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Integrating production, inventory and maintenance planning for a parallel system with dependent components

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ABSTRACT

This paper deals with the problem of integrating preventive maintenance and tactical production planning, for a production system composed of a set of parallel components, in the presence of economic dependence and common cause failures. Economic dependence means that performing maintenance on several components jointly costs less money and time than on each component separately. Common cause failures correspond to events that lead to simultaneous failure of multiple components due to a common cause. We use the β-factor model to represent common cause failures. This means that we assume two possible causes for system failure: the independent failure of single components, and the simultaneous common cause failure of all components. The suggested preventive maintenance is a *T*-age group maintenance policy in which components are cyclically renewed all together. Furthermore, between the periodic group replacements, minimal repairs are performed on failed components. We are given a set of products that must be produced by this parallel system in lots during a specified finite planning horizon. The objective is to determine an integrated lot-sizing and preventive maintenance strategy of the system that will minimize the sum of preventive and corrective maintenance costs, setup costs, holding costs, backorder costs and production costs, while satisfying the demand for all products over the entire horizon. Numerical examples are used to illustrate the proposed approach.

1. Introduction

It is well-known that production planning and preventive maintenance (PM) planning are mutually in conflict [1–5]. Because these activities are typically performed sequentially in practice, production and maintenance plans are often not optimal with respect to the objective minimizing the total maintenance and production cost. The integration of PM and production may reduce the total expected cost. Nourelfath et al. [6] have developed an integrated production and PM planning model for multi-state systems (MSS), which are composed of a set of binary-state components. They assumed that these components are stochastically and economically independent. Stochastic independence means that the condition of components does not influence the lifetime distribution of other components. Economic independence implies that the cost and the time of joint maintenance of a group of components are equal to the total cost and the time of individual maintenance of these components. The objective of the present paper is to extend the model in [6], by taking into account the presence of economic dependence and common cause failures (CCF). We consider that the system and its components are subject to both independent and CCF.

Economic dependence is common in most production systems. It can be either positive or negative. Positive economic dependence implies that costs can be saved when several components are jointly maintained instead of separately. Negative economic dependence between components occurs when maintaining components simultaneously is more expensive than maintaining components individually. We assume a positive dependence in this paper. That is, as the components of the parallel system are identical, it is possible to obtain savings of time and cost by grouping preventive maintenance. The term economies of scale is used to indicate that combining such preventive maintenance activities is cheaper than performing them on components separately. This means that the maintenance cost and time per component decrease with the number of maintained components. Economies of scale can result for example from preparatory activities that can be shared when several components are maintained simultaneously. Costs can be saved when preventive maintenance activities on different components are executed

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simultaneously. In fact, PM is planned in advance such that preparatory costs can be saved by simultaneous execution, and execution of a group of activities requires only one preparatory activity. A survey on economic dependence models can be found in [7]. This survey is emphasized on classifications and characteristics of maintenance policies. Wang [8] compared various existing maintenance policies for multi-component systems, and reviewed many papers dealing with group maintenance and opportunistic maintenance policies. There are three main categories of group replacement policy. A T-age policy (see, e.g., [9]) calls for replacement every T units of time. An m-failure policy (see, e.g., [10, 11]) calls for replacing the system at the time of the mth failure. A policy that combines characteristics of both of the above classes is the (m, T) policy, which calls for replacement at the time of the mth failure or at time T whichever occurs first (see [12, 13]).

Stochastic dependence occurs if the state of a component influences the lifetime distribution of other components, or if there are causes outside the system which bring about simultaneous failures and hence correlate the lifetimes, so-called common cause failures (CCF). A large amount of papers have been also devoted to characterize common cause failures and more generally failure dependencies. Vaurio [14, 15] applied the joint probability to measure CCF by evaluating the probabilistic correlation of the failures. Implicit methods that follow minimum-cut theory and probability equations, and explicit expressions derived from fault trees, are discussed. Watanabe et al. [16] used dynamic fault trees to simulate the failure process of a nuclear power plant, where the probabilistic correlation of the CCF is calculated by using simulation. Levitin [17] adapted the universal generating function technique of multi-state system reliability analysis to take into account CCF in system reliability estimation. Other studies have concerned the redundant dependency. Kotz et al. [18] have introduced the concept of 'quadrant dependent' to measure the effect of adding a component. The correlation effects between components are investigated upon some bivariate failure distributions. Barros et al. [19] considered a two-component parallel system, in which the failure rate of the operating component will increase, due to the additional loading induced by the other component's failure.

