Minimizing Average Interference through Topology Control

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Abstract. Reducing interference is one of the main challenges in wireless communication. To minimize interference through topology control in wireless sensor networks is a well-known open algorithmic problem. In this paper, we answer the question of how to minimize the average interference when a node is receiving a message. We assume the receiver-centric interference model where the interference on a node is equal to the number of the other nodes whose transmission ranges cover the node. For one-dimensional (1D) networks, we propose a fast polynomial exact algorithm that can minimize the average interference. For two-dimensional (2D) networks, we give a proof that the maximum interference can be bounded while minimizing the average interference. The bound is only related to the distances between nodes but not the network size. Based on the bound, we propose the first exact algorithm to compute the minimum average interference in 2D networks. Optimal topologies with the minimum average interference can be constructed through traceback in both 1D and 2D networks.

Keywords: Wireless Sensor Networks, Interference Minimization, Topology Control, Combinatorial Optimization.

1 Introduction

A wireless sensor network (WSN) consists of a set of nodes deployed across a region of interest. The nodes can adjust their transmission powers to achieve their desired transmission ranges with which a multi-hop network is then formed. WSNs have many applications in real life such as environmental monitoring, intrusion detection, and health care.

Energy is a precious resource in wireless sensor networks. One way to conserve energy, and to simultaneously improve communication efficiency, is to reduce *interference* due to concurrent transmissions of two or more nearby nodes. There exist numerous models for capturing the essence of interferences in a wireless network at various abstraction levels of interest. Two types of models that have been

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T. Erlebach et al. (Eds.): ALGOSENSORS 2011, LNCS 7111, pp. 115–129, 2012.

frequently studied in recent algorithmic research on wireless sensor networks are graph-based *protocol models* and SINR-based *physical models* [1]. Each type has its own merits. SINR-based protocols capture more accurately certain important wireless signal propagation characteristics([2]). The graph-based protocol models, although simplistic, are a good estimation of interference, which have been particularly popular with high-layer protocol designers.

One of the graph-based protocol models is the *sender-centric model*, where interference is computed for each edge [3–8]. The interference of an edge (u, v) is the number of other nodes that are covered by the disk centered at u or v with radius $|uv|$—that is, interference is considered at the sender but not the receiver. However, interference actually prevents correct data reception in the real networks. Thus, the authors in [9, 10] proposed *the receiver-centric model*, where the interference on a node is the number of other nodes whose transmission ranges cover the node. In Figure 1, the interference on the node v is 3 as all the other nodes can interfere with it. In this work, we consider ways to minimize the number of the other nodes that can interfere with a node when it is receiving a message. Therefore, the receiver-centric model is adopted.

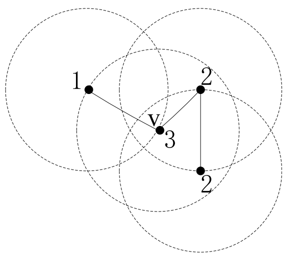

Fig. 1. The receiver-centric interference: the disk centered at a node is the node's transmission range; the number beside a node is interference on it

Generally, *topology control* refers to selecting a subset of the available communication links for data transmission, which helps save energy and reduce interference. The problem of minimizing (receiver-centric) interference through topology control is one of the most well-known open algorithmic problems in wireless communication. Researchers study the problem in two directions: minimizing the maximum interference and minimizing the average interference. Interference minimization is hard because it entails an unusually complicated combinatorial structure, and some intuitive ideas, such as low node degree, spare topology and Nearest Neighbor Forest (connecting each node to its nearest neighbor) can not guarantee low interference [4, 9].

In the literature, interference minimization is studied in both 1D and 2D networks. Despite their simplicity, 1D networks, i.e. the nodes are arbitrarily distributed along a line, have revealed many interesting challenges and features of the problem in general. Studying 1D networks is justified also from a practical point of view as some real networks are one-dimensional, such as the sensors deployed along a railway, a corridor, or inside a tunnel. For 1D networks, paper [9] bounded *the minimum maximum interference* (MMI) by $O(\sqrt{\Delta})$ and presented an approximation with ratio $O(\sqrt[4]{\Delta})$. Here Δ is the maximum node degree when each node is connected to all the other nodes within the maximum transmission range r_{max}. The only sub-exponential-time (but super-polynomial) exact algorithm to minimize the maximum interference was given in [11]. For 2D networks, the problem of computing the MMI was shown to be NP-complete in [12]. The algorithm in [13] could bound the maximum interference by $O(\sqrt{\Delta})$. For the problem of computing *the minimum average interference* (MAI), better results are known. In [11], a polynomial-time, $O(n^3\Delta^3)$ algorithm is proposed to minimize the average interference in a 1D network, where n is the network size. For 2D networks, the authors of [14] gave an asymptotically optimal approximation algorithm with an approximation ratio $O(\log n)$.

