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Reliability of k-out-of-n systems with phased-mission requirements and imperfect fault coverage

Liudong Xing [a,b,*], Suprasad V. Amari [c], Chaonan Wang [b]

[a] Collaborative Autonomic Computing Laboratory, School of Computer Science, University of Electronic Science and Technology of China, Chengdu 610054, China
[b] Electrical and Computer Engineering Department, University of Massachusetts, Dartmouth, MA 02747, USA
[c] Parametric Technology Corporation, Greensburg, PA 15601, USA

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ABSTRACT

In this paper, an efficient method is proposed for the exact reliability evaluation of k-out-of-n systems with identical components subject to phased-mission requirements and imperfect fault coverage. The system involves multiple, consecutive, and non-overlapping phases of operation, where the k values and failure time distributions of system components can change from phase to phase. The proposed method considers statistical dependencies of component states across phases as well as dynamics in system configuration and success criteria. It also considers the time-varying and phase-dependent failure distributions and associated cumulative damage effects for the system components. The proposed method is based on the total probability law, conditional probabilities and an efficient recursive formula to compute the overall mission reliability with the consideration of imperfect fault coverage. The main advantages of this method are that both its computational time and memory requirements are linear in terms of the system size, and it has no limitation on the type of time-to-failure distributions for the system components. Three examples are presented to illustrate the application and advantages of the proposed method.

1. Introduction

Operation of missions encountered in many real-world applications such as aerospace and nuclear power plants often involves multiple different tasks or phases that must be accomplished in sequence [1–4]. For example, an aircraft flight involves taxi, take-off, ascent, level-flight, descent, and landing phases [2,5]. During each phase, the system has to accomplish a specified task and may be subject to different stresses and environmental conditions as well as different reliability requirements. Thus, system configuration, success criteria, and component failure behavior may change from phase to phase [6]. For the above aircraft flight example, if there are two engines, one engine is usually required during the taxi phase, but both engines are necessary during the take-off phase. In addition, the engines are more likely to fail during the take-off period because they are generally under enormous stress in this phase as compared to other phases of the flight profile. Systems used in these missions are referred to as phased-mission systems (PMS).

An accurate reliability analysis of a PMS must address the above described dynamics in system configuration, success criteria, and component behavior. In addition, the analysis must consider the statistical dependencies of component states across phases, in particular, the state of a component at the beginning of a new phase being identical to the state at the end of the previous phase [6]. Consideration of these dynamics and dependencies poses unique challenges to existing reliability analysis methods. Considerable research efforts have been expended in the reliability analysis of PMS over the past four decades [7–15]. A state-of-the-art review of PMS reliability modeling and analysis techniques is provided in Ref. [16]. However, even with advances in computing technology, only small-scale PMS problems can be solved accurately due to high computational complexity of the existing methods.

Among various system structures for achieving fault tolerance, the k-out-of-n system structure has become a popular type of redundancy since it was introduced in 1961 by Birnbaum, Esary, and Saunders [17]. There are two types of k-out-of-n systems: k-out-of-n: G and k-out-of-n: F. The k-out-of-n: G system consists of n components and it functions if and only if (*iff*) at least k of the n components are functioning [18]. In other words, the system functions (fails) *iff* at most $n-k$ (at least $n-k+1$) components fail. Similarly for a k-out-of-n: F system, it fails *iff* at least k of the n components have failed. In this work, we focus on k-out-of-n: G systems, which are simply referred to as k-out-of-n systems hereafter.

* Corresponding author at: Electrical and Computer Engineering Department, University of Massachusetts, Dartmouth, MA 02747, USA. Tel.: +1 508 999 8883; fax: +1 508 999 8489.
E-mail address: lxing@umassd.edu (L. Xing).