Other papers dealing with joint optimization include [22–27]. In [22], the authors formulated the joint redundancy and replacement schedule optimization problem generalized to multistate system. In [23], the authors considered a preventive maintenance optimization problem for multi-state systems, for which the reliability is defined as the ability to satisfy given production demand. The authors of [24] presented a value-driven maintenance planning approach and applied it to approach to a production plant. In [25], the authors have shown the importance of linking maintenance and safety risks, and presented an approach to maintenance optimization where safety issues are important. The authors of [26] presented a dynamic modeling of the trade-off between productivity and safety in critical engineering systems. In [27], the authors presented an overall model for maintenance optimization. They developed an approach for identifying the optimal maintenance schedule for the components of a production system. Safety, health and environment objectives, maintenance costs and costs of lost production are all taken into account, and maintenance is thus optimized with respect to several objectives.

The existing approaches integrating PM and production planning have either considered the production system as one binary-state component [1–5], or overlooked the inclusion of economic dependence and CCF in the modeling of the system capacity [6]. However, because of economic dependence, there is often a great potential for cost savings by implementing a group maintenance policy. This happens every time costs (or times) can be saved when several components are jointly maintained instead of separately (in that case economies of scales can be obtained). On the other hand, recognition of CCF for optimal integrated planning is a crucial issue due to the significant impact these failures can have on the overall system capacity used to meet the required demand. Thus, for many planning problems, a modeling approach incorporating CCF in the system capacity evaluation is expected to be more realistic. For some industrial systems, an approach that considers CCF when evaluation the system capacity should, not only be the preferred approach but also the correct one. If we ignore CCF in our model, the estimated capacity will be overestimated. Since the available capacity is less than the expected one, the production plan could be unmet. This could increase the cost of backorders and may cause bad service levels. In addition, if we want to repair failed components after CCF, the maintenance time and cost may increase, for example because of the unavailability of enough maintenance crew to repair more than one component. The occurrence of a CCF results in a peak in manpower needs. Manpower restrictions may even be violated and additional labor needs to be hired, which is costly. Taking into account CCF means here that the capacity evaluation model includes CCF, and enough resources are available to deal with failures of multiple components.

Our model suggests a joint preventive maintenance and production planning model. At the production planning level, the production planning problem consists in a multi-product capacitated lot-sizing problem. At the preventive maintenance planning level, the maintenance policy suggests possible preventive replacements at the beginning of each production planning period, and minimal repair at machine failure. It is assumed that spare parts and maintenance crew are available at replacement times. This assumption can be justified by the fact that preventive maintenance is plannable. That is, new components can be ordered in time and also enough maintenance crew is available at the planned maintenance replacement times.

The remainder of the paper is structured as follows. Section 2 is devoted to how production and maintenance activities affect each other, in terms of time and cost. Section 3 presents an integrated production and PM planning model, which takes into account economic dependence and CCF. Then, a method is proposed in Section 4 to evaluate the times and the costs of preventive maintenance and minimal repair, and the average production system capacity in each period. Our solution method is applied to an illustrative example in Section 5, and conclusions are given in Section 6.

2. Integrating maintenance and production decisions

Preventive maintenance and production are mutually in conflict for two main reasons. First, since the time taken by PM activities could be used for production, production managers usually fail to realize the importance of PM. Second, delaying PM for production may increase the probability of failures, while maintenance managers try to reach high equipment availability. In practice, production and maintenance planning activities are usually performed independently. Therefore, it cannot be guaranteed that the obtained plans are optimal with respect to the objective minimizing the total maintenance and production cost. Better solutions could be obtained when maintenance planning is integrated with manufacturing activities. The integration of PM and production decisions may reduce not only the interruption time, but the total expected cost also. Depending on the production environments, this integration could be done either at the scheduling operational level (short-term) or at the tactical level (mid-term). The time horizons may vary for each planning level depending on the industry. Typical values are one week or less for operational planning, and one month or more for tactical planning.