Our Contribution: In this paper, we answer the question of how to minimize the average interference when a node is receiving a message.

1. To minimize the average interference in 1D networks, we propose an exact algorithm that substantially improves the time complexity from $O(n^3\Delta^3)$ [11] to $O(n\Delta^2)$.
2. In previous work, the MAI and the MMI were studied separately. We give a proof that the maximum interference can be bounded by $O(\log \lambda)$ while minimizing the average interference. Here $\lambda = \frac{min(d_{max}, r_{max})}{d_{min}}$, where d_{max} and d_{min} are the longest and shortest distance between two nodes respectively. The upper bound is only determined by the distances between nodes but not the network size.
3. Based on the upper bound, we propose an exact algorithm to compute the MAI in 2D networks exactly in time $n^{O(m \log \lambda)}$, where m is the minimum number of parallel lines so that all the nodes are located on the lines. Optimal topologies with the MAI can be constructed trough traceback. To the best of our knowledge, it is the first exact algorithm that computes the MAI in 2D networks.

The rest of the paper is organized as follows. We give some formal definitions in Section 2. In Section 3, we propose a fast exact algorithm to compute the MAI in 1D networks. The upper bound of the MMI while minimizing the average interference is proved in Section 4. Section 5 presents the exact algorithm to compute the minimum average interference in 2D networks. Section 6 concludes the paper and suggests some future work.

2 Problem Definitions

We model a wireless sensor network as an undirected graph $G = (V, E)$, where V is the set of nodes and E is the set of communication links. The nodes have the same maximum transmission radius, r_{max}. Each node can self-adjust its transmission radius from 0 to r_{max} in a continuous manner. An edge $(u, v) \in E$ exists only if both transmission radii, r_u and r_v, are not shorter than the

Euclidean distance $|uv|$. Therefore, in G, the transmission radius of a node is set to the distance to its farthest neighbor. (Two nodes are neighbors means there is an edge incident on them.) We assume the unit disk graph $UDG(V)$, in which each node connects to all the other nodes within a distance of r_{max}, is connected.

We adopt the receiver-centric interference model. The interference of a node v, denoted as $RI(v)$, is defined as the number of other nodes whose transmission ranges can cover v:

$$RI(v) = |\{u|u \in V \setminus \{v\}, |uv| \leq r_u\}|. \tag{1}$$

The average node interference in G, $RI_{avg}(G)$, can be defined as:

$$RI_{avg}(G) = \frac{\sum_{v \in V} RI(v)}{|V|}. \tag{2}$$

For a node v with transmission radius r_v, the interference created by v is defined as the number of the other nodes covered by the transmission range of v:

$$CI(v, r_v) = |\{u|u \in V \setminus \{v\}, |uv| \leq r_v\}|. \tag{3}$$

Therefore, we can have

$$RI_{avg}(G) = \frac{\sum_{v \in V} RI(v)}{|V|} = \frac{\sum_{v \in V} CI(v, r_v)}{|V|}. \tag{4}$$

It will not increase any interference by deleting an edge. Therefore, the optimal connected topology with minimum interference should be a spanning tree. Therefore, our problem can be defined as:

Problem 1. Given n nodes arbitrarily distributed in a 1D or 2D region, construct a spanning tree, $G = (V, E)$, to connect all the nodes with edges no longer than r_{max}, that induces the minimum average interference.

