Phase shift in an atom interferometer induced by the additional laser lines of a Raman laser generated by modulation

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The use of Raman laser generated by modulation for light-pulse atom interferometer allows to have a laser system more compact and robust. However, the additional laser frequencies generated can perturb the atom interferometer. In this article, we present a precise calculation of the phase shift induced by the additional laser frequencies. The model is validated by comparison with experimental measurements on an atom gravimeter. The uncertainty of the phase shift determination limits the accuracy of our compact gravimeter at 8×10^{-8} m/s^2. We show that it is possible to reduce considerably this inaccuracy with a better control of experimental parameters or with particular interferometer configurations.

I. INTRODUCTION

Light-pulse atom interferometry [1] is a promising technology to obtain highly sensitive and accurate inertial sensors. Laboratory experiments have already demonstrated state of the art performances for gravimeter [2, 3], gradiometer [4] and gyroscope [5, 6]. Inertial sensors start to be tested in mobile platforms such as a plane [7] or a truck [8]. An important research effort has still to be done in order to have instruments and particularly the laser system more compact and robust for practical applications like navigation [9], space mission [10–13], gravity mapping and monitoring [14], subsurface detection [15].

The key point of light-pulse atom interferometer is the stimulated Raman transition between two stable states of the atom [16]. This transition is driven with two counter-propagating lasers with a frequency difference corresponding to the energy difference between the stable states. This transition allows to make the equivalent of beam splitters and mirrors for matter waves. To generate this Raman laser, two phase locked lasers are generally used [17]. Another solution consists in a laser which is intensity or phase modulated at the frequency difference between the ground states. This technology gives a laser system more compact (only one laser is needed) and more robust (no phase lock). Moreover, the frequency noise of the laser is not reported into the Raman phase noise [18]. The drawback of this technique is the presence of additional laser frequencies which can perturb the atom interferometer. The realization of an atom interferometer using a modulated Raman laser has been demonstrated experimentally [7, 19] but with an additional phase shift which have not been quantified yet.

In this article, the perturbations induced on an atom interferometer by the additional frequencies present in a modulated Raman laser are studied in detail. In the first part, we show a precise calculation of a stimulated Raman transition with a modulated Raman laser. In the second part, we calculate for a Mach-Zehnder interferometer the phase shift induced by the additional laser frequencies. In a third part, we compare the results of our calculation to experimental measurements of the phase shift. Finally, we present the limitation in accuracy of an atom interferometer using a modulated Raman laser. We also show a configuration where the inaccuracy induced by the additional laser frequencies is considerably reduced.

II. STIMULATED RAMAN TRANSITION WITH A MODULATED LASER

The atom corresponds to a Λ-type three-level system with two lower levels $|a\rangle$ and $|b\rangle$ separated by an energy $\hbar G$ and an excited state $|i\rangle$ separated from the level $|b\rangle$ by $\hbar\omega_0$ (see Fig. 1). The atom interacts with a laser retro-reflected by a mirror at the position z_M. The spectrum of the laser is composed of lines separated in frequency by $\Delta\omega$. This kind of spectrum is obtained with a laser modulated in amplitude or in phase with a frequency $\Delta\omega$. The electric field seen by the atom can be written as:

$$E = \sum_{n=-\infty}^{\infty} E_n \cos\left((\omega_L + n\Delta\omega)(t - z/c) + \varphi_n\right) + E_n \cos\left((\omega_L + n\Delta\omega)(t + (z - 2z_M)/c) + \varphi_n\right) \quad (1)$$

When the frequency of modulation $\Delta\omega$ is close to the frequency difference of the two lower levels G, each couple of laser lines separated by $\Delta\omega \simeq G$ drives stimulated Raman transitions between the two lower states (see Fig. 1). In this article, one considers only two-photon Raman transition with counter-propagating beams with the higher frequency beam propagating downward (see Fig. 2). If the velocity of the atoms is big enough, the other kinds of Raman transition (co-propagating and counter-propagating with opposite direction) are out of resonance thanks to the Doppler effect and can be neglected. The couple of counter-propagating beams with frequencies $\omega_L + (n+1)G$ and $\omega_L + nG$ is coupling the state $|a, p\rangle$ to the state $|b, p + \hbar k_{\mathrm{eff}} + n\hbar\Delta k\rangle$ where $k_{\mathrm{eff}} = (2\omega_L + \Delta\omega)/c$ and $\Delta k = 2\Delta\omega/c$. The effective Rabi frequency Ω_n as-

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sociated with this transition is equal to :

$$\Omega_n = \frac{\Omega^*_{(n+1)ai}\Omega_{nbi}}{2(\Delta + n\Delta\omega)} = \frac{E_{n+1}E_n}{E_1 E_0}\frac{\Delta}{\Delta + n\Delta\omega}\Omega_0 \qquad (2)$$

where $\Delta = \omega_L - \omega_0$ and $\hbar\Omega_{nxi} = -\langle i|d_{xi}E_n|x\rangle$ with $x = a \,\text{or}\, b$.

