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热对流边界条件下含球形空洞流体饱和多孔介质的应力分析

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摘要: 基于热局部非平衡条件下饱和多孔介质热-弹性理论, 研究含球形空洞流体饱和多孔介质在热对流边界和完全接触型(固定)边界条件下的温度、孔隙压力以及固相热应力. 应用 Laplace 变换法获得了它们在 Laplace 变换域中的表达式, 数值分析和考察了在球形空洞边界附近处的温度、孔隙压力以及固相热应力的热局部非平衡影响效应. 数值结果表明, 对于热对流边界条件情况, 热局部非平衡影响效应是非常明显的, 特别在 Biot 数为中等值时, 热局部非平衡条件下孔隙压力以及径向应力和切向正应力绝对值的峰值都显著高于热局部平衡条件下所对应的值.

关键词: 饱和多孔介质; 热局部非平衡; Laplace 变换; 孔隙压力; 热应力

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Analysis of Thermal Stresses around Spherical Cavity in Saturated Porous Medium under Convective Heating/Cooling Boundary Conditions

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Abstract: Based on the local thermal non-equilibrium (LTNE) saturated thermo-poroelasticity theory, temperature, pore pressure and thermal stresses around a spherical cavity in an infinite fluid saturated porous medium are investigated, subject to convective heating/cooling on its impermeable and fixed boundary. Solutions of LTNE saturated thermo-poroelasticity are obtained by Laplace transform. Numerical analysis is performed to examine the effects of LTNE under convective heating/cooling boundary conditions on temperature, pore pressure and thermal stresses around the cavity. The results show that the LTNE effects become more pronounced when convective heat transfer boundary conditions are used. Thermally induced pore pressure and the magnitude of thermal stresses are significantly higher than the corresponding ones in classical saturated thermo-poroelasticity. This is particularly true under convective heating/cooling boundary conditions with moderate Biot numbers.

Key words: saturated porous medium; local thermal non-equilibrium; Laplace transform; pore pressure; thermal stress

在经典流体饱和多孔介质热-弹性理论中^[14],对于能量方程的简化通常采用热局部平衡条件假设,即假设在微观水平上固相骨架和孔隙流体之间没有热交换,在表征单元体内固相骨架和孔隙流体保持相同的温度.然而,当流体饱和多孔介质在承受突然加热或冷却时,固相骨架和孔隙流体之间的热交换不可能快速使流-固两相达到热局部平衡状态.有关热局部平衡条件假设适用性方面的研究工作以及热局部非平衡情况下的工程实际问题,可参见文献[5-8].对于热局部平衡条件假设,Minkowycz等^[5]引入一个判别参数 S_p ,并指出:仅当参数 S_p 较大时($S_p \gg 1$),热局部平衡条件假设是合理可行的,此时要求流-固两相之间有较大的热交换系数以及孔隙的表面积与体积之比较高.对于参数 S_p 较小的情况,则热局部非平衡现象(即基本单元体内固相骨架和孔隙流体具有不相同的温度)十分明显,采用热局部非平衡条件假设才能够更加准确地反映多孔介质的传热、孔隙压力以及热应力变化等特性.基于多孔介质理论,在假设流、固两相微观均不可压的基础上,Yang^[9]建立了热局部非平衡条件下饱和多孔弹性介质的热-流-固耦合模型.He等^[10]采用分离变量法,给出了热局部非平衡条件下多孔弹性有限长柱体的拟静态和动力响应解,讨论了热局部非平衡和平衡条件下固相变形和有效应力以及孔隙压力之间的差异.基于Biot理论框架,He等^[11-12]引入加权平均温度,建立了热局部非平衡条件下饱和多孔介质的热-弹性分析模型.对于空洞边处为给定温度边界条件的情况,文献[11-13]分别研究了含柱形和球形空洞流体饱和多孔弹性介质的热应力问题,给出了在短时间内温度场和应力的渐近解析式,讨论了热局部非平衡的影响效应^[11-13].