3 Minimizing Average Interference in 1D Networks

3.1 Independent Subproblems

For a 1D network, the n nodes in $V = \{v_0, v_1, ..., v_{n-1}\}$ are arbitrarily deployed along a line from left to right. We can view the line as an x-axis, and set $v_0 = 0$. For a segment $\overline{v_s v_t}$ on the line, where $s \leq t$, the nodes *located on* $\overline{v_s v_t}$ are $\{v_s, v_{s+1}, ..., v_{t-1}, v_t\}$; the nodes outside $\overline{v_s v_t}$ are the other nodes not including the ones that are on the line; the nodes *inside* $\overline{v_s v_t}$ are $\{v_{s+1}, , ..., v_{t-1}\}$.

We draw all the edges on one side of the line. A *cross* is defined as two edges that share at least a common point excluding their endpoints. Paper [11] presents the *no-cross property* as described in Theorem 1.

Theorem 1 (No-cross Property). *For any spanning tree connecting the nodes on a line with crosses, there is always another spanning tree to remove the crosses without increasing interference on any node.*

Based on the no-cross property, if there is an edge $(v_s v_t)$, $s < t$, the nodes inside the segment $\overline{v_s v_t}$ can not be adjacent to the nodes outside. Further, according to Equation 4, we compute the average interference by dividing the sum of the interference created by all the nodes by the network size. The interference created by a node is only related to its transmission radius and the positions of the other nodes. Recall that the node transmission radius is set to be the distance to its farthest neighbor, and the nodes are stationary after deployment. Therefore, for an edge (v_s, v_t), $s < t$, the total interference created by the nodes inside $\overline{v_s v_t}$ is independent of the topology of the nodes outside, and vice versa. Thus, we can now compute the MAI in 1D networks through dynamic programming.

3.2 Algorithms

For $s < t$, we define a topology $A(s,t)$, called an *arch*, for the nodes from v_s to v_t, such that 1) there is an edge (v_s, v_t); 2) $A(s,t)$ is a connected subgraph; and 3) there is no cross. In addition, several auxiliary functions are defined in Table 1.

Table 1. Definition of the functions $(s < t)$

Function	Definition
$f(s,t)$	In $A(s,t)$, returns the minimum total interference created by the nodes inside $\overline{v_s v_t}$
$f_1(s,p,m)$	In $A(s,t)$ and $s \leq p < m < t$, returns the minimum total interference created by nodes inside $\overline{v_s v_m}$ when v_p is the leftmost node adjacent to v_m.
$f_2(m,p,t)$	In $A(s,t)$ and $s < m < p \leq t$, returns the minimum total interference created by nodes inside $\overline{v_m v_t}$ when v_p is the rightmost node adjacent to v_m.
$f_1'(s,m)$	In $A(s,t)$ and $s \leq m < t$, returns the minimum total interference created by nodes $v_{s+1}, v_{s+2}, ..., v_m$.
$f_2'(m,t)$	In $A(s,t)$ and $s < m \leq t$, returns the minimum total interference created by nodes $v_m, v_{m+1}, ..., v_{t-1}$.
$g(p,m)$	When v_p is the leftmost node adjacent to v_m, returns the minimum total interference created by nodes $v_0, v_1, ..., v_{m-1}$.

As there is no cycle, in $A(s,t)$, there must be a node v_m $(s \leq m < t)$ such that no other links cross the line $x = \frac{v_m + v_{m+1}}{2}$ except (v_s, v_t) (Figure 2). So, we calculate

$$f(s,t) = \begin{cases} 0 & t \leq s+1 \\ min\{f_1'(s,m) + f_2'(m+1,t) | s \leq m < t\} & \text{otherwise.} \end{cases} \tag{5}$$

Here, we have

$$f_1'(s,m) = min\{f_1(s,p,m) + CI(v_m, |v_p v_m|) | s \leq p < m\}, \tag{6}$$

$$f_2'(m+1,t) = min\{f_2(m+1,p,t) + CI(v_{m+1}, |v_{m+1} v_p|) | m+1 < p \leq t\}. \tag{7}$$

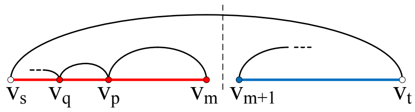

Fig. 2. The structure of $A(s,t)$: $f_1^{'}(s,m)$ is the minimum total interference created by the nodes on the red segment, and $f_2^{'}(m+1,t)$ that on the blue segment.