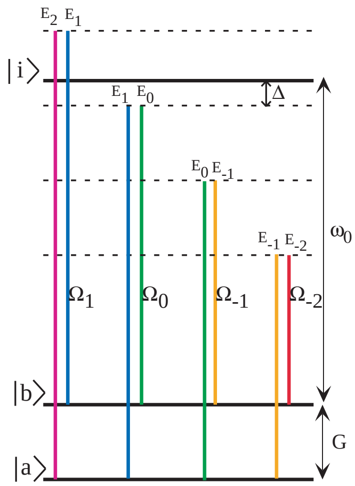

FIG. 1. Atomic level system and Raman transitions.

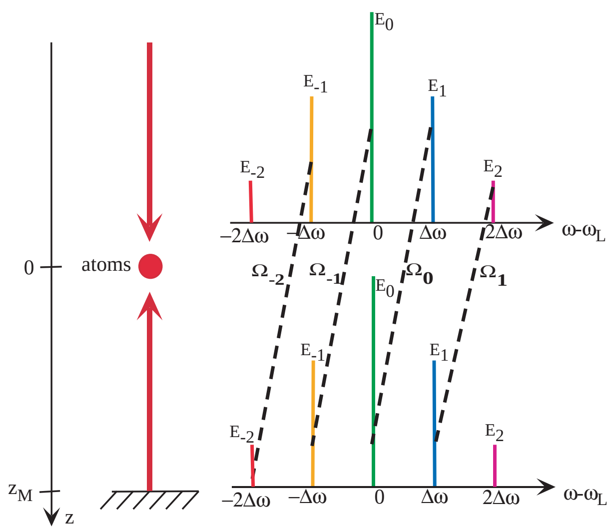

FIG. 2. Raman transition with a retro-reflected modulated laser. The dot lines correspond to the couple of laser lines driving the counter-propagating Raman transition considered in this article.

The family of quantum states coupled with these Raman transitions is therefore $|a,n\rangle = |a, p + n\hbar\Delta k\rangle$ and $|b,n\rangle = |b, p + \hbar k_{\text{eff}} + n\hbar\Delta k\rangle$ (see Fig. 3). Compared to the case of a Raman laser with only two laser lines where two quantum states are coupled, a modulated Raman laser couples an infinite number of quantum states. A Raman transition with a modulated laser is therefore not equivalent to a simple beam splitter but can be seen as a multiple beam splitter.

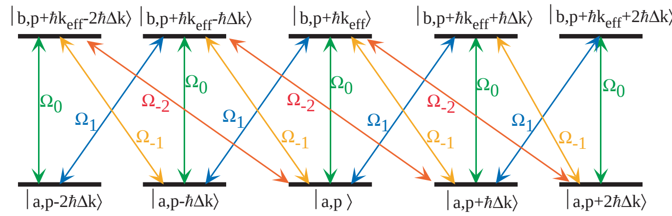

FIG. 3. Family of quantum states coupled with a modulated Raman laser.

The calculation of the transition matrix will be done within the following approximations. The kinetic energy difference induced by the term Δk will be neglected. In typical atom interferometer ($G/2\pi \lesssim 10\,\text{GHz}$, $v \lesssim 1\,\text{m/s}$), the Doppler effect associated with Δk is below 67 Hz and can be neglected for typical interaction time of $10\,\mu$s. We will therefore consider that the states $|a,n\rangle$ together with the states $|b,n\rangle$ are degenerated in energy. We also suppose that the Raman laser is at resonance i.e. the frequency of modulation $\Delta\omega$ is equal to the frequency difference between the states $|a,n\rangle$ and the states $|b,n\rangle$ eventually affected by the light shift. Within this approximation and by adiabatically eliminating the excited state and using the rotating wave approximation, the system can be described by the following effective Hamiltonian :

$$H = \frac{\hbar}{2}\sum_n\sum_m \Omega_m e^{i\phi_m}|b,n+m\rangle\langle a,n| + \Omega_m e^{-i\phi_m}|a,n-m\rangle\langle b,n| \qquad (3)$$

where ϕ_m is the phase of the Raman laser associated with the couple m of Raman laser :

$$\phi_m = \varphi_m - \varphi_{m+1} - (k_{\text{eff}} + m\Delta k)z_M + \frac{\Delta k}{2}z_M \qquad (4)$$

