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时标上一类三阶非线性动力方程的振动准则

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摘要:利用 Riccati 代换给出了时标上一类三阶非线性动力方程的振动准则,并推广了一些已知的三阶动力方程振动性的结果。

关键词:振动性;中立型;动力方程;时标

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Oscillation criteria of third-order nonlinear dynamic equations on a time scale

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Abstract: We present the oscillation criteria of third-order nonlinear neutral dynamic equations on a time scale *T*. We also generalize some known oscillation results of third-order equations.

Key words: oscillation; neutralization; dynamic equation; time scale

1 引言

最近有关时标的研究受到了广泛的关注,比如文献[1-5]都在致力于研究这一课题。但关于时标上三阶动力方程的研究比较少,文献[1]讨论了三阶的非线性动力方程(1.1)的振动性。本文利用 Riccati 代换给出方程(1.1)的一个新的振动准则,从而更加完善了[1]中的结果。

$$\{a(t)(|[r(t)(y(t)+p(t)y(t-\tau))^{\Delta}]^{\Delta}|^{\alpha-1}[r(t)(y(t)+p(t)y(t-\tau))^{\Delta}]^{\Delta})^{\gamma}\}^{\Delta}+q(t)f(t,y(t-\delta))=0 \quad (1.1)$$

其中 $t\in T_0=[t_x,\infty)\cap T$,$T$ 为任意时标,且 $\sup T=\infty$。设 γ,α 是正整数,τ 和 δ 是正常数,且 $\tau(t)=t-\tau<t$ 及 $\delta(t)=t-\delta<t$,$\tau(t):T\to T$ 及 $\delta(t):T\to T$,$t\in T$,$f:T\times R\to R$ 满足 $uf(t,u)>0,u\neq 0$。$m(t)$ 是时标 T 上正的且连续的函数,并 $|f(t,u)|\geqslant m(t)|u|^{\beta}$,其中 $\beta>0$。且令

$$x(t)=y(t)+p(t)y(t-\tau),\ R(t)=a(t)([r(t)(x(t))^{\Delta}]^{\Delta})^{\alpha\gamma},\ \varphi(t,t_1)=\frac{\int_{t_1}^{t}\left(\frac{1}{a(s)}\right)^{\frac{1}{\alpha\gamma}}\Delta s}{r(t)},$$

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$\psi(t,t_1)=\int_{t_1}^{t}\frac{\int_{t_1}^{u}\left(\frac{1}{a(s)}\right)^{\frac{1}{\alpha\gamma}}\Delta s}{r(u)}\Delta u Q(t)=m(t)q(t)(1-p(t-\delta))^{\beta}$, $\rho(t)=\begin{cases}e_1, \beta<\alpha\gamma\\ 1, \beta=\alpha\gamma\end{cases}$。

设以下条件在本文中成立：

$$a(t),r(t),p(t),q(t)\in C_{rd}(T,R^+),\ p(t)<1; \qquad (H_1)$$

$$\int_{t_0}^{\infty}\left(\frac{1}{a(t)}\right)^{\frac{1}{\alpha\gamma}}\Delta t\to\infty,\ \int_{t_0}^{\infty}\left(\frac{1}{r(t)}\right)\Delta t\to\infty,\ \int_{t}^{\infty}\frac{1}{r(v)}\int_{v}^{\infty}\left(\frac{1}{a(u)}\right)^{\frac{1}{\alpha\gamma}}\left(\int_{u}^{\infty}m(s)q(s)\Delta s\right)^{\frac{1}{\alpha\gamma}}\Delta u\Delta v\to\infty。 \qquad (H_2)$$

2 主要结果

引理 2.1[2] 设(1.1)在$[t_0,\infty)_T$上有正解$y(t)$，则存在充分大的$T\in[t_0,\infty)_T$，使得：
$x(t)>0$, $\{a(t)(|[r(t)(x(t))^{\Delta}]^{\Delta}|^{\alpha-1}[r(t)(x(t))^{\Delta}]^{\Delta})^{\gamma}\}^{\Delta}<0$, $[r(t)(x(t))^{\Delta}]^{\Delta}>0$,
$x^{\Delta}(t)>0, t\in[T,\infty)$ 或 $x^{\Delta}(t)<0, t\in[T,\infty)$。