对于含球形空洞流体饱和多孔介质的热应力分析,其工程背景源于放射性核废料的地下储存问题^[14],目前在热局部平衡条件下已有一些研究工作(包括考虑化学反应),如文献[2,14-17].由于核废料(或核废料储罐等)中某些物质发生化学反应而释放大量热量,导致周围岩石或工程土障温度的升高,因此,地下岩石结构或工程土障材料在热和机械荷载作用下的强度和评估是必须考虑的一个基本要素.本工作基于文献[11-12]的模型,重点讨论空洞边处为热对流边界条件以及完全接触不可渗透情况下(更加符合核废料储罐与周围土障完全接触的情况)流体饱和多孔介质的温度、孔隙压力以及固

相热应力在热局部非平衡条件下的影响效应.结合具体粘土材料,考察Biot数对温度场、孔隙压力以及固相热应力的影响.

1 基本控制方程

假设各向同性多孔弹性体内无热源,忽略固相变形和流场对流对温度场的影响,热局部非平衡条件下流体饱和多孔介质的温度场方程^[11,18]为

$$(1 - \phi)\rho_s c_s \frac{\partial \theta_s}{\partial t} = (1 - \phi)k_{Ts} \Delta \theta_s + h_{im}(\theta_f - \theta_s) = 0, \\ \phi \rho_f c_f \frac{\partial \theta_f}{\partial t} = \phi k_{Tf} \Delta \theta_f + h_{im}(\theta_s - \theta_f) = 0, \quad (1)$$

式中, t 为时间变量, Δ 为直角坐标系 $Oxyz$ 下的Laplace算子, $\theta_s(x, y, z, t)$ 和 $\theta_f(x, y, z, t)$ 分别为固相和流相的温度, ρ_s 和 ρ_f 分别为固相和流相的质量密度, c_s 和 c_f 分别为固相和流相的比热系数, k_{Ts} 和 k_{Tf} 分别为固相和流相的热传导系数, ϕ 为多孔介质的孔隙度, h_{im} 为流、固两相交界面之间的热交换系数.

基于Biot理论框架,在不计体力的影响下,固相位移和孔隙压力满足的基本方程^[11-13]为

$$G \Delta u_i + \frac{G}{1 - 2\nu} u_{k,ki} - \alpha p_{,i} - K \alpha_s \theta_{ave,i} = 0, \quad (2) \\ \frac{\alpha}{BK} (1 - B\alpha) \frac{\partial p}{\partial t} + \alpha \frac{\partial \varepsilon_{kk}}{\partial t} = \\ \frac{k}{\mu} p_{,kk} + \{ \alpha \alpha_s + \phi(\alpha_f - \alpha_s) \} \frac{\partial \theta_{ave}}{\partial t}, \quad (3)$$

式中, $\theta_{ave} = (1 - \phi)\theta_s + \phi\theta_f$.对于热局部非平衡情况,加权平均温度 θ_{ave} 的物理含义就是在体积上加权平均的结果,类似于复合材料理论中有关弹性常数的计算方法^[19].另外,式(2)和(3)中的有关物理量和参数的定义可参见文献[13].

2 含球形空洞饱和多孔介质的温度场

现考虑一含球形空洞的无限大流体饱和多孔介质,如图1所示, a 为球的半径, r 为径向坐标.固相和流相温度的初始条件以及边界条件分别为

$$\theta_s(r, t) |_{t=0} = \theta_f(r, t) |_{t=0} = 0, \quad a \leq r < \infty, \quad (4) \\ \begin{cases} \theta_s(r, t) |_{r=a} = \theta_a H(t), & t > 0, \\ -k_{Tf} \frac{\partial \theta_f(r, t)}{\partial r} \Big|_{r=a} = h_{\pi}(\theta_a - \theta_f) |_{r=a}, & t > 0, \\ \theta_s(r, t) |_{r=\infty} = 0, \theta_f(r, t) |_{r=\infty} = 0, & t > 0, \end{cases} \quad (5)$$

式中, $H(t)$ 为Heaviside函数, θ_a 为温度增量, h_{π} 为多孔介质中流体与核废料储罐(热源)之间的热

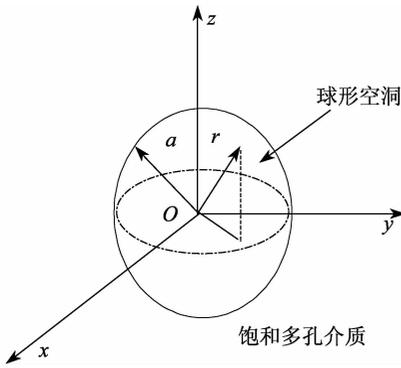


图1 含球形空洞的无限大流体饱和多孔介质

Fig. 1 A spherical cavity in an infinite fluid saturated porous medium

交换系数. 这里不考虑核废料的温度衰减问题, 原因是热局部非平衡效应发生在短时间内. 另外, 当 Biot 数 ($ah_{\text{TF}}/k_{\text{TF}}$) 很大时, 式(5)将退化到第一类边界条件情况(固相和流相在边界处均为给定温度)^[12-13].