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Both series systems and parallel systems are special cases of the k-out-of-n systems. The k-out-of-n redundancy has found a wide range of applications in both industrial and military systems. Examples include cables in a bridge, a data processing system with multiple video displays, communication systems with multiple transmitters, and airplanes with multi-engine systems [19,20]. Although the k-out-of-n systems have been extensively studied in the literature, the focus of these studies is on the binary-state systems subject to single-phase mission requirements. However, many of the practical applications of k-out-of-n systems, such as space systems [21], airborne weapon systems [22], and distributed computing systems [6,23], are subject to phased-mission requirements where the number of working components required (k values) can change from phase to phase. In addition, imperfect fault coverage could happen during the mission. In particular, the automatic recovery mechanism that is designed into the k-out-of-n system to tolerate faults can fail, such that the system cannot adequately detect, locate, and recover from a fault occurring in an online component [2]. The uncovered fault can lead to an overall system failure despite the presence of sufficient redundancies because the imperfect recovery causes the number of on-line components to be less than k. A practical example would be a multi-engine aircraft, in particular, a two-engine aircraft requiring at least one of the two engines to be operational during the taxi, level-flight, descending, and landing phases ($k=1$), but both engines to be operational during the take-off and ascending phases ($k=2$). During the level-flight phase, in the case of the first engine failure, a recovery process is immediately initiated to switch in the other engine so that the aircraft can continue to operate correctly. But if the recovery mechanism fails, the whole mission fails despite the existence of the other operational engine. Other examples of phased mission system with imperfect coverage and phased dependent k values are Aerospace Computing Systems discussed in Ref. [21] and Automatically Reconfigurable Modular Multiprocessor System (ARMMS) from Marshall Space Flight Center of NASA [23]. The imperfect fault coverage introduces two failure modes for a component that must be considered for the accurate system reliability analysis: *covered* failure that affects only a single component, and *uncovered* failure that can propagate through the system and lead to the failure of the entire system.

In this paper, an efficient method is proposed for the exact reliability evaluation of k-out-of-n phased-mission systems with the consideration of the imperfect fault coverage effect. The proposed method is based on the total probability law, conditional probabilities and an efficient recursive formula to compute the reliability of the entire mission. The method is applicable to any arbitrary failure distributions for the system components.

The remainder of the paper is organized as follows. Section 2 presents an overview of the problem to be solved including a system description, assumptions, and problem inputs. Section 3 presents the method for computing the conditional reliability/unreliability of a component at a particular phase given that the component is functioning at the beginning of the phase. Section 4 describes the proposed phased-mission reliability evaluation method. Section 5 illustrates the application and advantages of the proposed method using three examples. Lastly, Section 6 concludes the paper.

2. Problem statement

This paper considers the problem of evaluating the reliability of k-out-of-n phased-mission systems subject to the imperfect fault coverage behavior. The k values and failure time distributions for the system components can vary with the phases. The cumulative damage effects of the system components are also considered. The assumptions and inputs for the problem are listed in the following subsections.

2.1. System description and assumptions

1) The system mission consists of M consecutive and non-overlapping phases. Phase durations are deterministic.
2) The system uses a k-out-of-n active redundancy structure where the k values can change with the phases. In other words, the number of good components required can vary with the phases.
3) The system has n identical components. The component failures are s-independent within each phase. Dependencies exist among different phases, and failure modes for the same component.
4) The components can have phase-dependent and time-varying failure distributions.
5) An uncovered component fault causes the overall system failure, even in the presence of adequate remaining redundancies. The probability that a fault is covered given that the fault occurs, denoted by c, is given as a fixed probability. c is also known as a fault coverage factor [2]. Thus the probability that an uncovered fault occurs is $(1-c)$.
6) The system is not repairable during the mission; once a component transfers from the operation mode to a failure mode (covered or uncovered), it will remain in that failure mode for the rest of the mission time.
7) The overall mission is considered to be failed if the system fails during any one of the phases. In other words, for the entire mission to be successful, the system must operate successfully during all the phases. Refer to Ref. [5] for the generalized combinatorial phase requirements where the failure of the mission is expressed as a logical combination of phase failures.
8) The system is coherent, meaning that each component contributes to the system state, and the system state worsens (at least does not improve) as the number of failed components increases.

2.2. Problem inputs

The following lists all the required input parameters for solving the problem.