Specifically, we show how to compute $f_1(s,p,m)$ efficiently (Figure 2). When $p = s$, $f_1(s,p,m) = f(s,m)$. For $p > s$,

$$f_1(s,p,m) = min\{f_1(s,q,p) + f(p,m) + CI(v_p, max(|v_pv_q|, |v_pv_m|))\ |s \le q < p\}.$$

By setting

$$Case_1 = min\{f_1(s,q,p)+CI(v_p,|v_pv_q|)\ |s \le q < p\ \&\ |v_pv_q| \ge |v_pv_m|\}+f(p,m), \tag{8}$$

$$Case_2 = min\{f_1(s,q,p)\ |s \le q < p\ \&\ |v_pv_q| < |v_pv_m|\}+CI(v_p,|v_pv_m|)+f(p,m), \tag{9}$$

we have when $p > s$,

$$f_1(s,p,m) = min\{Case_1, Case_2\}. \tag{10}$$

In Equations 8 and 9, the values of q are continuous numbers. Therefore, we can use RMQ (Range Minimum Query) [15] to compute them. $f_2(m+1,p,t)$ can be computed similarly.

With $f(s,t)$, the function $g(p,m)$ can be computed as:

$$g(p,m) = \begin{cases} f(0,m) + CI(v_0, |v_0v_m|) & p = 0, \\ min\{g(q,p) + CI(v_p, max(|v_pv_q|, |v_pv_m|)) + f(p,m)|0 \le q < p\} \\ & 0 < p < m \le n-1. \end{cases} \tag{11}$$

Finally, the minimum average interference, AVG_{min}, can be calculated as:

$$AVG_{min} = \frac{min\{g(p,n-1) + CI(v_{n-1}, |v_{n-1}v_p|)|0 \le p < n-1\}}{n}. \tag{12}$$

3.3 Analysis

Our algorithm actually compares the average interference on all the spanning trees without a cross, which guarantees the output is optimal with the MAI. Further, our methods have also been verified by comparing the results with the outputs generated by the brute-force search, which runs slowly in time $O(n^{\Delta})$.

According to the process of dynamic programming, the computation of the different functions $f_1(s,p,m)$ and $f_2(m,p,t)$ (as defined in Table 1) contributes the main part of the time complexity. Δ is the maximum number of neighbors for a node constrained by the maximum transmission radius r_{max}. v_t is a neighbor of v_s. For a given s, there are at most Δ different choices of t and at most $t-s$ choices of m. Since all the nodes are deployed along a line, $t-s \leq \Delta$. Also, for a given m, there are at most Δ choices of p as v_p is a neighbor of v_m. Therefore, the total amount of different functions $f_1(s,p,m)$ is $O(n\Delta^2)$. A similar result can be achieved for $f_2(m,p,t)$. Thus, the time complexity to compute the MAI in 1D networks is $O(n\Delta^2)$. The optimal spanning tree can be computed through traceback efficiently. Because of limited space, we omit the details of the traceback here.

4 Bound on MMI while Minimizing Average Interference

In this section, we derive an upper bound on the MMI while minimizing the average interference.

4.1 Preliminaries

Firstly, we define the following property, dubbed the EX property which stands for 'mutual EXclusion of the long edges'.

Definition 1 (EX property). *For four nodes a, b, c, and d, if* $\min(|ab|,|cd|) > \max(|ad|,|bc|)$*, the edges (a,b) and (c,d) are not in a spanning tree simultaneously. It also holds when $a = d$.*

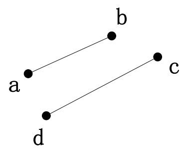

Fig. 3. Four nodes in $T = (V,E)$

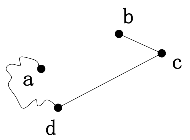

Fig. 4. Replace (a,b) with (b,c)

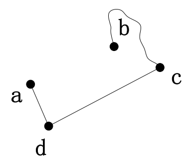

Fig. 5. Replace (a,b) with (a,d)

Next, we show that we can always find an optimal spanning tree with the MAI that satisfies the EX property.

