Does Lorentz Force Law Contradict the Principle and Theories of Relativity for Uniform Linear Motion?

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Abstract

I show that no force or torque is generated in cases involving a charge and a magnet with their relative velocity zero, in any inertial frame of reference. A recent suspicion of an anomalous torque and conflict with relativity in this case is rested. What is distilled as 'Lorentz force' in standard electrodynamics, with relative velocity as the parameter, is an under-representation of two distinct physical phenomena, an effect due to Lorentz contraction and another due to the Ampere current-current interaction, rolled into one due to prejudice from special relativity applied only to linear motion. When both are included in the analysis of the problem there is no anomalous force or torque, ensuring the validity of Poincare's principle of relativity. The issue of validity of electrodynamics without the concept of absolute rest, however, is subtle and empirically open when general noninertial motion is considered, as I will discuss in another paper.

The modern theory of electrodynamics was built by Maxwell by distilling various experimental results and then including the phenomenon of displacement current he discovered from a consistency analysis. This was well before the theories of relativity by Lorentz, Poincaré and Einstein came into discussion. The relativity principle expounded by Poincaré ensures the impossibility of detecting uniform linear motion by experiments in electrodynamics

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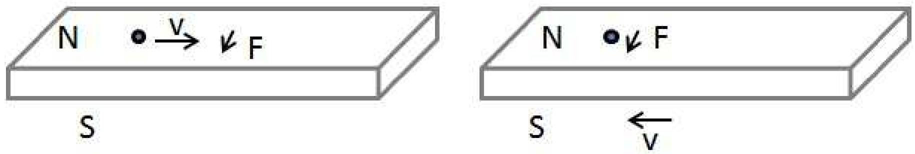

Figure 1: The charge moving at velocity $\vec{v}$ is physically equivalent to magnet moving at velocity $-\vec{v}$ in Einstein's special relativity. Both forces are given by the Lorentz force law with $\vec{v}$ as their relative velocity.

and mechanics [1]. Thus the state of uniform rectilinear motion is equivalent to the state of rest. It was Einstein who asserted, in the opening paragraph of his 1905 special relativity paper [2], that *the reciprocal electrodynamic action of a magnet and a conductor depends only on the relative motion of the conductor and the magnet*. See figure 1 for the relevant situation. He further wrote, "examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium", suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest".

While analyzing the cases involving a conductor and magnet in various states of relative motion between them, I had realized that current electrodynamics, as represented in Maxwell's equations and the Lorentz force law, is actually an under-representation of the fundamental physical phenomena involved [3]. In particular, the Lorentz force law and its relativistic transformations *involving only the relative velocity* between charges and magnets mix up two distinct physical phenomena into a single expression that does not adequately represent the physical effects for general motion involving non-rectilinear motion. Something has been lost in translation by Maxwell and Lorentz, from the body of results arrived at by painstaking experiments over decades by Faraday, Ampere, Weber and others. However, that shows up only in the case of non-rectilinear motion and the subject of this paper is the analysis that shows that there is no torque or force when a charge and a magnet that are relatively stationary are observed from a uniformly moving frame.

The physical phenomena that are relevant are (a) Ampere's current-current interaction and (b) the modification of charge densities involved in the effective equivalent currents in a magnet due to Lorentz contraction in motion. *These are two different physical phenomena, with different physical origins and mathematical expressions, which coincide exactly for the case of force on a charge moving uniformly relative to a magnet.* Usually, the force is written as the Lorentz force,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

Since $\vec{v}$ is interpreted as the *relative velocity* between the magnet and the charge, there is no force when a charge is at rest relative to a magnet and no force appears when the system is viewed from another frame moving relative to the physical system. However, instead of using the formula, if one tries to calculate the force from the interactions of the charges and currents involved in the problem, there could be an apparent conflict that is spurious if one of two fundamental physical effects we discussed is not taken into account. Indeed, the current-current interaction is often forgotten in discussions since it is assumed that Maxwell's equations contain fully its manifestations. This is perhaps true, except that there is certainly one ambiguous element in the formulation of modern electrodynamics, and that is the interpretation of the velocity that appears in the transformation equations: *relative or absolute*? When the Lorentz force law is discussed in moving frames, usually only the Lorentz contraction effect is considered. In particular, the case of a charge and a magnet there is a spurious force or torque unless the current-current interaction is included [3]. This seems to be the case in a news and analysis article announcing a serious conflict between relativity and the Lorentz force law, in the case of a charge and a magnet that are relatively stationary, when observed from a moving frame [4]. The magnet is equivalent to a current loop involving two opposite currents separated by distance. In a moving frame Lorentz contraction generates asymmetric charge densities and an electric field (see below). If this is the only effect that is considered as the basis of Lorentz force, an anomalous torque is indicated and if true, it would conflict with relativity since a real force or torque and resultant motion (dynamics) cannot appear within this physical system merely due to uniform motion of an observer.