For equipment that is not highly reliable, PM schedules may be weekly or even daily. In these environments, it is necessary to use a job-to-job PM planning tool. This issue has been dealt with in Ref. [2], where the authors have developed a model integrating job sequencing and PM decisions. Considering that preventive maintenance, and repair affect both available production time, and the probability of machine failure, the model in Ref. [2] coordinates PM planning decisions with single-machine scheduling decisions so that the total expected weighted completion time of jobs is minimized.

For highly reliable equipment, PM schedules may be performed at a lower frequency (monthly, quarterly or even semi-annually). As a result, PM activities should be integrated with tactical production planning. The objective of this paper is to develop an integrated production and PM planning model dealing with tactical aggregate production planning decisions. At the tactical level, it is often dealt with items from a product family viewpoint. A product family is defined as a grouping of end items that share a common manufacturing set-up. Set-up is the process of actually converting the equipment. This may be achieved by adjusting the equipment to correspond to the next product family or by changing non-adjustable "change parts" to accommodate the product family. As already suggested by the Total Productive Maintenance approach [21], the successful implementation of a maintenance program requires that its tasks be considered as parts of the production plan rather than as interruptions to that plan. Within this in mind, we consider that preventive maintenance activities are performed by machine operators responsible of set-up activities. The set-up activities are achieved at the planning periods. Thus, knowing that the production and maintenance requirements share common labor and time resources, PM tasks can be advantageously integrated to these set-up activities at the beginning of planning periods. In this case, because PM tasks are executed by machine operators responsible of set-up activities, the time and the cost of PM actions will be clearly lower than interrupting production to PM tasks during a production cycle. The next section presents the proposed integrated model for tactical production and PM.

3. The mathematical model

3.1. The system description

We consider a production system containing n parallel components, also called machines. The system fails if all its components fail. We assume that the states of the components are binary (i.e., either good or failed). Each component i ($i=1, 2, \ldots, n$) is characterized by its own nominal performance. Failures of some components lead to the degradation of the entire system performance g_i. Therefore, the system can be seen as multi-state: it can perform its task with various distinguished levels of performance rates, ranging from perfect functioning up to complete failure. The performance measure used for this system is the capacity, which is represented by the production rate (i.e., number of produced items per time unit). Common cause failures must be taken into account in the system capacity computation.

The system produces a set of items or products P during a given planning horizon H including T periods. All periods have the same fixed length L. For each product $p \in P$, a demand d_{pt} is to be satisfied at the end of period t ($t=1, 2, \ldots, T$).

3.2. The maintenance policy

As economic and stochastic dependences are considered, the best maintenance policy is not one of considering each component separately and maintenance decisions will not be independent. We use the β-factor model to represent common cause failures. This means that we assume two possible causes for system failure: the independent failure of single components, and the simultaneous common cause failure of all components. Planned preventive maintenance and unplanned corrective maintenance can be performed on components. The suggested PM is a T-age group maintenance policy in which components are cyclically renewed all together. This means that PM is assumed to restore periodically all components to "as good as new" conditions, so that the age of components becomes zero. The replacement interval is αL with $\alpha=1, 2, \ldots, T-1$. That is, the components can be replaced at times αL, $2\alpha L$, etc. The cyclic PM actions coincide with the planning periods, and a PM replacement can be cyclically performed at the beginning of any planning period, except for the first period (where the components are considered as new), and at the end of the last period. The PM decision variable is defined by α. Furthermore, between the periodic group replacements, minimal repairs are performed at failures. If any single component fails, it is minimally repaired; and if all components fail (due to a CCF), they are minimally repaired all together. Minimal repair means that a component is restored to an operating condition, without altering its age.

The expected maintenance cost is the sum of preventive and corrective maintenance costs. The average production capacity of the system during a period t depends on α, and it is denoted by $G_t(\alpha)$. All required corrective and preventive maintenance times and costs are assumed to be known. The expected maintenance cost during the planning horizon is the sum of preventive and corrective maintenance costs: it depends on α and it is denoted by $CM(\alpha)$. Similarly, the expected maintenance time is denoted by $TM(\alpha)$.