1) Mission time t.
2) Number of phases M.
3) Duration of each phase i: τ_i.
4) Number of system components n.
5) Failure criteria for each phase, described in terms of the number of good components required in each phase i: k_i.
6) Fault coverage factor c_i for a component during phase i.
7) Acceleration factor during phase i: α_i.
8) The baseline failure time distribution of each component in each phase and related parameter values, from which the cumulative failure probability of the component at the end of phase j, $Q_j(t)$ can be derived.

Note that in this paper, we consider the above listed parameters as given input parameters of the problem, and we focus on the system-level reliability evaluation.

3. Evaluation of conditional component reliabilities

In this section, we describe the method for evaluating the conditional reliability/unreliability of a component at a phase given that it is functioning at the beginning of the phase. Those conditional probabilities are used in the proposed approach for the reliability analysis of k-out-of-n PMS with imperfect fault coverage.

We use the concept of equivalent age associated with the cumulative exposure model (CEM) to account for effects of phase-dependent stress on the failure properties of the components [24]. Let $F_j(t)$ be the stress dependent failure distribution of a component in phase j. Let F be the baseline distribution and α_j be the acceleration factor during the phase j. Note that our method has no limitation on the type of distribution for F. If the life–stress relationship follows the accelerated failure time model (AFTM), then $F_j(t)$ can be represented as [24–26]

$$F_j(t) = F(\alpha_j t) \tag{1}$$

Let τ_j be the duration of phase j, and Q_j and P_j be the cumulative failure probability and reliability of the component at the end of phase j, respectively. According to the CEM [26], we have

$$\begin{aligned} Q_j &= F(\alpha_1\tau_1 + \cdots + \alpha_j\tau_j) \\ P_j &= 1 - Q_j \end{aligned} \tag{2}$$

where, by definition: $Q_0=0$ and $P_0=1$. Let f_j be the probability that a component first fails in phase j. It can be calculated as

$$f_j = Q_j - Q_{j-1} \tag{3}$$

To consider the imperfect fault coverage, let f_{jc} represent the probability that the component first fails in phase j and the failure is covered. Let c_j be the coverage factor during phase j. Then f_{jc} can be calculated as

$$f_{jc} = f_j \cdot c_j \tag{4}$$

For the perfect coverage case, $c_j=1$ and $f_{jc}=f_j$. Thus, the probability that the component first fails in phase j and the failure is uncovered, denoted by f_{ju}, can be calculated as

$$f_{ju} = 1 - f_{jc} \tag{5}$$

The probability that a component fails uncovered during the mission is

$$S_u = \sum_{j=1}^{M} f_{ju} \tag{6}$$

Since there are n identical components, the probability that no component experiences an uncovered failure during the mission, denoted by P_u, can be calculated as

$$P_u = (1 - S_u)^n \tag{7}$$

Let Q_{jc} be the cumulative failure probability of the component at the end of phase j given that no uncovered failure happens. It can be calculated as

$$Q_{jc} = \frac{\sum_{i=1}^{j} f_{ic}}{1 - S_u} \tag{8}$$

For the perfect coverage case, $Q_{jc}=Q_j$. The reliability of the component at the end of phase j given that no uncovered failure happens can be calculated as $P_{jc}=1-Q_{jc}$.

Let q_j be the conditional unreliability of the component in phase j given that it is working at beginning of the phase and no uncovered failure happens. It can be calculated as

$$q_j = \frac{f_{jc}}{1 - Q_{(j-1)c}} \tag{9}$$

Similarly, the conditional reliability of the component in phase j given that it is working at beginning of the phase and no uncovered failure happens, denoted by p_j, can be calculated as

$$p_j = 1 - \frac{f_{jc}}{1 - Q_{(j-1)c}} = \frac{1 - Q_{jc}}{1 - Q_{(j-1)c}} = \frac{P_{jc}}{P_{(j-1)c}} \tag{10}$$

4. The proposed method for reliability analysis of *k*-out-of-*n* PMS

The system under consideration consists of n identical components. It requires at least k_j working components in phase j for the successful operation in that phase. In other words, the system is considered to be failed if there are at least $m_j=(n-k_j+1)$ failed components during phase j. The system is considered to be failed if it fails in any one of the phases.