Now I show that this paradox arose precisely because of not taking into account the fundamental phenomenon of current-current interaction in the moving frame. Including that effect makes the torque vanish. Indeed, the system considered in the news analysis is a variant of an example I have analyzed earlier with the difference that he was considering a system where the charge is separated from the magnet and a torque is generated, whereas I have been dealing with examples where the charge is right near the surface of the large magnet [3]. (This is because we are attempting to make measurements on the dynamics of charges near moving magnets in various configurations and the possibility of good measurements reduces with increasing distance between the charge and the magnet). In the particular case of linear motion, the analysis can be done either with relative velocities or with velocities with respect to a preferred absolute frame and the results are identical, reiterating the fact that no experiment in linear motion can be used as a demarcating test between different theories of relativity obeying the relativity principle. Three such theories are the Lorentz-Poincaré relativity theory [1, 5], Einstein's special relativity [2] and the new paradigm of Cosmic Relativity [6, 7] that I have been advocating in which all relativistic effects are due to the gravitational effects of cosmic matter and cosmic frame is a preferred frame perfectly replacing the old ether.

Consider a charged particle in close vicinity of a bar magnet with large polar area, close to one of its poles. Moving the charge relative to the magnet will make it deflect transversely and moving the magnet also will produce the same physical effect (figure 1). The force is given in both cases, as in modern electrodynamics, by the Lorentz force law,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{2}$$

with $E = 0$ in this case. This is the example that guided Einstein to (hastily) conclude that *all electrodynamic phenomena* are completely symmetric in relative motion. Now consider the entire system of the magnet and the charge at rest. In special relativity, $\vec{v}$ is the relative velocity between the magnet and the charge, as originally asserted by Einstein. When the relative velocity is zero, there is no force on the charge and it remains at rest. However, viewed from a frame moving at velocity $\vec{V}$, the magnet becomes charged, with opposite charges on its two sides parallel to the motion of the observer due to Lorentz contraction (figure 2).

This is because the magnet is equivalent to a current loop and on one side the velocity of the charged particles corresponding to the current add to the motion of the observer and on the other side, it subtracts. If the speed of the moving charges in the magnet is u, the Lorentz contraction factors for the two opposite currents are different, $\left(1 - (V+u)^2/c^2\right)^{1/2}$ and $\left(1 - (V-u)^2/c^2\right)^{1/2}$. For low velocities involved for both the current and

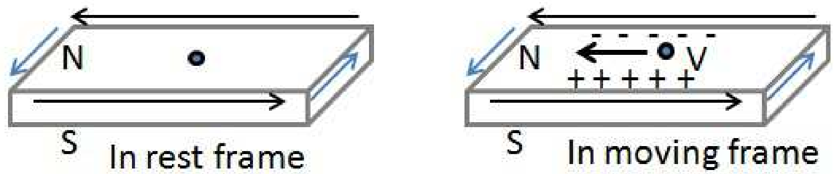

Figure 2: Lorentz contraction of currents in the magnet generates asymmetric charge density and electric field. The long arrows depict the current elements. In a frame moving right with velocity $\vec{V}$, the originally stationary charge becomes a current with velocity $-\vec{V}$.

the observer, these can be written as

$$\begin{aligned} (1) \quad \Delta l_1 &= -L(V^2+u^2+2uV)/2c^2 \\ (1) \quad \Delta l_2 &= -L(V^2+u^2-2uV)/2c^2 \end{aligned} \tag{3}$$

and the difference for low velocities is $-2LVu/c^2$. (the similarity to the Sagnac term is not accidental. See reference [6, 8]). The change in the linear density will be then $-2Vu/c^2$. This multiplied by the negative charge density λ is the difference in charge densities, creating a transverse electric field $\vec{E}$ in the direction $\vec{V} \times (\nabla \times \vec{u})$, of magnitude

$$E = 2\lambda Vu/2\pi\varepsilon_0 rc^2 \tag{4}$$

However, now we have a conflict with principle of relativity if there is only the Lorentz contraction, since the charge should start moving due to this electric field, which is unphysical since an observer's mere motion should not physically displace the charge relative to the magnet! *However, note that in the same frame, the charge becomes a current* $-q\vec{V}$ and there are forces of opposite sign to that of the Lorentz contraction from the Ampere current-current interaction that exactly bring the net force to zero. Parallel current attract and opposite ones repel, where one current $i_q = qV$ is from the moving test charge and the other two oppositely directed ones i_m from the current loop in the magnet. This current-current force is exactly same in magnitude and opposite in direction to the one generated by the Lorentz contraction. The two cancel, and the charge remains at rest, as if there is no force. The Ampere interaction from the two opposite currents on the current