If a free fall frame is used, one should replace in this expression z_M by $z_M - \frac{1}{2}g\,t^2$. To compensate the Doppler shift induced by gravity, one generally applies a frequency chirp α to the Raman frequency. In this case, one should add to the phase ϕ_m the term $-\alpha\,t^2/2$. The terms $\frac{\Delta k z_M}{2}$ and $k_{\text{eff}}z_M$ are constant and can be ignored in the following. If the intensity or phase modulated laser propagates through a dispersive medium, $\varphi_{m+1} - \varphi_m$ is not null and is proportional to m. This term is equivalent to a modification of the mirror distance z_M and can be ignored. Finally, the expression of ϕ_m can be written into a term independent of m and a term proportional to m :

$$\phi_m = A + mB \qquad (5)$$

with

$$A = \frac{1}{2}\left(k_{\text{eff}}g - \alpha\right)\,t^2$$
$$B = \Delta k\left(\frac{1}{2}g\,t^2 - z_M\right) \qquad (6)$$

The evolution operator of the atom interacting with the Raman laser is given by:

$$U = e^{-i\frac{Ht}{\hbar}} \tag{7}$$

One can show that the elements of the evolution operator can be written as :

$$\begin{aligned}
\langle a, n+m|U|a,n\rangle &= t_m\, e^{i\,m\,B}\\
\langle b, n+m|U|b,n\rangle &= t_{-m}\, e^{i\,m\,B}\\
\langle b, n+m|U|a,n\rangle &= -i\, r_m\, e^{i\,A}\, e^{i\,m\,B}\\
\langle a, n+m|U|b,n\rangle &= -i\, r_{-m}\, e^{-i\,A}\, e^{i\,m\,B}
\end{aligned} \tag{8}$$

The coefficient r_m and t_m are real and can be calculated numerically by truncating the number of states coupled and the number of laser lines. Typically, we perform the calculation with 22 quantum states and 5 laser lines. The calculation of the transition amplitude is done for the interaction times: $\tau = \frac{\pi}{2\Omega_0}$ and $\tau = \frac{\pi}{\Omega_0}$ corresponding to $\pi/2$ and π pulses without additional laser lines (beam splitter and mirror). The result of the calculation is presented in the table I for the case of stimulated Raman transition on Rubidium 87.

We can notice that the probability transition into parasite states ($n \neq 0$)is small but not negligible. We should therefore take into account the additional paths engendered by these parasite transitions in order to determine precisely the phase of an atom interferometer.

n	$t_n^{\frac{\pi}{2}}$	$r_n^{\frac{\pi}{2}}$	t_n^{π}	r_n^{π}
-2	$\sqrt{1.1\times10^{-5}}$	$-\sqrt{8.3\times10^{-5}}$	$\sqrt{1.3\times10^{-4}}$	$-\sqrt{1.5\times10^{-4}}$
-1	$\sqrt{1.2\times10^{-3}}$	$-\sqrt{3.2\times10^{-3}}$	$\sqrt{9.2\times10^{-3}}$	$-\sqrt{1.1\times10^{-3}}$
0	$\sqrt{0.497}$	$\sqrt{0.497}$	$-\sqrt{3.0\times10^{-6}}$	$\sqrt{0.979}$
1	$\sqrt{1.2\times10^{-3}}$	$-\sqrt{1.5\times10^{-4}}$	$\sqrt{9.2\times10^{-3}}$	$\sqrt{9.1\times10^{-4}}$
2	$\sqrt{1.1\times10^{-5}}$	$-\sqrt{1.8\times10^{-8}}$	$\sqrt{1.3\times10^{-4}}$	$\sqrt{1.3\times10^{-5}}$

TABLE I. Transition amplitude with a phase modulated Raman laser with $G/2\pi = 6.8\,GHz$, $\Delta/2\pi = 0.7\,GHz$, $E_n = J_n(1.25)$ where J_n is the Bessel function of the first kind of order n.

III. PHASE SHIFT IN A MACH ZEHNDER INTERFEROMETER INDUCED BY THE ADDITIONAL LASER LINES OF A MODULATED RAMAN LASER

In this section, we present the calculation of a Chu-Bordé interferometer using a modulated laser for stimulated Raman transition. This interferometer consists of three Raman laser pulses of duration τ, 2τ and τ ($\Omega_0\tau = \pi/2$) separated in time by T. This interferometer is equivalent to an optical Mach Zehnder interferometer where the first and last pulses act as beam splitters and the second pulse acts as a mirror. The evolution operator of this interferometer is given by:

$$U = U_3 \cdot U_L \cdot U_2 \cdot U_L \cdot U_1 \tag{9}$$

where U_n is the evolution matrix for the n^{th} laser pulse which has been calculated in the previous section and U_L is the free evolution during the time T. The free falling frame will be used for the calculation. The gravitational potential is therefore not included in the free evolution and one has only the internal energy and the kinetic energy :

$$U_L = \sum_n e^{-i\omega_{an}T}|a,n\rangle\langle a,n| + e^{-i\omega_{bn}T}|b,n\rangle\langle b,n| \tag{10}$$

with:

$$\begin{aligned}
\omega_{an} &= \frac{(p+n\hbar\Delta k)^2}{2\hbar M}\\
\omega_{bn} &= \frac{(p+n\hbar\Delta k+\hbar k_{eff})^2}{2\hbar M} + G
\end{aligned} \tag{11}$$

where M is the mass of the atom.