引入如下无量纲参数:

$$x = \frac{r}{a}, \quad \tilde{t} = \frac{\kappa t}{a^2}, \quad b_1 = \frac{\phi \rho_f c_f}{(1-\phi)\rho_s c_s}, \quad b_2 = \frac{\phi k_{\text{TF}}}{(1-\phi)k_{\text{Ts}}},$$

$$b_3 = \frac{\kappa}{\kappa_s}, \quad b_{4f} = \frac{k_{\text{TF}}}{ah_{\text{TF}}}, \quad \tilde{h} = \frac{a^2 h_{\text{int}}}{(1-\phi)b_3 k_{\text{Ts}}}, \quad (6)$$

式中, κ 为热局部平衡条件下流体饱和多孔介质的热扩散系数^[11], $\kappa_s = k_{\text{Ts}}/(\rho_s c_s)$ 为固相的热扩散系数. 对于空间球对称问题, 温度场方程(1)的无量纲形式可写为

$$\frac{\partial}{\partial \tilde{t}} \left(\frac{\theta_s}{\theta_a} \right) - \frac{1}{b_3} \Delta \left(\frac{\theta_s}{\theta_a} \right) - \tilde{h} \left(\frac{\theta_f}{\theta_a} - \frac{\theta_s}{\theta_a} \right) = 0,$$

$$b_1 \frac{\partial}{\partial \tilde{t}} \left(\frac{\theta_f}{\theta_a} \right) - \frac{b_2}{b_3} \Delta \left(\frac{\theta_f}{\theta_a} \right) + \tilde{h} \left(\frac{\theta_f}{\theta_a} - \frac{\theta_s}{\theta_a} \right) = 0, \quad (7)$$

式中, $\Delta(\cdot) = \frac{\partial^2(\cdot)}{\partial x^2} + \frac{2}{x} \frac{\partial(\cdot)}{\partial x}$. 对式(7)关于变量 \tilde{t}

作 Laplace 变换, 并应用初始条件(4), 得

$$s \left(\frac{\bar{\theta}_s}{\theta_a} \right) - \frac{1}{b_3} \Delta \left(\frac{\bar{\theta}_s}{\theta_a} \right) - \tilde{h} \left(\frac{\bar{\theta}_f}{\theta_a} - \frac{\bar{\theta}_s}{\theta_a} \right) = 0,$$

$$b_1 s \left(\frac{\bar{\theta}_f}{\theta_a} \right) - \frac{b_2}{b_3} \Delta \left(\frac{\bar{\theta}_f}{\theta_a} \right) + \tilde{h} \left(\frac{\bar{\theta}_f}{\theta_a} - \frac{\bar{\theta}_s}{\theta_a} \right) = 0. \quad (8)$$

另外, 边界条件(5)经 Laplace 变换后, 可得

$$\left. \left(\frac{\bar{\theta}_s(x,s)}{\theta_a} \right) \right|_{x=1} = \left[\frac{\bar{\theta}_f(x,s)}{\theta_a} - b_{4f} \frac{\partial}{\partial x} \left(\frac{\bar{\theta}_f(x,s)}{\theta_a} \right) \right] \Big|_{x=1} = \frac{1}{s},$$

$$\left. \left(\frac{\bar{\theta}_s(x,s)}{\theta_a} \right) \right|_{x \rightarrow \infty} = \left. \left(\frac{\bar{\theta}_f(x,s)}{\theta_a} \right) \right|_{x \rightarrow \infty} = 0, \quad (9)$$