3.3. The integrated model

For such a system, the objective is to develop an integrated production and preventive maintenance planning model. The production planning part corresponds to a multi-product capacitated lot-sizing problem. At this level, the decisions involve determination of quantities of items (lot sizes) to be produced in each period. Lot-sizing is one of the most important problems in production planning. Almost all manufacturing situations involving a product-line contain capacitated lot-sizing problems, especially in the context of batch production systems. The setting of lot sizes is usually considered as a decision related to tactical planning, which is a medium-term activity. In aggregate planning, the lot sizing models are extended by including labor resource decisions [1, 20]. Tactical planning bridges the transition from the strategic planning level (long-term) to the operational planning level (short-term).

The objective function in the integrated problem is a non linear equation minimizing the sum of maintenance and production costs, while satisfying the demand for all products over the entire horizon. The constraints are related to the dynamics of the inventory and the backorder, the capacity, the setup, and the available total maintenance time.

Before introducing the model, we present the following notations:

h_{pt} inventory holding cost per unit of product p by the end of period t

b_{pt} backorder cost per unit of product p by the end of period t
s_{pt} fixed set-up cost of producing product p in period t
π_{pt} variable cost of producing one unit of product p in period t
x_{pt} quantity of product p to be produced in period t
I_{pt} inventory level of product p at the end of period t
B_{pt} backorder level of product p at the end of period t
Set_{pt} binary variable, which is equal to 1 if the setup of product p occurs at the end of period t, and 0 otherwise
d_{pt} demand to be satisfied at the end of period t
G_t average production capacity of the system during a period t.

The integrated model is mathematically formulated as follows:

$$\text{minimize} \sum_{p\in P}\sum_{t=1}^{T}(h_{pt}I_{pt}+b_{pt}B_{pt}+\pi_{pt}x_{pt}+s_{pt}Set_{pt})+CM(\alpha), \quad (1)$$

$$\textit{subject to } I_{pt}-B_{pt}=I_{pt-1}-B_{pt-1}+x_{pt}-d_{pt}, p\in P, t=1,2,\ldots,T, \quad (2)$$

$$x_{pt}\le\left(\sum_{q\ge t}d_{pq}\right)y_{pt}, \quad p\in P,\ t=1,2,\ldots,T, \quad (3)$$

$$\frac{\sum_{p\in P}x_{pt}}{L}\le G_t(\alpha),\ t=1,2,\ldots,T \quad (4)$$

$$TM(a)\le TM_0, \quad (5)$$

$$x_{pt},I_{pt},Bp\in\mathbb{N}; Set_{pt}\in\{0,1\}, \quad \alpha=1,2,\ldots,T-1. \quad (6)$$

The objective function (1) consists of a total maintenance costs $CM(\alpha)$, a total holding cost of the inventory $\sum_{p\in P}\sum_{t=1}^{T}h_{pt}I_{pt}$, a backorder cost (backlogs are allowed) $\sum_{p\in P}\sum_{t=1}^{T}b_{pt}B_{pt}$, a total production cost $\sum_{p\in P}\sum_{t=1}^{T}\pi_{pt}x_{pt}$, and a total setup cost $\sum_{p\in P}\sum_{t=1}^{T}s_{pt}Set_{pt}$. The first constraint (2) relates inventory or backorder at the start and end of period t to the production and demand in that period. There is no optimal solution where $I_{pt}>0$ and $B_{pt}>0$ simultaneously, since the objective function can be improved by decreasing both I_{pt} and B_{pt} until one becomes zero. Eq. (2) ensures simply that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. The second constraint (3) forces $x_{pt}=0$ if $Set_{pt}=0$ and frees $x_{pt}\ge 0$ if $Set_{pt}=1$. In Eq. (3), the quantity $(\sum_{q\ge t}d_{pq})$ is an upper bound of x_{pt}. Eq. (4) corresponds to the available production capacity constraint. Eq. (5) specifies the available total maintenance time constraint when available.

To solve the integrated model (1)–(6), a method is presented in the next section to evaluate the values of $CM(\alpha)$, $TM(\alpha)$, and $G_t(\alpha)$.

4. Evaluation method

4.1. Evaluation of times and costs

In order to evaluate the expected maintenance costs and times, we need to estimate the number of failures for each component i, and the number of cyclic preventive replacements for the group denoted by v.