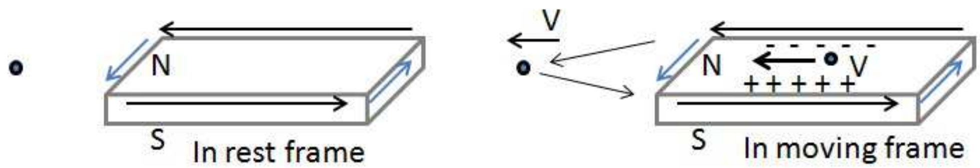

Figure 3: Spurious torque on the magnet due to a charge on the far left, considering Lorentz contraction alone in the problem in the moving frame. The current-current interaction cancels this.

of the charge q, now moving with velocity $\vec{V}$ in the new reference frame is

$$F_A = \frac{2\mu_0}{2\pi}\frac{i_q i_m}{r} = \frac{\mu_0}{\pi}\frac{qVi_m}{r} = \frac{\mu_0}{\pi}\frac{qV\lambda u}{r} \tag{5}$$

and the (oppositely directed) force from Lorentz contraction and the electric field of the modified charge density of the currents in the magnet is

$$F = qE = \frac{q\lambda V u}{\pi\varepsilon_0 c^2 r} = \frac{\mu_0}{\pi}\frac{qV\lambda u}{r} = \frac{\mu_0}{\pi}\frac{i_q i_m}{r} \tag{6}$$

if we identify $\mu_0 = 1/\varepsilon_0 c^2$, which is the case in electrodynamics! If the charge, instead of being near the pole of the magnet, where the field is uniform, is far away as illustrated in reference [4] then we get a (spurious) nonzero torque from just the Lorentz contraction (figure 3).

The problem posed as a paradox conflicting relativity is completely solved if the torque is recalculated with the current-current interaction added. That generates an equal and opposite torque to what was calculated from Lorentz contraction and the net torque is zero. The factual situation is that there are two equal and opposite forces (or forces and torques in the general case), which balance in the case of linear motion. This should not be trivialized, as is done in standard electrodynamics, to a 'zero force' situation since this balance is broken in the case of rotation motion as we shall see [9]. A 'zero' as genuine nothing and as a delicate and special balance of two equals are completely different, conceptually and physically.

It is clear that, for the case depicted in fig. 3, one may take the limit of the current loop area shrinking to infinitesimal values with the magnetic moment remaining constant and the analysis, with the current-current interaction included, gives zero torque in all frames in the presence of the charge.

I may add here is that what we have done is to derive the relation between μ_0, ε_0 and c from the principle of relativity with Ampere's current-current interaction and the Lorentz contraction as the ingredients, without entering the plane of the electrodynamic wave equation. It is very important to recognize that these phenomena of linear motion does not allow us to demarcate between special relativity and an absolute frame theory as the empirically valid theory because both incorporate Poincaré's principle of relativity. Explanations can be found for all electrodynamic phenomena in linear motion in both theories by interpreting the velocities involved as absolute velocities in absolute frame theories, and as relative velocity between the magnet and the conductor in special relativity. Both predict zero force and zero torque in the problems discussed above.

Even in the simpler problem of a single conductor and an external charge, the paradox appears if the current-current interaction is ignored. But such anomalies are spurious since in the same moving frame the charge is transformed to a current that interacts with the current in the conductor cancelling the force from Lorentz contraction. Principle of relativity in rectilinear motion remains intact. However, the respite is temporary, as I will elaborate in another paper dealing with cases of uniform rotational motion [9]. It is also important to recognize that the wise do not consider that modern electrodynamics as discussed in standard text is the last word on the electrodynamics of the real world and several issues related to the exact nature of the current-current interaction, unipolar induction, action and reaction etc. are being actively discussed and experimented with, albeit in a relatively small yet competent community.

Disclaimer: The discussion in this paper pertains to the news and analysis, reference [4], and not directly to the original paper, which was not available at the time of writing or submitting this paper. The paper by M. Mansuripur is now available in arXive.org as arXiv:1205.0096.

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