Let x_j be the number of components that have failed before the end of phase j, where $j=1, 2, \ldots, M$. Hence, the system is considered to be successful if $x_j < m_j$ for all values of j. The system reliability can be calculated as the sum of the probabilities of all combinations of x_j values: $(x_1, x_2, \ldots, x_M)$ where $x_j < m_j$ for all values of j. These individual probabilities can be calculated using the multinomial distribution. However, this method is computationally inefficient because the number of combinations increases exponentially. A similar computation is involved in the reliability analysis of a generalized multi-state k-out-of-n system (GMSS) model, and this model has been studied extensively by several researchers for more than a decade [27]. Recently, a fast and robust algorithm was proposed to analyze the GMSS model by utilizing the properties of an embedded Markov chain associated with the sequence of x_j values: $(x_1, x_2, \ldots, x_M)$ [27]. The speed and efficiency of the algorithm in Ref. [27] is compared with the existing methods for the GMSS model using several published benchmark problems. For small-scale problems, this algorithm is 150 times faster than the existing methods. For large-scale problems, it is 841,000 times faster. This enormous efficiency improvement motivates us to adapt the embedded Markov chain-based computation method for GMSS models to solve the PMS problems in this work.

Let $Z_{j,i}$ be the probability of the system state such that $x_j=i$ and $x_l < m_l$ for all $l<j$. That is,

$$Z_{j,i} = \Pr\{x_j = i;\ x_{j-1} < m_{j-1}; \cdots; x_1 < m_1\} \tag{11}$$

Using the Markov property of the x_j sequence [27], Eq. (11) can be calculated as

$$Z_{j,i} = \sum_{a=0}^{m_{j-1}-1} Z_{(j-1),a} \cdot \Pr\{x_j = i | x_{j-1} = a\} \tag{12}$$

where,

$$Z_{1,i} = \Pr\{x_1 = i\} = \binom{n}{i} (q_1)^i (p_1)^{n-i} \tag{13}$$

$$\Pr\{x_j = i | x_{j-1} = a\} = \begin{cases} 0 & \text{if } i < a \\ \binom{n-a}{i-a} (q_j)^{i-a} (p_j)^{n-i} & \text{if } i \geq a \end{cases} \tag{14}$$

where q_j and p_j are defined in Eq. (9) and (10). Eq. (12) forms the basic recursion for system reliability calculations. To improve the efficiency of the calculations and reduce the storage requirements, we use the following recursive relationships:

$$\Pr\{x_j = i | x_{j-1} = 0\} = \frac{n-i+1}{i} \cdot \frac{q_j}{p_j} \cdot \Pr\{x_j = i-1 | x_{j-1} = 0\} \tag{15}$$

$$\Pr\{x_j = i | x_{j-1} = a\} = \frac{i-a+1}{n-a+1} \cdot \frac{1}{q_j} \cdot \Pr\{x_j = i | x_{j-1} = a-1\} \tag{16}$$

$$\Pr\{x_j = 0 | x_{j-1} = 0\} = (p_j)^n \tag{17}$$

Once we calculate $Z_{M,i}$ values using the recursive formulas, we can calculate the system reliability R_c for the perfect coverage case (i.e., the case conditioned on no component experiences an uncovered failure during the mission) as

$$R_c = \sum_{i=0}^{m_M - 1} Z_{M,i} \tag{18}$$

Considering the imperfect fault coverage, the system reliability R can be calculated using the total probability theorem as

$$\begin{aligned} R &= \Pr(\text{system functions} \mid \text{at least one uncovered failure}) \\ &\quad \times \Pr(\text{at least one uncovered failure}) \\ &\quad + \Pr(\text{system functions} \mid \text{no uncovered failure}) \\ &\quad \times \Pr(\text{no uncovered failure}) \\ &= 0 \times [1 - P_u] + R_c \times P_u \\ &= R_c \times P_u \end{aligned} \tag{19}$$

where R_c and P_u can be calculated using Eq. (18) and (7), respectively.