We assume that the atom is initially in the internal state $|a\rangle$ and has a momentum probability amplitude $\varphi(p)$:

$$|\Psi_0\rangle = \int \varphi(p)|a,p\rangle dp \tag{12}$$

The momentum probability amplitude $\varphi(p)$ will be modeled by a gaussian wave packet with a mean momentum p_0, a width given by the temperature of the atoms T_a and centered at the position $z = 0$.

$$\varphi(p) = \frac{1}{(2\pi\, M\, k_B T_a)^{1/4}} \exp\left(-\frac{(p-p_0)^2}{4\, M\, k_B T_a}\right) \tag{13}$$

After the interferometer sequence, the probability to be in the state $|a\rangle$ is given by :

$$P_a = \int |\langle a,p|U|\Psi_0\rangle|^2 dp \tag{14}$$

By inserting equation (12) into the expression of P_a and using the family of quantum states coupled by U (see Fig. 3), one obtains:

$$P_a = \int \left|\sum_n \varphi(p+n\hbar\Delta k)\langle a,p|U|a,p+n\hbar\Delta k\rangle\right|^2 dp \tag{15}$$

We assume now that the atomic coherence length $l_c = h/\sqrt{M\, k_B\, T_a}$ is small compared to the microwave wavelength $2\pi/\Delta k$. This approximation is completely valid for atoms at a temperature of $\sim 1\mu$K ($l_c \sim 0.5\,\mu$m) and with $G/2\pi \lesssim 10\,$GHz ($2\pi/\Delta k \gtrsim 1.5\,$cm). Within this approximation, one can write $\varphi(p+n\hbar\Delta k) \simeq \varphi(p)$ and one obtains :

$$P_a = \int \left|\sum_n \langle a,p|U|a,p+n\hbar\Delta k\rangle\right|^2 |\varphi(p)|^2 dp \tag{16}$$

By decomposing U in free evolution and laser interaction (Eq. (9)), one obtains :

$$P_a = \int |\varphi(p)|^2 |C_{ab} + C_{ba} + C_{aa} + C_{bb}|^2 dp \qquad (17)$$

with:

$$C_{ij} = \sum_{n'',n',n} e^{-i\,(\omega_{jn''}+\omega_{in'})T}\, \langle a,0|U_3|j,n''\rangle\langle j,n''|U_2|i,n'\rangle \langle i,n'|U_1|a,n\rangle \qquad (18)$$

In this expression, C_{ij} represents the probability amplitude of an atom to follow the path $a \to i \to j \to a$ where i,j = a or b. We will consider only the interference between the term C_{ab} and C_{ba}. The interferences with the terms C_{aa} and C_{bb} vanish if the path separation $D = \hbar k_{eff} T/M$ is much bigger than the coherence length of the atoms. This assumption is perfectly verified in practical situation with $T > 1\,\mathrm{ms}$ ($D > 100\,\mu\mathrm{m}$) and atoms at a temperature $T_a \sim 1\mu\mathrm{K}$ ($l_c \sim 0.5\,\mu\mathrm{m}$). We will consider therefore only the interferences between the term C_{ab} and C_{ba} and the expression of P_a becomes:

$$P_a = \int |\varphi(p)|^2\, (|C_{aa}|^2 + |C_{bb}|^2 + |C_{ab}|^2 + |C_{ba}|^2 + C_{ab}\cdot C_{ba}^* + C_{ab}^*\cdot C_{ba})\, dp \qquad (19)$$

By inserting the equations (18), (11) and (8) in the previous expression and by neglecting the phase terms proportional to $\frac{\hbar\Delta k^2}{2M}T$ ($\sim 10^{-6}$ for ^{87}Rb and $T = 48\,\mathrm{ms}$), we obtain :

$$P_a = P_0 + \frac{C}{2} cos((k_{eff}g - \alpha)T^2 + \Delta\varphi) \qquad (20)$$

where P_0 is the mean value of P_a, C is the contrast and $\Delta\varphi$ is the phase shift induced by the additional laser lines.