If we denote by $\lfloor x\rfloor$ the greatest integer lower bound of x, we write the variable v as:

$$v=\frac{T-1}{\alpha}. \quad (7)$$

We define also the variable ψ that characterizes the remaining time from the last PM action until the end of the planning horizon H:

$$\psi=T-\alpha v. \quad (8)$$

Fig. 1 illustrates the definitions of the variables v, and ψ for a simple example that consists of one component, and a planning horizon $H=11$ months. There are 11 periods ($T=11$) of 1 month each (i.e., $L=1$ month). As shown in the figure, when this component is preventively replaced each 3 months, we have $\alpha=3$, three preventive replacements ($v=3$), and $\psi=2$.

The failure rate of all components simultaneously is denoted by $r_{cc}(y)$, and the failure rate of a component i is denoted by $r_i(y)$. A total failure rate is defined as the sum of the component independent failure rate, $r_i(y)$, and the failure rate due to the common cause effect, $r_{cc}(y)$:

$$r_{tot}(y)=r_i(y)+r_{cc}(y). \quad (9)$$

The "β-factor" is then defined as the ratio of the common cause failure rate to the total failure rate of the component:

$$\beta=\frac{r_{cc}(y)}{r_{tot}(y)}=\frac{r_{cc}(y)}{r_{cc}(y)+r_i(y)}. \quad (10)$$

Rearranging Eqs. (9) and (10), we get:

$$r_{cc}(y)=\frac{\beta}{\beta-1}. \quad (11)$$

Because we assume repair is minimal (bad as old), we can model the occurrence of failures during $[0,\alpha\ L[$ using a non-homogeneous Poisson process. Then, the total expected number of failures during $[0,\alpha\ L[$ is given by:

$$M_i^{tot}[0,\alpha L[=M_i^{cc}[0,\alpha L[+M_i[0,\alpha L[, \quad (12)$$

where

$$M_i^{cc}[0,\alpha L[=\int_0^{\alpha L}r_{cc}(y)\mathrm{d}y, \quad (13)$$

and

$$M_i[0,\alpha L[=\int_0^{\alpha L}r_i(y)\mathrm{d}y. \quad (14)$$

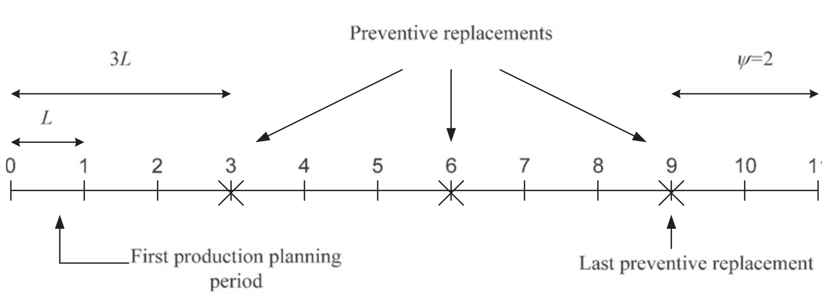

Fig. 1. Illustrative example.

Eq. (13) gives the expected number of CCF (i.e., simultaneous common cause failure of all components), while Eq. (14) represents the expected number of independent failures for component i. The failure rates $r_i(y)$ and $r_{cc}(y)$ are obtained from the probability density functions $f_i(y)$ and $f_{cc}(y)$ as follows:

$$r_i(y) = \frac{f_i(y)}{\int_y^\infty f_i(x)\mathrm{d}x}, \tag{15}$$

$$r_{cc}(y) = \frac{f_{cc}(y)}{\int_y^\infty f_{cc}(x)\mathrm{d}x}. \tag{16}$$

Knowing that components are renewed at times αL, $2\alpha L$, …, $v \alpha L$, the total expected number of failures of component i during the planning horizon H is estimated as:

$$N_i^{tot} = vM_i^{tot}[0,\alpha L[+M_i^{tot}[0,\psi L[. \tag{17}$$

Using Eqs. (12)–(14), we can distinguish between the expected number of common cause failures, N_i^{cc}, and expected number of independent failures, N_i, as follows:

$$N_i^{cc} = vM_i^{cc}[0,\alpha L[+M_i^{cc}[0,\psi L[, \tag{18}$$

$$N_i = vM_i[0,\alpha L[+M_i[0,\psi L[. \tag{19}$$

Let introduce now the following given times and costs:

- CMR_i is the expected minimal repair cost of component i (independent failure).
- CMR is the expected cost when all components are minimally repaired (CCF).
- CPM is the expected preventive maintenance cost (components are renewed all together).
- TMR_i is the expected minimal repair time of component i (independent failure).
- TMR is the expected time required for minimal repair of all components (CCF).
- TPM is the expected preventive maintenance time.

