SPICE model of memristive devices with threshold

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Abstract—**Although memristive devices with threshold voltages are the norm rather than the exception in experimentally realizable systems, their SPICE programming has not yet been suggested. Here, we show how to implement such systems in the SPICE environment. We expect this implementation to find widespread use in circuits design and testing.**

Introduction: In the last few years, circuit elements with memory, namely, memristive [1], memcapacitive and meminductive [2] systems have attracted considerable attention from different disciplines due to their capability to store and manipulate information on the same physical platform [3]. However, when combined into complex circuits, progress in this field significantly relies on the available tools at our disposal. One such tool is the SPICE simulation environment, commonly used in circuit simulations and testing. While several SPICE models of memristive [4], [5], [6], [7], [8], memcapacitive [6] and meminductive [6] elements are already available, they typically [4], [5], [6], [7], [8] rely on physical models without a threshold (see, e.g., Refs. [9], [10]).

Threshold-type switching is instead an extremely important common feature of memristive devices (for examples, see Ref. [3]) and, due to physical constraints, likely to be common in memcapacitive and meminductive elements as well. Indeed, it is the threshold-type switching which is responsible for non-volatile information storage, serves as basis for logic operations [11], [12], etc., and therefore, it can not be neglected. In the present paper we introduce a SPICE model for a memristive device with threshold voltage that has been proposed by the present authors [13]. Using this type of memristive devices, we have already demonstrated and analyzed several electronic circuits including a learning circuit [13], memristive neural networks [14], logic circuits [12], analog circuits [15] and circuits transforming memristive response into memcapacitive and meminductive ones [16]. These previous results thus demonstrate the range of applicability of the selected physical model. As a consequence, we expect its SPICE implementation to find numerous applications as well.

SPICE model: The memristive devices with threshold that we consider are those described by [13]

$$I = X^{-1}V_M, \tag{1}$$
$$\dot{X} = (\beta V_M + 0.5(\alpha - \beta)[|V_M + V_t| - |V_M - V_t|]) \times$$
$$\theta\left(\frac{X}{R_{on}} - 1\right)\theta\left(1 - \frac{X}{R_{off}}\right) \equiv f(V_M)\theta(..)\theta(..), \tag{2}$$

where V_t is the threshold voltage, R_{on} and R_{off} are limiting values of the memristance $R_M \equiv X$, and the θ-functions

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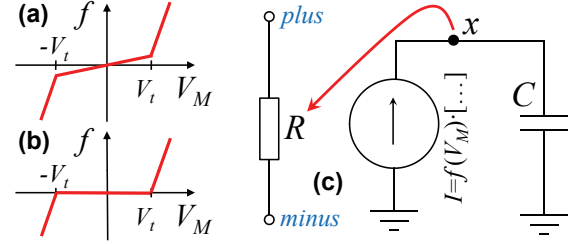e-mail: diventra@physics.ucsd.edu.

Fig. 1. Sketch of the function $f(V_M)$ for (a) $\alpha > 0$ and $\beta > 0$ and (b) $\alpha = 0$ and $\beta > 0$. (c) Schematic of the SPICE model. The memristive device functionality is organized as a subcircuit consisting of a behavioral resistor R, current source ↑ and capacitor C. The voltage across the capacitor (at the node x) defines the resistance of R.

(step functions) symbolically show that the memristance can only change between R_{on} and R_{off}. The important model parameters are the coefficients α and β that characterize the rate of memristance change at $|V_M| < V_t$ and $|V_M| > V_t$, respectively. These two coefficients define the slopes of the $f(V_M)$ curve below and above the threshold (see Fig. 1). When $\alpha = 0$ (Fig. 1(b)), the device state changes only if $|V_M| > V_t$.