$$\begin{aligned} P_0 =& \left|\sum_{m,m',m''} t_m^{\frac{\pi}{2}}\, t_{m'}^{\pi}\, t_{m''}^{\frac{\pi}{2}}\, e^{-\theta(m'+2m'')^2}\, e^{i\Delta k(m\,z_A + m' z_C + m'' z_F)}\right|^2 \\ &+ \left|\sum_{m,m',m''} r_m^{\frac{\pi}{2}}\, t_{m'}^{\pi}\, r_{-m''}^{\frac{\pi}{2}}\, e^{-\theta(m'+2m'')^2}\, e^{i\Delta k(m\,z_A + m' z_B + m'' z_D)}\right|^2 \\ &+ \left|\sum_{m,m',m''} t_m^{\frac{\pi}{2}}\, r_{m'}^{\pi}\, r_{-m''}^{\frac{\pi}{2}}\, e^{-\theta(m'+2m'')^2}\, e^{i\Delta k(m\,z_A + m' z_C + m'' z_E)}\right|^2 \\ &+ \left|\sum_{m,m',m''} r_m^{\frac{\pi}{2}}\, r_{-m'}^{\pi}\, t_{m''}^{\frac{\pi}{2}}\, e^{-\theta(m'+2m'')^2}\, e^{i\Delta k(m\,z_A + m' z_B + m'' z_E)}\right|^2 \end{aligned}$$

$$\begin{aligned} \frac{C}{4} e^{i\Delta\varphi} =& \sum_{m,m',m''} t_m^{\frac{\pi}{2}}\, r_{m'}^{\pi}\, r_{-m''}^{\frac{\pi}{2}}\, e^{-\theta(m'+2m'')^2}\, e^{-i\Delta k(m\,z_A + m' z_C + m'' z_E)} \\ &\times \sum_{m,m',m''} r_m^{\frac{\pi}{2}}\, r_{-m'}^{\pi}\, t_{m''}^{\frac{\pi}{2}}\, e^{-\theta(m'+2m'')^2}\, e^{i\Delta k(m\,z_A + m' z_B + m'' z_E)} \end{aligned} \qquad (21)$$

with:

$$\begin{aligned} z_A &= -z_M \\ z_B &= -z_M + \frac{p_0 + \hbar k_{eff}}{M} T + \frac{1}{2} g\, T^2 \\ z_C &= -z_M + \frac{p_0}{M} T + \frac{1}{2} g\, T^2 \\ z_D &= -z_M + \frac{p_0 + \hbar k_{eff}}{M} 2T + \frac{1}{2} g\, (2T)^2 \\ z_E &= -z_M + \frac{2\,p_0 + \hbar k_{eff}}{M} T + \frac{1}{2} g\, (2T)^2 \\ z_F &= -z_M + \frac{p_0}{M} 2T + \frac{1}{2} g\, (2T)^2 \\ \theta &= \frac{\Delta k^2 T^2 k_B T_a}{2M} \end{aligned}$$

In this expression, we constat that the additional laser lines are responsible for a decrease of the interferometer contrast. For practical case, this contrast diminution is small ($\sim 3\%$) and does not affect the performance of the inertial sensor. We find also that the phase of the interferometer is modified. Without additional laser lines, we obtain the classical result $(k_{eff}g - \alpha)T^2$. The presence of the additional lines gives an additional phase shift $\Delta\varphi$ which induces if not corrected an error on the acceleration measurement. This phase shift depends on the parameters of the atom interferometer: Δ, T, p_0, T_a ,E_n , z_M. In the expression (21), one can see that the phase shift has a periodic dependence with the mirror position z_m and the initial velocity p_0/m with respectively a period of $2\pi/\Delta k$ and $2\pi/(\Delta k T)$.

With the expression (21), it is possible to numerically calculate the phase shift induced by the additional laser lines. For a ^{87}Rb atom interferometer with the parameters $T = 48\,\mathrm{ms}$ and $\Delta/2\pi = 0.7\,\mathrm{GHz}$, the phase shift is between $\pm 160\,\mathrm{mrad}$ depending on the mirror position z_M. This phase shift which corresponds to an acceleration of $\pm 4\times 10^{-6}\,\mathrm{m/s^2}$ has to be evaluated precisely in order to reach the state of the art accuracy of $\sim 10^{-8}\,\mathrm{m/s^2}$ [2]. And consequently, the parameters needed in the calculation of the phase shift have to be known precisely.

IV. SIMPLIFIED EXPRESSION OF THE PHASE SHIFT IN THE LIMIT OF LOW TEMPERATURE

It is possible to simplify the expression of the additional phase $\Delta\varphi$ by assuming that the spatial separation

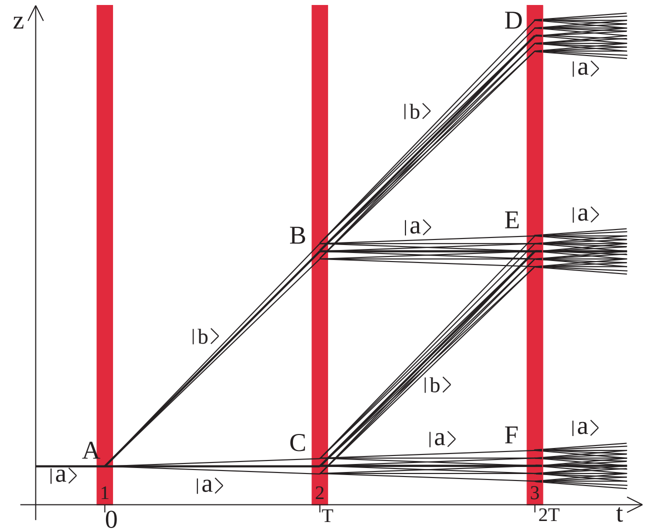

FIG. 4. Spatio-temporal diagram of an atomic Mach-Zehnder interferometer using a modulated Raman laser