The expected maintenance cost during the planning horizon is then given by:

$$CM(\alpha) = N_i^{cc}CMR + \sum_{i=1}^{n} N_iCMR_i + vCPM. \tag{20}$$

Using Eqs. (18) and (19) in Eq. (20), we obtain:

$$CM(\alpha) = (vM_i^{cc}[0,\alpha L[+M_i^{cc}[0,\psi L[)CMR + \sum_{i=1}^{n}(vM_i[0,\alpha L[+M_i[0,\psi L[)CMR_i + vCPM. \tag{21}$$

Similarly, the expected maintenance time during the planning horizon is:

$$TM(\alpha) = (vM_i^{cc}[0,\alpha L[+M_i^{cc}[0,\psi L[)TMR + \sum_{i=1}^{n}(vM_i[0,\alpha L[+M_i[0,\psi L[)TMR_i + vTPM. \tag{22}$$

4.2. Evaluation of $G_t(\alpha)$

In order to evaluate the average production rate of the system, it is necessary to estimate the average availability of each component i per period. We assume that the length L is large enough, so that we can consider that a stationary regime is reached during each period. This assumption is realistic since a typical value for a tactical planning period in industry is one month or more. We denote by A_i^t the steady-state availability during a period t. This availability depends on the expected number of failures during $[(t-1)L, tL[$ and on the occurrence or no of a preventive maintenance at the beginning of period t. In fact, within the time period $[(t-1)L, tL[$, the component is expected to fail a number of times and be minimally repaired. Furthermore, every time a preventive replacement is performed, the component is unavailable. We define the binary variable Δ_t as follows: Δ_t is equal to 1 if a PM is performed at the beginning of period t, and it is equal to 0 otherwise. Let define also the total expected number of component i failures during the interval $[(t-1)L, tL[$ as follows:

$$M_i^{tot}[(t-1)L,tL[= M_i^{cc}[(t-1)L,tL[+M_i[(t-1)L,tL[, \tag{23}$$

where

$$M_i^{cc}[(t-1)L,tL[= \int_{(t-1)L}^{tL} r_{cc}(y)\mathrm{d}y, \tag{24}$$

and

$$M_i[(t-1)L,tL[= \int_{(t-1)L}^{tL} r_i(y)\mathrm{d}y. \tag{25}$$

Eq. (24) gives the expected number of CCF (i.e., simultaneous common cause failure of all components), while Eq. (25) represents the expected number of independent failures for component i.

The availability A_i^t is then:

$$A_i^t = \frac{L - TPM\Delta_t - TMR_iM_i[(t-1)L,tL[- TMRM_i^{cc}[(t-1)L,tL[}{L}. \tag{26}$$

Eq. (26) gives an expected value of the component availability over the time horizon L beginning from time $(t-1)L$ until time tL. Once the average availability is calculated for each period and for each component, the average production rate $G_t(\alpha)$ of the parallel system can be obtained easily. To illustrate this, let consider a parallel system consisting of two independent binary-state components. Assume $t=1$ and the nominal production rates (in items per time unit) are $g_1=10$ and $g_2=15$. By using the method described above, the availability of each component is assumed to be evaluated as $A_1^1=0.9$ and $A_2^1=0.85$. The number of the possible combinations of the states of components is 4. In Table 1, we give the production rate and the probability for each state. We obtain for this example an average production rate of 25 p_1+10 p_2+15 p_3=21.75 items per time unit.

5. Solution method and numerical examples

The proposed solution method consists in enumerating all PM alternatives as follows. In the mixed-integer non linear problem formulated by (1)–(6), for each product p and for each period t, the decision variable are x_{pt}, I_{pt}, B_{pt}, Set_{pt} and α. For a given solution α, the values of $CM(\alpha)$, $TM(\alpha)$ and $G_t(\alpha)$ can be evaluated by using the results of the previous section. Knowing these values, the problem (1)–(6) becomes a mixed integer linear production planning problem corresponding to the capacitated lot-sizing problem, which can be solved using any selected existing algorithm. This pure production planning problem can be solved also

Table 1
Example of a system with two parallel components.