The SPICE model for these devices is formulated following the general idea of Ref. [4]: the memristive system is realized as a sub-circuit combining a behavioral resistor R (a resistor whose resistance can be specified by an expression), a current source ↑, and a capacitor C. Table I presents the code of the sub-circuit. Its second line (*Bx ...*) defines the current source with the current specified through ternary functions. (A ternary function is defined in the code as $a ? b : c$, which means "IF a, THEN b, ELSE c" [17].) The purpose of these functions is to limit R_M between R_{on} and R_{off}. The third line of the code in Table I specifies the capacitor C (*Cx ...*) with an initial condition. The fourth line (*Rmem ...*) defines the behavioral resistor whose resistance takes the same numerical value as the voltage across the capacitor. The next line (*.func ...*) provides the function f according to Eq. (2).

Example: Let us consider a memristive device with threshold directly connected to a sinusoidal voltage source of 5V amplitude is shown in Fig. 2. The circuit simulations were performed as a transient analysis of the circuit taking into account initial conditions (the *uic* option of *.tran*) within the *ngspice* circuit simulator. Basically, the simulation results presented in Fig. 2 are consistent with the general properties of memristive devices [1], [2]. In particular, the $I - V$ curves are pinched hysteresis loops whose area reduces with increasing applied voltage frequency. Moreover, in the inset of Fig. 2 we plot the resistance R_M as a function of V that clearly demonstrates that R_M changes only if $|V| > V_t = 4.6$V.

Conclusions: We have developed and tested a SPICE model of memristive devices with threshold voltage. In this model,

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.subckt memristor plus minus PARAMS: Ron=1K Roff=10K Rinit=5K alpha=0 beta=1E13 Vt=4.6
Bx 0 x I='(f1(V(plus)-V(minus))>0) && (V(x)<Roff ? {f1(V(plus)-V(minus))}: (f1(V(plus)-V(minus))<0) && (V(x)>Ron) ? {f1(V(plus)-V(minus))}: {0}'
Cx x 0 1 IC={Rinit}
Rmem plus minus r={V(x)}
.func f1(y)={beta*y+0.5*(alpha-beta)*(abs(y+Vt)-abs(y-Vt))}
.ends
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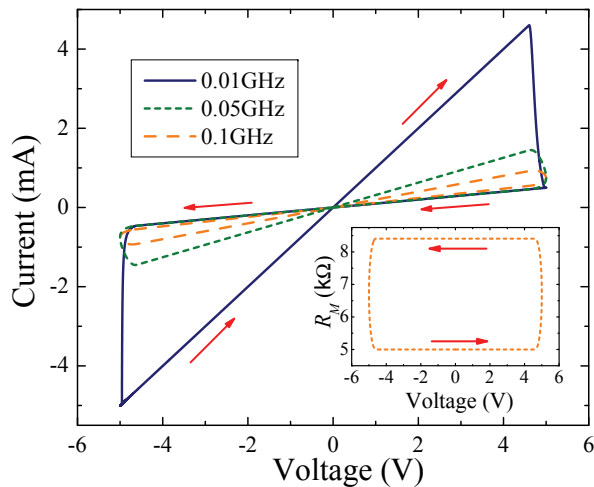
TABLE I
SPICE CODE IMPLEMENTING EQS. (1),(2).

Fig. 2. Frequency-dependent hysteresis loops for a threshold type memristive device. The simulations parameters are: the applied voltage amplitude $V_0 = 5V$, $R_{on} = 1k\Omega$, $R_{off} = 10k\Omega$, $R_M(t = 0) = 5k\Omega$, $\alpha = 0$, $\beta = 10^{10}k\Omega/(V\ s)$, $V_t = 4.6V$. The inset (calculated at 0.1GHz sine voltage frequency) demonstrates that the memristance changes only when the absolute value of the applied voltage exceeds the threshold voltage V_t.

the limiting conditions for the memristance are realized using ternary functions which adhere more closely to the actual physical situation, compared with the window functions approach previously suggested [10]. The memristive device is realized as a sub-circuit consisting of several elements. While the present physical model of the memristive device is based on a single internal state variable, X, it can be easily generalized to more complex physical models involving several internal state variables. Finally, we note that threshold models of memcapacitive and meminductive systems in SPICE can be implemented in a similar way.

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