between parasite paths $\hbar\Delta kT/m$ is small compared to the atomic coherence length l_c :

$$\theta = \frac{\Delta k^2 T^2 k_B T_a}{2M} \ll 1 \quad (22)$$

For typical Rubidium atom interferometer parameters ($T = 50$ ms, $T_a = 2\,\mu$K, $G/2\pi = 7\,$GHz), $\theta = 0.02$. Within this approximation, the equation (21) can be simplified and one obtains :

$$\Delta\varphi = \varphi_p(z_A) + \varphi_p(z_E) - \varphi_p(z_B) - \varphi_p(z_C) \quad (23)$$

where

$$\varphi_p(z) = \arg\left(\sum_m \Omega_m e^{im\Delta k\, z}\right) \quad (24)$$

In the next section, we will validate our model by comparing the calculated phase shift with experimental measurements.

V. COMPARISON WITH AN EXPERIMENT

Our experimental apparatus consists in a compact atom gravimeter using ^{87}Rb. For this atom, the two ground states used for the Raman transition are $5^2S_{1/2}$ F=1 and F=2 separated in frequency by 6.8 GHz. The laser used in our atom interferometer is similar to the one described in [19]. The laser consists in a DFB laser at 1560 nm frequency doubled in a PPLN crystal. A fiber phase modulator at 1560 nm is used to generate sidebands at 6.8 GHz for the Raman transition. The source of cold atoms is a magneto-optical trap of ^{87}Rb. After a stage of sub-Doppler cooling and a Zeeman selection, the atoms are in the state $F = 1$, $m_F = 0$ at a temperature of 1.75 μK. The atoms interact with a vertical Raman laser retro-reflected on a mirror placed on a passive vibration isolation table. The detuning of the Raman laser compared to the excited state $5^2P_{3/2}$ F'=2 is equal to -0.7 GHz. The interferometer sequence starts 9.5 ms after the beginning of the atoms fall and consists in three pulses of duration 10, 20 and 10 μs separated by a time T which is chosen between 5 and 48 ms. During the interferometer a frequency chirp α is applied to the Raman frequency in order to compensate the Doppler effect coming from the gravitational acceleration. After the interferometer sequence, the relative atomic populations in the states F=2 and F=1 are detected by fluorescence. Interference fringes can be measured by varying the chirp α applied to the Raman frequency. The gravity acceleration is obtained by measuring the relative atomic population on each side of the central fringe ($\alpha_0 = k_{\text{eff}} g$) [17]. In order to validate our theoretical model described in the previous section, we measured the gravity for different times T and for two different distances of the mirror compared to the atoms (see Fig. 5). Each measurement of gravity corresponds to an averaging over 1000 atoms drops. Systematic effects on the gravity measurement depending on T can perturb the comparison theory/experiment of the phase shift due to additional laser lines. The systematic effects depending on k_{eff} (first order light shift, magnetic field) are canceled by alternating the sign of k_{eff} [20]. The only important systematic effect which does not depend on the sign of k_{eff} and has a dependance in T is the two photons light shift [21, 22]. In the result presented, we take into account this effect by subtracting to the data the predicted error induced by the two-photon light shift.

The different parameters used in the calculation of the phase shift induced by the additional lines have been evaluated. The relative intensities of the laser lines of the Raman laser I_n have been measured with an optical Fabry-Perot interferometer at 780 nm. The relative phase of the laser lines are determined by the fact that the laser is phase modulated : $E_n = \sqrt{I_n}$ for $n > 0$ and $E_n = (-1)^n\sqrt{I_n}$ for $n < 0$. By taking into account the two excited states coupled by the laser with two different Clebsch-Gordan coefficients, we obtain for the effective Rabi frequencies :

$$\frac{\Omega_n}{\Omega_0} = \frac{E_{n+1}E_n}{E_1E_0}\frac{\frac{1}{\Delta_2+nG}+\frac{1/3}{\Delta_1+nG}}{\frac{1}{\Delta_2}+\frac{1/3}{\Delta_1}} \quad (25)$$

where Δ_2 (resp. Δ_1) is the Raman detuning compared to the excited state $F' = 2$ (resp. 1). The initial velocity of the atoms is deduced from the gravity and from the delay between the beginning of the atomic fall and the first Raman pulse. The temperature of the atoms has been measured with Raman spectroscopy and is equal to 1.75 μK. In our experiment, we can not evaluate precisely the distance between the atoms and the mirror (z_M). This parameter will be adjustable in our calculation in order to obtain the best fit of our experimental data. For the sec-