式中, s 表示 Laplace 变换变量, 上标“ $-$ ”表示相应变量的 Laplace 变换. 联合方程组(8), 经简单运算, 可转化为

$$\left(\frac{\partial^2}{\partial x^2} + \frac{2}{x} \frac{\partial}{\partial x} - \lambda_1 \right) \left(\frac{\partial^2}{\partial x^2} + \frac{2}{x} \frac{\partial}{\partial x} - \lambda_2 \right) \left(\frac{\bar{\theta}_s}{\theta_a} \right) = 0,$$

$$\left(\frac{\bar{\theta}_f}{\theta_a} \right) = \left(1 + \frac{s}{\tilde{h}} \right) \left(\frac{\bar{\theta}_s}{\theta_a} \right) - \frac{1}{b_3 \tilde{h}} \left(\frac{\partial^2}{\partial x^2} + \frac{2}{x} \frac{\partial}{\partial x} \right) \left(\frac{\bar{\theta}_s}{\theta_a} \right), \quad (10)$$

式中,

$$\lambda_{1,2} = b_3 (A_1 s + A_2 \pm \sqrt{A_3 s^2 + A_4 s + A_2^2}),$$

$$A_1 = \frac{1}{2} \left(1 + \frac{b_1}{b_2} \right), \quad A_2 = \frac{\tilde{h}}{2} \left(1 + \frac{1}{b_2} \right), \quad A_3 = \frac{1}{4} \left(\frac{b_1}{b_2} - 1 \right)^2,$$

$$A_4 = \frac{\tilde{h}}{2} \left(\frac{1}{b_2} - 1 \right) \left(\frac{b_1}{b_2} - 1 \right). \quad (11)$$

对于常微分方程组(10), 应用边界条件和有界性条件(9), 可求得解为

$$\frac{\bar{\theta}_s(x,s)}{\theta_a} = c_1 \frac{1}{x} e^{-x\sqrt{\lambda_1}} + c_2 \frac{1}{x} e^{-x\sqrt{\lambda_2}},$$

$$\frac{\bar{\theta}_f(x,s)}{\theta_a} = c_1 \left(1 + \frac{s - \lambda_1/b_3}{\tilde{h}} \right) \frac{1}{x} e^{-x\sqrt{\lambda_1}} +$$

$$c_2 \left(1 + \frac{s - \lambda_2/b_3}{\tilde{h}} \right) \frac{1}{x} e^{-x\sqrt{\lambda_2}}, \quad (12)$$

式中,

$$c_1 = \frac{1}{sc_3} \{ s - \lambda_2/b_3 + b_{4f}(1 + \sqrt{\lambda_2})(\tilde{h} + s - \lambda_2/b_3) \} e^{\sqrt{\lambda_1}},$$

$$c_2 = \frac{1}{sc_3} \{ s - \lambda_1/b_3 + b_{4f}(1 + \sqrt{\lambda_1})(\tilde{h} + s - \lambda_1/b_3) \} e^{\sqrt{\lambda_2}},$$

$$c_3 = \frac{(\lambda_1 - \lambda_2)}{b_3} - b_{4f}(1 + \sqrt{\lambda_1})(\tilde{h} + s - \lambda_1/b_3) +$$

$$b_{4f}(1 + \sqrt{\lambda_2})(\tilde{h} + s - \lambda_2/b_3). \quad (13)$$

对于热局部平衡情况, 即 $\theta_s \equiv \theta_f = \theta$, 这里仍考虑对流边界条件, 流体饱和多孔介质无量纲温度场的 Laplace 变换解为

$$\frac{\bar{\theta}(x,s)}{\theta_a} = \frac{1}{s[1 + b_5(1 + \sqrt{s})]} \frac{1}{x} e^{-(x-1)\sqrt{s}}, \quad (14)$$

式中, 无量纲参数 $b_5 = k_{\text{Tm}}/(ah_{\text{Tm}})$, k_{Tm} 为热局部平衡条件下流体饱和多孔介质的热传导系数, h_{Tm} 为球洞周边处多孔介质与核废料储罐之间的热交换系数. 对于式(12)和(14), 一般情况下很难求出 Laplace 逆变换的解析表达式, 通常需要采用反演法进行数值计算^[20].