引理 2.2[2] 设(1.1)在$[t_0,\infty)_T$上有正解$y(t)$且在$[t_1,\infty)$上有$x^{\Delta}(t)<0$，则有$\lim\limits_{t\to\infty}y(t)=0$。

引理 2.3[2] 设(1.1)在$[t_0,\infty)_T$上有正解$y(t)$且在$[t_1,\infty)$上有$x^{\Delta}(t)>0$，则

$$x^{\Delta}(t)\geqslant(R(t))^{\frac{1}{\alpha\gamma}}\varphi(t,t_1),\ x(t)\geqslant(R(t))^{\frac{1}{\alpha\gamma}}\psi(t,t_1)。$$

定理 2.1 $\limsup\limits_{t\to\infty}\int_{t_1}^{t}\{Q(s)\Phi(s)-M(s)\}\Delta s\to\infty$，其中

$M(t)=\frac{(\lambda-1)}{\lambda^{1+\frac{1}{\beta}}}\frac{\{\Phi^{\Delta}(t)\}^{1+\frac{1}{\beta}}[2^{1-\beta}(\mu(t-\delta))^{\beta-1}\Phi(t)]^{-\frac{1}{\beta}}}{\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1}}$，$\alpha\gamma\geqslant\beta,\beta\geqslant1$，则(1.1)的解振动或渐进趋于零。

证明 设(1.1)没有振动解，不妨令$y(t)$是(1.1)的一个最终正解，则由引理2.1，
(i) $x^{\Delta}(t)<0, t\geqslant t_1$，由引理2.2知(1.1)的每一个解渐近趋近于零；
(ii) $x^{\Delta}(t)>0, t\geqslant t_1$，令$w(t)=\frac{\Phi(t)R(t)}{x^{\beta}(t-\sigma)}$，且由$y(t)\geqslant(1-p(t))x(t)$有

$$w^{\Delta}(t)\leqslant-Q(t)\Phi(t)+\frac{w^{\sigma}(t)\Phi^{\Delta}(t)}{\Phi^{\sigma}(t)}-\frac{R^{\sigma}(t)\Phi(t)(x^{\beta}(t-\delta))^{\Delta}}{(x^{\beta}(t-\delta))^{\sigma}x^{\beta}(t-\delta)};$$

由$x^{\beta}-y^{\beta}\geqslant2^{1-\beta}(x-y)^{\beta}$, $\beta\geqslant1$，有$(x^{\beta}(t-\delta))^{\Delta}\geqslant2^{1-\beta}(\mu(t-\delta))^{\beta-1}(x^{\Delta}(t-\delta))^{\beta}$。

再由引理2.1，引理2.2以及引理2.3有$\frac{x^{\Delta}(t-\delta)}{x^{\sigma}(t-\delta)}\geqslant\frac{\varphi(t-\delta,t_1)w^{\sigma}(t)(R^{\sigma}(t))^{\frac{\beta-\alpha\gamma}{\alpha\gamma}}\psi^{\sigma}(t-\delta,t_1)}{\Phi^{\sigma}(t)}$，

则有

$$w^{\Delta}(t)\leqslant-Q(t)\Phi(t)+\frac{w^{\sigma}(t)\Phi^{\Delta}(t)}{\Phi^{\sigma}(t)}-\frac{2^{1-\beta}(\mu(t-\delta))^{\beta-1}(w^{\sigma}(t))^{1+\beta}\Phi(t)}{(\Phi^{\sigma}(t))^{\beta+1}}\times(\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1})^{\beta}。$$

令$1+\beta=\lambda$，$A=[2^{1-\beta}(\mu(t-\delta))^{\beta-1}\Phi(t)(\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1})^{\beta}]^{\frac{1}{\lambda}}\frac{w^{\sigma}(t)}{\Phi^{\sigma}(t)}$，

$$B=\left\{\frac{\Phi^{\Delta}(t)}{\lambda\Phi^{\sigma}(t)}\left[\frac{2^{1-\beta}(\mu(t-\delta))^{\beta-1}\Phi(t)(\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1})^{\beta}}{(\Phi^{\sigma}(t))^{\lambda}}\right]^{\frac{1}{-\lambda}}\right\}^{\frac{1}{\lambda-1}},$$