States	Production rates	Probabilities
{10,15}	25	$p_1 = A_1^1A_2^1 = 0.765$
{10,0}	10	$p_2 = A_1^1(1-A_2^1) = 0.135$
{0,15}	15	$p_3 = (1-A_1^1)A_2^1 = 0.085$
{0,0}	0	$p_4 = (1-A_1^1)(1-A_2^1) = 0.015$

by using the mixed integer solver of a commercial optimization package.

5.1. Example 1

To illustrate this solution method, let consider a parallel system containing 2 binary-state components. Table 2 gives individual characteristics of these components, while Table 3 provides data related to grouped minimal repair and PM. We remark that the parameters in Table 3 are lower than the sum of the PM individual costs and times given in Table 2.

The planning horizon H is 5 months composed of 5 periods ($L=1$ month). The system has to produce two kinds of products in lots so that the demands are satisfied. For each product, the periodic demands are presented in Table 4. Table 5 gives the holding, backorder, set-up and production costs for each product. These costs are the same for all periods.

For the independent failure of single components, we assume that the lifetime of component 1 is distributed according to a second order Gamma distribution, and the lifetime of component 2 is distributed according to Weibull distribution with parameters (2, 2). Furthermore, suppose the time to common cause failure is governed by a Weibull probability distribution with parameters (3, 3).

Because components are replaced all together, with two components and five periods, there are 5 possible PM solutions α, $\alpha \in \{1, 2, 3, 4, 5\}$. Since the planning horizon is 5 months, $\alpha=5$ means that no PM action is performed. Table 6 presents for each α the values of the total maintenance cost, the total production cost and the total cost (i.e., the sum of total maintenance and production costs). The total cost of an optimal integrated production and maintenance plan is reduced to 48,977 $ and is obtained for $\alpha=2$. Table 7 shows the optimal production plan for the two products when integrating production and PM planning.

The plan obtained by the proposed approach is indeed more realistic that that obtained by the approach developed in [6], since it takes into account CCF and economic dependence. When solving the same example by the approach in [6], we obtained a total cost of 48,772.5 $ (see [6] for more details). Even if this cost is lower than 48,977 $, it is important to note that the effective cost should be higher since the CCF are not taken into account by the model in [6]. In fact, by ignoring CCF, the estimated capacity is overestimated, which increases the cost of backorders. Furthermore, as we have not taken into account CCF in our PM planning, the available resources could be insufficient to repair multiple components, which may increase the maintenance time and cost. Recall that taking into account CCF not only means that the capacity evaluation model includes CCF, but this means also that enough maintenance crew to repair more than one component. As the occurrence of a CCF may result in a peak in unplanned manpower needs, additional labor needs to be hired, which could be costly. The next example considers more than two components to demonstrate the advantages of the proposed method.

Table 2
Individual characteristics of the components.

Component i	g_i (items/month)	CPM_i ($)	CMR_i ($)	TPM_i (month)	TMR_i (month)
1	50	1500	1000	0.020	0.1
2	55	1700	1250	0.025	0.15

Table 3
Characteristics of the components in case of grouped maintenance.

CPM ($)	CMR ($)	TPM (month)	TMR (month)
2000	1500	0.030	0.18

Table 4
Demands of products.

Period t	Demand of product 1 d_{1t} (items)	Demand of product 2 d_{2t} (items)
1	50	50
2	48	49
3	48	50
4	47	47
5	48	48

Table 5
Cost data of products.

Product p	Holding cost h_{pt} ($)	Backorder cost b_{pt} ($)	Set-up cost s_{pt} ($)	Production cost π_{pt} ($)
1	40	120	500	70
2	40	120	500	70

Table 6
Evaluation of costs for each PM alternative.