Theorem 2. *For a set of nodes V deployed in a 2D plane, there is always a spanning tree, $T_{ex} = (V, E_{ex})$, with the MAI that satisfies the EX property.*

Proof. For a spanning tree $T = (V,E)$ with the MAI, if it satisfies the EX property, we set $T_{ex} = T$ and we have the proof. If not, we can construct T_{ex} as follows. For each set of four nodes a, b, c and d such that $\min(|ab|,|cd|) > \max(|ad|,|bc|)$ and $(a,b) \in E, (c,d) \in E$ (Note that here a and d can be the same node.) (Figure 3),

1. if a has a path to d in the graph $T_1(V, E - \{(a,b),(c,d)\})$, we set $E' = E - (a,b) + (b,c)$ (Figure 4);
2. if a does not have a path to d in the graph $T_1(V, E - \{(a,b),(c,d)\})$, we set $E' = E - (a,b) + (a,d)$ (Figure 5).

Firstly, we show that T_{ex} is a spanning tree. According to the construction of T_{ex}, in case 1, as a and d have a path, the four nodes are still connected and $|E_{ex}| = |E| = n - 1$; therefore, T_{ex} is a spanning tree. The same result can be obtained similarly for case 2. Secondly, we show that T_{ex} also has the MAI. In case 1, we delete (a,b) and add (b,c). As $|bc| < |ab|$ and $|bc| < |cd|$, the modification does not increase the transmission radii of any node, which means that the total interference created by the nodes is not increased. The same conclusion applies to case 2. Thus, T_{ex} is a spanning tree with the MAI that satisfies the EX property. The theorem is proved.

□

As T_{ex} satisfies the EX property, we have

Corollary 1. *For two regions S_1 and S_2 of diameters d_1 and d_2 respectively, there is at most one edge $(u,v) \in E_{ex}$ such that $|u,v| > max(d_1,d_2)$ with $u \in S_1$ and $v \in S_2$. (Figure 6).*

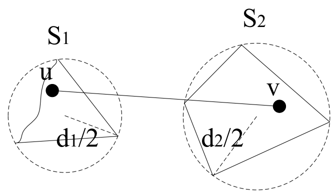

Fig. 6. There is at most one edge $(u,v) \in E_{ex}$ where $u \in S_1$, $v \in S_2$ and $|u,v| > max(d_1,d_2)$

4.2 The Upper Bound

According to Corollary 1, we can bound the maximum interference in T_{ex} as described in Theorem 3.

Theorem 3. *In the spanning tree T_{ex}, the maximum interference is bounded by $O(log\lambda)$, where $\lambda = \frac{min(d_{max}, r_{max})}{d_{min}}$. d_{max} and d_{min} are the longest and shortest distance between any two nodes respectively.*

Proof. For any node $v \in E_{ex}$, the set H contains the other nodes that can interfere with v. We separate the elements in H into subsets according to their transmission radii as follows:

$$h_i = \{u|u \in H \quad and \quad (1+\epsilon)^{i-1} d_{min} \le r_u < (1+\epsilon)^i d_{min}\}, \quad i = 1,2,3... \tag{13}$$

where ϵ is a positive constant. The subsets have the following properties:

$$H = \sum_i h_i \quad and \quad \{h_i \cap h_j = \varnothing \quad if \quad i \ne j\}. \tag{14}$$

Since the possible longest transmission radius in T_{ex} is $\lambda \times d_{min}$, we have the maximal i, denoted as i_{max} as

$$(1+\epsilon)^i \leq \lambda \Rightarrow i_{max} = O(\log \lambda). \tag{15}$$

As the transmission radii of the nodes in h_i are smaller than $(1+\epsilon)^i d_{min}$, the nodes and their neighbors are all inside the circle[1] $c(v, 2(1+\epsilon)^i d_{min})$. We use a set of squares, the length of whose edges is $\frac{\sqrt{2}}{4}(1+\epsilon)^{i-1} d_{min}$, to fully cover the area inside the circle $c(v, 2(1+\epsilon)^i d_{min})$. So, the number of the squares needed is

$$c_0 = (\lceil \frac{2 \times 2(1+\epsilon)^i d_{min}}{\frac{\sqrt{2}}{4}(1+\epsilon)^{i-1} d_{min}} \rceil)^2 = (\lceil 8\sqrt{2}(1+\epsilon) \rceil)^2. \tag{16}$$

For each node $u \in h_i$, since $r_u \geq (1+\epsilon)^{i-1} d_{min}$, u must have an edge $(uu') \in E_{ex}$ which lies inside the circle $c(v, 2(1+\epsilon)^i d_{min})$ such that $|uu'| \geq (1+\epsilon)^{i-1} d_{min}$.

