *Discounting and power series.* We'll often come across an infinite sum in the form of a power series, and need to know if it converges. If $|x| < 1$, then

$$\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + \cdots = \frac{1}{1-x}.$$

- *Jensen's Inequality.*

  Suppose that $x$ is a random variable, with mean $\mu$ and variance $\sigma^2$. Something else, say $y$, depends on $x$ and the relationship is nonlinear, say some function $y = f(x)$. Then we cannot find the moments of $y$ without knowing the probability density function of $x$. It is helpful to remember something that is *not* true. When $f$ is nonlinear,

$$E(y) \equiv E[f(x)] \neq f[E(x)] \equiv f(\mu).$$

Instead,

$$E[f(x)] > f[E(x)] \quad \text{if } f'' > 0,$$

$$E[f(x)] < f[E(x)] \quad \text{if } f'' < 0,$$

which is called *Jensen's Inequality.*

If we are given the complete density of $x$ then we can work out the density of $y$ and hence find its moments precisely.

- *Lag operator.* When we keep track of macroeconomic variables in discrete time, observations at different dates can be denoted with the lag operator, $L$. It is defined this way:

  $$Lx_t \equiv x_{t-1},$$

  so that

  $$L^i x_t \equiv x_{t-i}.$$

  The characteristic equation of a difference equation is simply a polynomial in the lag operator.

- *Covariance decomposition.* Suppose that $x$ and $y$ are random variables and that we need to study the forecast or expectation of their product. Then we use the covariance decomposition, which is:

  $$E(xy) = E(x)E(y) + cov(x, y).$$