PM solutions α	Total production cost ($)	Total maintenance cost ($)	Total cost ($)
1	38,950	11,375	50,325
2	39,110	9,867	**48,977**
3	40,460	10,522	50,982
4	43,780	13,621	57,401
5	45,180	17,965	63,145

5.2. Example 2

Let consider now a parallel system containing 4 binary-state components. Table 8 gives individual characteristics of these components, while Table 9 provides data related to grouped minimal repair and PM. We remark that the parameters in Table 9 are lower than the sum of the PM individual costs and times given in Table 8.

As in Example 1, the planning horizon H is 5 months composed of 5 periods ($L=1$ month); the system has to produce two kinds of products in lots so that the demands are satisfied; for each product, the periodic demands are presented in Table 4; and Table 5 gives the holding, backorder, set-up and production costs for each product (these costs are the same for all periods).

For the independent failure of single components, we assume that the lifetime of components 1 and 2 are distributed according to a second order Gamma distribution, and the lifetime of components 3 and 4 are distributed according to Weibull distribution with parameters (2, 2). Furthermore, suppose the time to common cause failure is governed by a Weibull probability distribution with parameters (3, 3).

From Table 10, we remark that separate production optimization and group maintenance optimization leads to a total cost of 66,783.5 $ for $\alpha=2$. In this separate optimization, we first optimize maintenance plan, then we optimize the production plan (taking into account this optimal maintenance plan). On the other hand, the total cost of an optimal integrated production and maintenance plan is reduced to 65,467.5 $ and is obtained for $\alpha=1$. The difference between the two plans illustrates that the

Table 7
Optimal production plan when integrating production and PM.

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	51	1	0	1	50	0	0	1
2	47	0	0	1	48	0	1	1
3	48	0	0	1	51	0	0	1
4	47	0	0	1	47	0	0	1
5	48	0	0	1	48	0	0	1

Table 8
Individual characteristics of the components (Example 2).

Component i	g_i (items/month)	CPM_i ($)	CMR_i ($)	TPM_i (month)	TMR_i (month)
1	25	1200	1000	0.02	0.1
2	22	1300	1250	0.02	0.15
3	25	1200	1000	0.02	0.1
4	23	1400	1250	0.02	0.15

Table 9
Characteristics of the components in case of grouped maintenance (Example 2).

CPM ($)	CMR ($)	TPM (month)	TMR (month)
2250	1500	0.050	0.3

Table 10
Separate and integrated optimization solutions.

PM solutions α	Total production cost ($)	Total maintenance cost ($)	Total cost ($)
1	49,890	15,577.5	65,467.5
2	51,530	15,253.5	66,783.5

Table 11
Optimal production plan when integrating production and PM (Example 2).

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	41	0	9	1	50	0	0	1
2	41	0	16	1	49	0	0	1
3	40	0	24	1	50	0	0	1
4	71	0	0	1	19	0	28	1
5	48	0	0	1	42	0	34	1

proposed integrated optimization method is better than separate production optimization and group maintenance optimization. Table 11 shows the optimal production plan for the two products when integrating production and PM planning.

Finally, this example has been solved using the method developed in [6], which suggests performing preventive maintenance on components separately using age-based replacement (i.e., preventively replacing a component when it reaches a pre-specified replacement age). In this case, the optimal integrated production and maintenance plan is obtained for a total optimal cost of 69,615 $, and it suggests that no preventive maintenance is performed for components 1 and 2, and it is performed each 2 months for components 3 and 4, while each component is minimally repaired at failure. The difference between the two plans illustrates that the proposed maintenance method is better than performing preventive maintenance on components separately using age-based replacement.

6. Conclusion

Because production and preventive maintenance are mutually in conflict, their planning integration may reduce the total expected cost. Assuming that components are stochastically and economically independent, Nourelfath et al. [6] have developed an integrated production and preventive maintenance planning model for multi-component systems. In this paper, we extended the model in [6] by taking into account the presence of economic dependence and common cause failures in parallel systems. We used the β-factor model to represent common cause failures. The suggested PM is a T-age group maintenance policy in which components are cyclically renewed all together. Furthermore, between the periodic group replacements, minimal repairs are performed on failed components. A method was proposed to evaluate the times and the costs of PM and minimal repair, and the average production system capacity per period. For each chosen PM solution, the problem was solved as a multi-product capacitated lot-sizing problem. An issue currently under investigation consists in extending the proposed model to deal with non-cyclical preventive maintenance, while taking into account dependences.

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