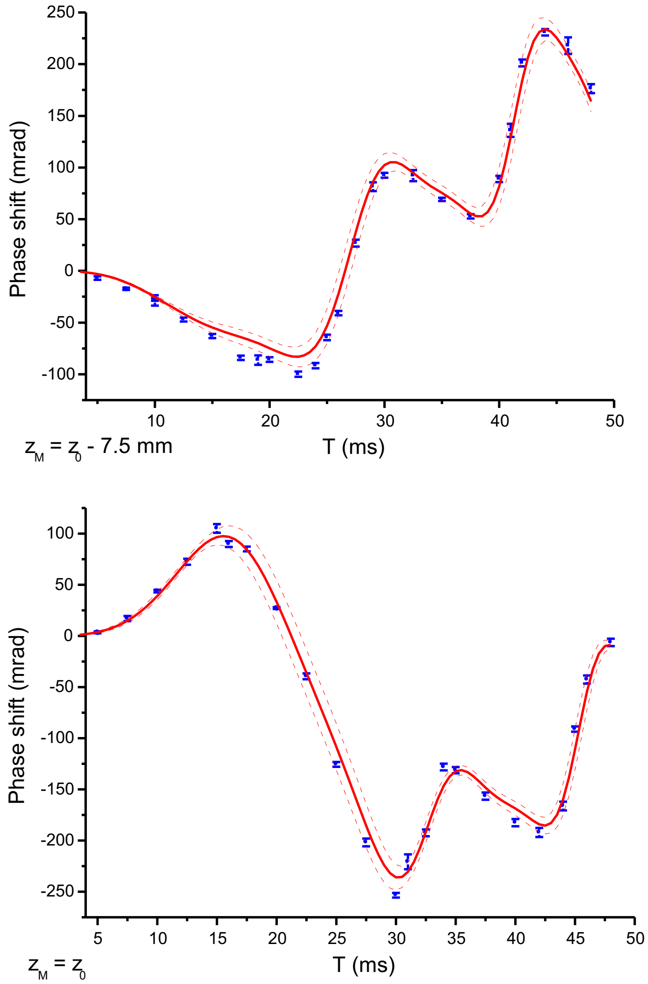

FIG. 5. Measurement of the phase shift versus T for two different mirror distances and comparison with the theory. The points are the experimental measurements. The solid line is the result of the calculation of the phase shift due to the additional laser lines. The only adjustable parameter is the mirror distance z_M for the first graph. The dashed lines correspond to the error of the calculated phase shift due to experimental uncertainties of the interferometer parameters (see Table II).

ond set of measurements, the position of the mirror has been changed by a known quantity (7.5 mm). Therefore the position of the mirror is not adjusted for the second set of measurements. For the experimental uncertainties of the parameters listed in table II, the phase shift can be estimated with an accuracy of 3.1 mrad for the first set of measurements ($z_M = z_0$) and with an accuracy of 10.3 mrad for the second set of measurements ($z_M = z_0 - 7.5\,\mathrm{mm}$). We notice that the uncertainty on the phase shift depends on the mirror distance z_M. To obtain the best accuracy on the acceleration measurement, one has therefore to choose the mirror distance that minimized the uncertainty of the phase shift. In our case, the optimum distance is obtained for $z_M \simeq z_0$ (first set of measurement).

X	δX	$\delta\Delta\varphi$ ($z_M = z_0$)	$\delta\Delta\varphi$ ($z_M = z_0 - 7.5mm$)
z_M	0.3 mm	2.4 mrad	2.9 mrad
p_0/m	5 mm·s^{-1}	1.6 mrad	7.7 mrad
T_a	0.5 μK	0.96 mrad	0.97 mrad
I_1/I_0	0.02	0.18 mrad	3.6 mrad
I_{-1}/I_0	0.02	0.18 mrad	3.7 mrad
I_2/I_0	0.004	0.05 mrad	1.6 mrad
I_{-2}/I_0	0.004	0.13 mrad	1.2 mrad
I_3/I_0	0.0015	0.26 mrad	2.2 mrad
I_{-3}/I_0	0.0015	0.16 mrad	0.20 mrad
total		3.1 mrad (8.3×10^{-8} m/s^2)	10.3 mrad (2.8×10^{-7} m/s^2)

TABLE II. Experimental uncertainties on the parameters of the atom interferometer and resulted uncertainties on the estimation of the phase shift due to additional laser lines ($G/2\pi = 6.8\,\mathrm{GHz}$, $\Delta_2/2\pi = -0.7\,\mathrm{GHz}$, $T = 48\,\mathrm{ms}$, $p_0/M = 9.3\,\mathrm{mm/s}$, $T_a = 1.75\,\mu\mathrm{K}$, $I_{-3,-2,-1,1,2,3}/I_0 = 0, 0.066, 0.614, 0.636, 0.062, 0$).