利用不等式$A^{\lambda}-\lambda AB^{\lambda-1}+(\lambda-1)B^{\lambda}\geqslant0, \lambda>1$，有

$$\frac{w^{\sigma}(t)\Phi^{\Delta}(t)}{\Phi^{\sigma}(t)}-\frac{2^{1-\beta}(\mu(t-\delta))^{\beta-1}(w^{\sigma}(t))^{1+\beta}\Phi(t)}{(\Phi^{\sigma}(t))^{\beta+1}}(\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1})^{\beta}$$

$$\leqslant\frac{(\lambda-1)}{\lambda^{1+\frac{1}{\beta}}}\left\{\frac{\Phi^{\Delta}(t)}{\Phi^{\sigma}(t)}\right\}^{1+\frac{1}{\beta}}\left[\frac{2^{1-\beta}(\mu(t-\delta))^{\beta-1}\Phi(t)(\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1})^{\beta}}{(\Phi^{\sigma}(t))^{\lambda}}\right]^{-\frac{1}{\beta}},$$

则 $w^{\Delta}(t) \leqslant -Q(t)\Phi(t) + \frac{(\lambda-1)}{\lambda^{1+\frac{1}{\beta}}}\frac{\{\Phi^{\Delta}(t)\}^{1+\frac{1}{\beta}}[2^{1-\beta}(\mu(t-\delta))^{\beta-1}\Phi(t)]^{-\frac{1}{\beta}}}{\varphi(t-\delta,t_1)\rho(t)(\psi^{\sigma}(t-\delta,t_1))^{\beta-1}} = -Q(t)\Phi(t) + M(t)$。将上式从 t_1 到 t 积分,$w(t) \leqslant w(t_1) - \int_{t_1}^{t}[Q(s)\Phi(s) - M(s)]\Delta s$,此与假设矛盾,定理得证。

参考文献:

[1]王军华,王俊俊.时标上一类三阶中立型动力方程的振动性[J].郑州大学学报(工学版),2010,31(4):121-123.

[2]SAKER S H. Oscillation criteria of second-order half-linear dynamic equations on time scales[J]. J Comp Appl Math,2005,177(2): 375-387.

[3]HASSAN T S. Oscillations of third order delay dynamic equations on time scales [J]. Methematical and Computer Modelling, 2009,49 (7-8): 1573-1586.

[4]SAKER S H. AGARWAL R P, O'REGAN D. Oscillation of second-order damped dynamic equations on time scales[J]. Journal of Mathematical Analysis and Applications, 2007,330 (2):1317-1337.

[5]CHEN D. Oscillations and asymptotic behavior for nth-order nonlinear neutral delay dynamic equations on time scales[J]. Acta Appl Math,2008,109(3):703-719.

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[3]SUN Y P. Positive solutions of singular third-order three-point boundary value problem[J]. Journal of Mathematical Analysis and Applications,2005,306(3):589-603.

[4]SANG Y,SU H. Several suffcient conditions of solvability for a nonlinear higher-order three-point boundary value problem with all derivatives[J]. Applied Mathematics and Computation,2009,215(3):1154-1160.

[5]ANDERSON D R. Green´s function for a third-order generalized right focal problem[J]. Journal of Mathematical Analysis and Applications,2003,288(1):1-14.

[6]MA R. Positive solutions of nonlinear m-point boundary value problems[J]. Computers & Mathematics with Applications,2001,42(6):755-756.

[7]BAI C,FANG J. Existence of multiple positive solutions for nonlinear m-point boundary value problems[J]. Journal of Mathematical Analysis and Applications,2003,81(1):76-85.

[8]GE W,REN J. Fixed point theorems in double cones and their applications to nonlinear boundary value problems[J]. Chinese Annals of Mathematics,2006,27(2):155-168.

[9]JI D,FENG M,GE W. Multiple positive solutions for multipoint boundary value problems with sign chaning nonlinearity[J]. Applied Mathematics and Computation,2008,196(2):515-520.

[10]GUO D,LAKSHMIKANTHAN V. Nonlinear Problem in Abstract Cones[M]. San Diego:Academic Press,1988.