The diameter of each square is $\frac{(1+\epsilon)^{i-1} d_{min}}{2}$. According to Corollary 1, for each pair of the squares, s_1 and s_2, there is at most one edge $(v_1 v_2)$ such that $|v_1 v_2| \geq (1+\epsilon)^{i-1} d_{min}$ and $v_1 \in s_1$, $v_2 \in s_2$. Therefore, the number of nodes in h_i is:

$$|h_i| \leq 2 \times \binom{c_0}{2} = c_1 \tag{17}$$

where c_1 is a constant. Based on Equation 15, the interference on the node v is

$$RI(v) = |H| = \sum_i |h_i| \leq c_1 \times i_{max}. \tag{18}$$

According to Equations 15 and 18, we have

$$RI(v) = O(\log \lambda). \tag{19}$$

Therefore, the maximum interference in T_{ex} is bounded by $O(\log \lambda)$. The theorem is proved.

□

Based on the above theorem, we have the following corollary:

Corollary 2. *In 2D networks, it is possible to bound the MMI by $O(\log \lambda)$ while minimizing the average interference.*

5 Minimizing Average Interference in 2D Networks

5.1 Basic Ideas

Given n nodes arbitrarily deployed in a 2D region, we can simply find the minimum number, denoted as m, of parallel lines so that all the nodes are located

[1] $c(v, r)$ stands for a circle centering at point v with radius of r.

on the lines (Figure 7). We set a parallel line as the x-axis, and list the n nodes from left to right as $V = \{v_0, v_1, ..., v_{n-1}\}$, where for two nodes $v_i = (x_i, y_i)$ and $v_j = (x_j, y_j)$,

$$i < j \quad iff \quad x_i < x_j \ or \ \{x_i = x_j \ and \ y_i < y_j\}. \tag{20}$$

According to Equation 18, we can construct the topology with the MAI while the maximum interference does not exceed $k = \min(c_1 \times i_{max}, n - 1)$. Here, we restrict the maximum interference because it is a critical parameter to determine the time complexity of our algorithm which will be analyzed in Section 5.3.

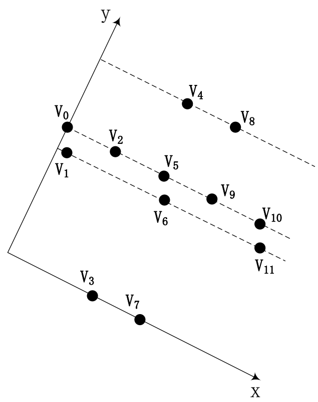

Fig. 7. 12 nodes deployed in a 2D region with the minimum number of parallel lines covering them.

We assume a virtual line *clin* that separates the nodes into the left and the right parts. Initially, there is only v_0 that is on the left of *clin*. We move rightward (and rotate if necessary) the line to include one more node on its left each time until all the nodes are on the left of *clin*. When moving *clin* to include v_p $(0 \le p < n)$ in the left part, we compute the minimum total interference created by the nodes inside $[0, p]$.[2] while the maximum interference does not exceed k and the total topology for the n nodes is connected. Here, the nodes left of *clin* may connect to and interfere with the nodes on the right, and vice versa. When computing the topology for the nodes left of *clin*, we need to assume a topology on the right and take the mutual interference into account. Thus, for an interval $[s, t]$ $(0 \le s \le t \le n - 1)$, we define the following items:

- c[s,t]: record how the nodes inside $[s, t]$ interfere with the nodes outside. $c[s, t]$ contains the nodes and their transmission radii that can interfere with the nodes outside $[s, t]$.
- $s[s, t]$: record all the connected components of the nodes in $c[s, t]$.

[2] For an interval $[s, t]$, $s \le t$, the nodes inside $[s, t]$ are the ones from v_s to v_t. The nodes outside $[s, t]$ are the ones left of $[s, t]$ (the nodes from v_0 to v_{s-1}) and right of $[s, t]$ (the nodes from v_{t+1} to v_{n-1}).

As the maximum interference does not exceed k, we call $c[s,t]$ *valid* if and only if there are no more than k nodes inside $[s,t]$ that interfere with the same node outside $[s,t]$. With the above definitions, we now introduce the algorithms to compute the MAI while the maximum interference does not exceed k.