On Fig. 5, we compare the measured phase shift on our atom interferometer with the calculated phase shift induced by the additional laser lines. We obtain an excellent agreement between theory and experiment. The differences are compatible with the error bars. We validate our model of phase shift calculation up to an accuracy of few mrad. For a more precise validation, one needs a better estimation of the atom interferometer parameters and better gravity measurements. For our compact gravimeter, the uncertainty of the phase shift induced by the additional laser lines limits the accuracy of our acceleration measurement at $8 \times 10^{-8}\,\mathrm{m/s^2}$. This level of accuracy is close to the state of the art of $10^{-8}\,\mathrm{m/s^2}$ and satisfies most of the applications in gravimetry. In the next part, we will show some methods which reduce the inaccuracy coming from the use of a modulated Raman.

VI. METHODS FOR REDUCING THE INACCURACY INDUCED BY ADDITIONAL LASER LINES

If one wants to improve the accuracy limitation induced by the additional laser lines, a first possibility is to have a better control of the interferometer parameters. On table II, one can see that for our gravimeter, the inaccuracy is limited by the uncertainty of the mirror position and of the atoms velocity. The mirror position is not well controlled in our experiment because the mirror is on an anti-vibration table which can freely move vertically. If the entire experiment is placed on a vibration isolation table, the mirror position is fixed and the fluctuation of the mirror distance is only given by the magneto-optical trap position fluctuations which are in the order of $20\,\mu$m [23]. The control of the atoms velocity can be improved with a Raman velocity selection [24]. A control of the mean velocity and the velocity width under 0.1 mm/s can be easily achieved. With these two improvements of the parameters control, one obtains for our compact gravimeter ($T = 48$ms) a phase shift uncertainty of 0.45 mrad corresponding to an acceleration accuracy of $1.2 \times 10^{-8}\,\mathrm{m/s^2}$.

Another solution to improve the accuracy limitation is to find a configuration of the interferometer where the phase shift is reduced. On the approximated expression of the phase shift (23), one can see that the phase shift is equal to zero when $z_A - z_C$ and $z_B - z_E$ are multiple of the microwave wavelength $2\pi/\Delta k$. As z_A, z_B, z_C, z_E are the position of the atoms at different points of the interferometer, this condition is equivalent to say that the distances between the position of the atoms at the moment of the three Raman pulses is multiple of $2\pi/\Delta k$. The parameters of the atom interferometer which satisfy this condition are :

$$T = \sqrt{n' \frac{2\pi}{\Delta k\, g}} \tag{26}$$

$$v_0 = \left(n - \frac{n'}{2}\right) \sqrt{\frac{1}{n'} \frac{2\pi\, g}{\Delta k}} \tag{27}$$

where n, n' are integers.

If this condition is satisfied, the exact phase shift should be minimized. We perform thus the calculation of the exact phase shift for a time $T = \sqrt{2\pi/\Delta k\, g} = 47.3\,\mathrm{ms}$ and for a velocity $v_0 = 0.5\sqrt{2\pi g/\Delta k} = 0.232\,\mathrm{m/s}$ which satisfy the previous condition. For the experimental parameters described in the previous section, one obtains a phase shift between $\pm 2.6\,\mathrm{mrad}$ depending on the mirror position. For the uncertainties of the interferometer parameters given in table II and for the optimum mirror distance, one obtains an uncertainty on the phase shift estimation of $76\,\mu$rad corresponding to an accuracy in acceleration of $2.0 \times 10^{-9}\,\mathrm{m/s^2}$. We obtain the same accuracy of $2.0 \times 10^{-9}\,\mathrm{m/s^2}$ if we use our atom gravimeter in micro-gravity [7]. In this case, the atoms are always at the same position and the phase shift induced by the additional laser lines is also minimized. With better control of the atom interferometer parameters, the use of a Raman modulated laser should not be a limitation to reach the extreme accuracy needed for future space mission [11–13].

VII. CONCLUSION

In this article, we have showed that the additional laser lines present in a Raman laser generated by modulation induce in an atom interferometer a supplementary phase shift. We have presented a model which allows a precise calculation of this phase shift. An excellent agreement between experimental measurements and calculation has validated our model. For our compact gravimeter, the uncertainty of the parameters needed for the phase shift calculation limits the accuracy at the level of $8 \times 10^{-8}\,\mathrm{m/s^2}$. However, with a better control of the interferometer parameters or with particular configurations, the inaccuracy is reduced below $10^{-8}\,\mathrm{m/s^2}$ and does not prevent to reach the state of the art in gravity accuracy.

The calculation presented here can be easily extrapolated to other inertial sensors like gyroscopes or gradiometers. In conclusion, the use of a modulated Raman laser allows to have a laser system more compact and robust and does not prevent to reach an accuracy at a level of $10^{-8}\,\mathrm{m/s^2}$.

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