The computational complexity of the recursive method for the reliability analysis of k-out-of-n phased mission systems proposed in this section is $O(nmM)$, where m is the mean value of the vector $\boldsymbol{m} = [m_1, m_2, \ldots, m_M]$.

5. Numerical examples

The proposed method for the reliability analysis of k-out-of-n PMS has been implemented using Matlab. The application and advantages of the method is illustrated through the analysis of three examples, as detailed in the following subsections.

5.1. Example 1 ($n=5$; $M=4$)

Consider a multi-processor computer system consisting of 5 processors. It is used for a scientific computation task involving 4 phases. Depending on the workload of each phase, the system functions correctly when at least 2, 4, 3 and 2 processors are functioning in phase 1, 2, 3, and 4, respectively. Such a system can be modeled as a k-out-of-n phased-mission system with $n=5$ components and $M=4$ phases. The duration of phases and the phase-dependent system parameters (k_j, α_j and c_j values) are shown in Table 1.

We analyze this example system using the proposed method for three different cases, where Weibull, log normal, and exponential distributions are respectively assumed for the baseline failure time distribution of each component. Note that there are many different failure time distributions that can be used to model component reliabilities [28]. In this work, we use the most commonly used and most widely applicable distributions for illustrating the flexibility of the proposed method on handing different types of distributions.

Case 1. The baseline failure time distribution of each component is Weibull with $\eta=1000$ and $\beta=2$. The cumulative distribution function for the Weibull distribution is shown in Eq. (20).

$$F(t;\eta,\beta) = 1 - \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{20}$$

Case 2. The baseline failure time distribution of each component is Log normal with $\mu=7$ and $\sigma=1$. The cumulative distribution function for the Log normal distribution is shown in Eq. (21).

$$F(t;\mu,\sigma) = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\ln t - \mu}{\sqrt{2\sigma^2}}\right) \tag{21}$$

where erf(●) is error function.

Case 3. The baseline failure time distribution of each component is exponential with $\lambda=0.0005$. The cumulative distribution function for the exponential distribution is shown in Eq. (22).

$$F(t;\lambda) = 1 - \exp(-\lambda t) \tag{22}$$

The system reliability results and CPU time in seconds, without and with the consideration of imperfect fault coverage, are shown in Table 2.

5.2. Example 2 ($n=100$; $M=200$)

In this section, we consider a large-scale PMS problem to demonstrate the efficiency of the proposed method. The system has 100 components and 200 phases. Hence, $n=100$, $M=200$. Such a large-scale problem can exist in applications such as computer networks, computer clusters, and cloud computing systems. In particular, the system can correspond to a large computer cluster with 100 connected computers that work together to accomplish a specific task. Assume there are 200 different tasks that must be finished in non-overlapping consecutive phases. Depending on the nature of the task involved in each phase, the minimum number of computers required to be functioning is different from phase to phase. Similarly, such a large-scale problem can occur in automotive and power systems industry due to repeated phases caused by repeated flights and varying demands between inspection and maintenance intervals. For example, aircraft flight involves multiple phases where the total aircraft flight duration is much smaller than the periodic check intervals of the aircraft subsystems. Therefore, the aircraft can contain latent failures at the beginning of a flight. For an accurate analysis of this system, we should analyze aircraft system with multiple flights with repeated phases between the inspection checks. Hence, the reliability analysis of this system may involve more than 100 phases. In the power systems, the demand of the system can vary with peak and off-peak hours as well as the summer and winter seasons. Therefore, within a maintenance renewal period of a power plant, there exists several operational demand phases. To address these challenges, the PMS model should able to handle a large number of phases.