5.2 Algorithms to Compute MAI

We define a function $F(p, c[0,p], c[p+1, n-1], s[0,p]), 0 \leq p < n-1$, to construct a topology minimizing the interference created by the nodes inside $[0,p]$ while satisfying the following conditions:

1. the interference from nodes inside $[0,p]$ to the nodes inside $[p+1, n-1]$ is the same as that recorded in $c[0,p]$;
2. the interference from nodes inside $[p+1, n-1]$ to the nodes inside $[0,p]$ is the same as that recorded in $c[p+1, n-1]$;
3. the connectivity of the nodes in $c[0,p]$ is the same as that recorded in $s[0,p]$;
4. all the nodes inside $[0,p]$ but not in $c[0,p]$ have a path to at least one node in $c[0,p]$;
5. the interference on each node inside $[0,p]$ does not exceed k.

If F returns $+\infty$, it means there is no such topology that satisfies all the conditions. Here, conditions 1, 2 and 5 are to guarantee that the maximum interference in the final topology does not exceed k. Conditions 3 and 4 are for the requirement of connectivity. Specifically, condition 4 is to guarantee that the nodes in $[0,p]$ but not in $c[0,p]$ can connect to the nodes in $[p+1, n-1]$ through the nodes in $c[0,p]$. The function F can be calculated in Algorithm 1. In Algorithm 1, $R(v) = \{|uv| | u \in V \; and \; |uv| \leq r_{max}\}$, which is the set of potential transmission radii of u. Lines 1–5 are the boundary condition. Lines 7–10 are to enumerate the possible situations. Line 11 is to connect v_p to nodes in $[0, p-1]$ to maintain connectivity. In Line 12, $c'[0,p]$ and $s'[0,p]$, which are defined as the same as $c[0,p]$ and $s[0,p]$ respectively, are computed based on $c[0,p-1]$, $s[0,p-1]$ and the newly added edges in Line 11. Line 13–16 are to check all the conditions and compute the minimum total interference.

The MAI of all the nodes can be computed in the algorithm MAI-GRID (Algorithm 2). MAI-GRID checks the interference on v_{n-1} and makes sure that all the nodes in $s[0, n-2]$ have a path to v_{n-1} such that the network connectivity is maintained. MAI-GRID computes the minimum total interference by the sum of interference created by nodes in $[0, n-2]$ and the interference created by v_{n-1}. After computing MAI-GRID, we can also construct the optimal spanning tree with the MAI through traceback. Figure 8 is an example of the optimal topology.

5.3 Analysis

Based on the definition of the function F, Condition 4 and the operation of connecting v_{n-1} to all the nodes within its transmission range (Line 5 in Algorithm 2) guarantee the connectivity of our output; Condition 5 and the check of

Algorithm 1. Compute $F(p, c[0,p], c[p+1, n-1], s[0,p])$

if $p = 0$ **then** /* the boundary condition */
- **if** there are more the k nodes in $c[p+1, n-1]$ that can interference with v_0 **then**
  - **return** $+\infty$;
- **else**
  - **return** $CI(v_0, r_{v_0})$;

total $\leftarrow +\infty$;
foreach valid $c[0, p-1]$ **do**
- **foreach** valid $c[p, n-1]$ **do**
  - **foreach** $s[0, p-1]$ **do**
    - **foreach** $r_{v_p} \in R(v_p)$ **do**
      - Connect v_p to the nodes in $\{v | v \textit{ is inside } [0, p-1] \textbf{ and } |vv_p| \leq \min(r_v, r_{v_p})\}$;
      - Compute $c'[0,p]$ and $s'[0,p]$;
      - **if** $c[0,p] = c'[0,p]$ **and** $s[0,p] = s'[0,p]$ **and** all the nodes in $[0,p]$ but not in $c[0,p]$ have a path to at least one node in $c[0,p]$ **and** the interference on v_p does not exceed k **then**
        - $tmp \leftarrow F(p-1, c[0,p-1], c[p, n-1], s[0, p-1]) + CI(v_p, r_{v_p})$;
        - **if** $tmp < total$ **then**
          - $Total \leftarrow tmp$;

return Total;

Algorithm 2. MAI-GRID: compute the MAI in a grid network

$k \leftarrow \min(c_1 \times i_{max}, n-1)$, *total* $\leftarrow +\infty$;
foreach valid $c[0, n-2]$ **do**
- **foreach** $s[0, n-2]$ **do**
  - **foreach** $r_{v_{n-1}} \in R(v_{n-1})$ **do**
    - Connect v_{n-1} to the nodes in $\{v | v \textit{ is inside } [0, n-2] \textbf{ and } |vv_{n-1}| \leq \min(r_v, r_{v_{n-1}})\}$;
    - $c[n-1, n-1] = \{v_{n-1}, r_{v_{n-1}}\}$;
    - **if** the interference on v_{n-1} does not exceed k **and** all the nodes in $s[0, n-2]$ has a path to v_{n-1} **then**
      - $t \leftarrow F(n-2, c[0, n-2], c[n-1, n-1], s[0, n-2]) + CI(v_{n-1}, r_{v_{n-1}})$;
      - **if** $t < total$ **then**
        - $Total \leftarrow t$;

return $\frac{total}{n}$;

the interference on v_{n-1} (Line 7 in Algorithm 2) guarantee the maximum interference of our output does not exceed k. Further, our algorithm actually compares all the possible connected topology with the maximal interference equal or smaller than k. Therefore, our method output the optimal topology with the MAI while the maximum interference does not exceed k. The correctness of the algorithms has been established through comparing our results with the outputs of the brute-force search which runs in time $O(n^{\Delta})$.

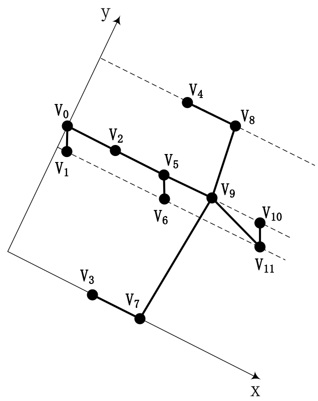

Fig. 8. The optimal topology with the MAI, which is $\frac{29}{12}$

The main complexity to construct the optimal spanning tree is to compute the F functions. In our optimal topologies, the maximum interference does not exceed k. If there are more than mk nodes in $c[s,t]$ that interfere with the nodes on the left of $[s,t]$, there must be a parallel line, and the rightmost node left of $[s,t]$ on the line will experience interference larger than k. Therefore, in a valid $c[s,t]$, there are at most $\min(mk,n)$ nodes that can interfere with one node left of $[s,t]$. Similarly, there are at most mk nodes interfering with one node right of $[s,t]$. The number of different transmission radii of a node v is at most Δ. Therefore, the number of valid $c[0,p]$ is $O((n\Delta)^{mk})$. A similar result can be achieved for $c[p+1,n-1]$. The number of variations of $s[0,p]$ is $O((mk)^{mk})$. As $\Delta \leq n-1$ and $k = O(log\lambda)$, the time complexity to construct the optimal spanning tree with the MAI is $n^{mO(\log \lambda)}$.

As the minimum number of parallel lines to cover all the nodes can be linear to n, $m = O(n)$. Therefore, the time complexity is still exponential in the worst cases. However, in some cases when the nodes are deployed along a few parallel lines, e.g. m is a small constant, our algorithm runs fast.

6 Conclusion

In this paper, we study how to minimize the average interference while preserving connectivity through topology control in wireless sensor networks. In 1D networks, based on the no-cross property and dynamic programming, we propose a fast exact algorithm to compute the minimum average interference. In 2D networks, using computational geometry, we prove that the minimum maximum interference can still be bounded while minimizing the average interference. Moreover, we propose exact algorithms to compute the minimum average interference in 2D networks. In this work, we assume that the interference range is the same as the transmission range. It is meaningful in the future to study interference minimization in networks where the interference range is larger than the transmission range. Other future work directions include interference minimization in 3D networks, and how to reduce interference for network properties besides connectivity, such as planarity, low node degree and small spanner.

Acknowledgements. The research was supported in part by the National Basic Research Program of China grants 2007CB807900 and 2007CB807901, the National Natural Science Foundation of China grants 61073174, 61033001, and 61061130540, the Hi-Tech research and Development Program of China grant 2006AA10Z216, and Hong Kong RGC-GRF grants 714009E and 714311.

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