For illustration purpose, we used modulo operator (mod), i.e., remainder, to generate non-monotonic values for the input parameters. Specifically, the duration of phase j is: $\tau_j = 1 + \text{mod}(j, 10)$. Hence, the total mission duration is: $t = 1100$. The k value for phase j is: $k_j = 10 + \text{mod}(j, 75)$. Further, $m_j = n - k_j + 1$. The acceleration factors during phase j are: $\alpha_j = 1 + 0.1 \times \text{mod}(j, 10)$. The coverage factors during phase j are: $c_j = 1 - 0.002 \times \text{mod}(j, 10)$. This way to define input parameter values allows the inputs to

Table 1
Phase-dependent requirements and parameters.

Phase	Phase 1	Phase 2	Phase 3	Phase 4
Duration	20	60	80	40
k_j	2	4	3	2
α_j	1	1.5	2	0.5
c_j	0.99	0.98	0.98	0.99

Table 2
Results for example 1.

		Perfect coverage	Imperfect coverage
Weibull	CPU time	5.41e−5	6.76e−5
	Reliability	0.99570	0.98843
Log normal	CPU time	7.84e−5	9.86e−5
	Reliability	0.99447	0.98620
Exponential	CPU time	4.74e−5	6.71e−5
	Reliability	0.96435	0.95350

Table 3
Results for example 2.

		Perfect coverage	Imperfect coverage
Weibull	CPU time	0.0319	0.0323
	Reliability	0.99964	0.86049
Log normal	CPU time	0.0319	0.0321
	Reliability	0.99993	0.87837
Exponential	CPU time	0.0321	0.0321
	Reliability	0.99981	0.89800

Table 4
Results for example 3.

		Perfect coverage	Imperfect coverage
Weibull	CPU time	0.317	0.317
	Reliability	0.99925	0.89843
Log normal	CPU time	0.315	0.318
	Reliability	0.99994	0.91225
Exponential	CPU time	0.316	0.317
	Reliability	0.99979	0.91309

reproduced exactly for verification and future research comparisons. We also study three cases for this example.

Case 1: The baseline failure time distribution of each component is Weibull with $\eta=5000$ and $\beta=2$.
Case 2: The baseline failure time distribution of each component is Log normal with $\mu=8.75$ and $\sigma=1$.
Case 3: The baseline failure time distribution of each component is exponential with $\lambda=0.00005$.

Using the proposed method, the system reliability results and CPU time in seconds, without and with the consideration of imperfect fault coverage, are obtained and shown in Table 3.

5.3. Example 3 ($n=200$; $M=500$)

To further illustrate the efficiency of the proposed method, we analyze another large example. The system has 200 components and 500 phases. Hence, $n=100$ and $M=500$. Similar to Example 2, this system can be used to model a larger computer cluster with 200 connected computers working on 500 different tasks. The coverage factors during phase j are $c_j=1-0.0002\times \mathrm{mod}(j,\ 10)$. Other parameters are the same as in Example 2. Again, three cases are studied.

Case 1: The baseline failure time distribution of each component is Weibull with $\eta=6000$ and $\beta=2$.
Case 2: The baseline failure time distribution of each component is Log normal with $\mu=8.75$ and $\sigma=1$.
Case 3: The baseline failure time distribution of each component is exponential with $\lambda=0.0001$.

The results are shown in Table 4.

6. Conclusions

We presented a recursive method for the reliability analysis of k-out-of-n systems subject to the phased-mission requirements and imperfect fault coverage behavior. The proposed method has no limitation on the type of failure distributions for the system components. The proposed method, which is based on conditional probabilities and total probability law, is computationally efficient. As illustrated by the numerical examples, the method can be used to find the reliability of large-scale k-out-of-n systems subject to time-dependent and phase-dependent failure parameters in negligible CPU times. Hence, within a reasonable computational time, the method can evaluate the phased-mission system reliability for several alternative configurations with different redundancy levels and/or different types of components. Therefore, the method can be integrated with optimization algorithms, such as simulated annealing and genetic algorithms [29] that involve computing the system reliability for different potential configurations to find the optimal configurations for phased mission systems [29], which will be our future work. Another direction of our future work is to consider multi-state k-out-of-n phased-mission systems subject to imperfect fault coverage, where the system components can exhibit multiple performance levels (corresponding to different states), ranging from perfect operation to complete failure [30–33].

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