3 含球形空洞饱和和多孔介质的孔隙压力和固相热应力

对于含球形空洞无限大流体饱和和多孔弹性体(见图1),设球洞边处核废料储罐与周围土障完全接触(固定),孔隙流体不可渗透.流相和固相的初始条件和边界条件可写为

$$\begin{aligned} \sigma_r(r,t)|_{t=0} = u(r,t)|_{t=0} = 0, \quad p(r,t)|_{t=0} = 0, \\ a \leq r < \infty, \\ u(r,t)|_{r=a} = 0, \quad \sigma_r(r,t)|_{r \rightarrow \infty} \rightarrow 0, \quad t > 0, \\ \left. \frac{\partial p(r,t)}{\partial r} \right|_{r=a} = 0, \quad p(r,t)|_{r \rightarrow \infty} \rightarrow 0, \quad t > 0, \end{aligned} \quad (15)$$

式中, u 为固相的径向位移, $\sigma_r(r,t)$ 为固相的径向应力.

对于空间球对称问题,基本方程(2)和(3)可转化为非耦合形式^[12-13],然后,分别求解孔隙压力和固相位移.类似上节的求解过程,首先可获得流体孔隙压力的 Laplace 变换解(无量纲形式解)为

$$\begin{aligned} \frac{\bar{p}(x,s)}{c'\theta_a} = & \left\{ \frac{m_1(s)}{(\tilde{c}\lambda_1 - s)} \frac{(1 + \sqrt{\lambda_1})}{(1 + \sqrt{s/\tilde{c}})} + \right. \\ & \left. \frac{m_2(s)}{(\tilde{c}\lambda_2 - s)} \frac{(1 + \sqrt{\lambda_2})}{(1 + \sqrt{s/\tilde{c}})} \right\} \frac{1}{x} e^{-(x-1)\sqrt{s/\tilde{c}}} - \\ & \frac{m_1(s)}{(\tilde{c}\lambda_1 - s)} \frac{1}{x} e^{-(x-1)\sqrt{\lambda_1}} - \\ & \frac{m_2(s)}{(\tilde{c}\lambda_2 - s)} \frac{1}{x} e^{-(x-1)\sqrt{\lambda_2}}, \end{aligned} \quad (16)$$

式中,

$$\begin{aligned} \tilde{c} = c/\kappa, \quad c = \frac{k}{\mu S}, \quad c' = \frac{1}{S} \left[\frac{2(1-2\nu)}{3(1-\nu)} \alpha \alpha_s + \phi(\alpha_f - \alpha_s) \right], \\ S = \frac{\alpha}{BK} \left(1 - \frac{4\eta B}{3} \right), \quad \eta = \frac{1-2\nu}{2(1-\nu)} \alpha, \end{aligned} \quad (17)$$

$$\begin{aligned} m_1(s) = & \frac{1}{c_3} \{ s - \lambda_2/b_3 + b_{4f}(1 + \sqrt{\lambda_2}) \cdot \\ & (\tilde{h} + s - \lambda_2/b_3) \} \left\{ \frac{\tilde{h} + \phi(s - \lambda_1/b_3)}{\tilde{h}} \right\}, \\ m_2(s) = & -\frac{1}{c_3} \{ s - \lambda_1/b_3 + b_{4f}(1 + \sqrt{\lambda_1}) \cdot \\ & (\tilde{h} + s - \lambda_1/b_3) \} \left\{ \frac{\tilde{h} + \phi(s - \lambda_2/b_3)}{\tilde{h}} \right\}. \end{aligned} \quad (18)$$

对于热局部平衡情况,同理可获得流体饱和和多孔介质流体孔隙压力的 Laplace 变换解为

$$\frac{\bar{p}^{(eq)}(x,s)}{c'\theta_a} = \left\{ \frac{1}{(\tilde{c}-1)[1+b_5(1+\sqrt{s})]} \frac{(1+\sqrt{s})}{(1+\sqrt{s/\tilde{c}})} \right\}.$$

$$\begin{aligned} \frac{1}{s} \left(\frac{1}{x} e^{-(x-1)\sqrt{s/\tilde{c}}} \right) - \left\{ \frac{1}{(\tilde{c}-1)[1+b_5(1+\sqrt{s})]} \right\} \\ \frac{1}{s} \left(\frac{1}{x} e^{-(x-1)\sqrt{s}} \right). \end{aligned} \quad (19)$$

在获得流体孔隙压力以及固相和流相的温度场后,进而可推导出多孔弹性介质的固相位移以及径向应力和切向正应力(无量纲形式解),分别为

$$\begin{aligned} \frac{u(x,\tilde{t})}{a} = & \frac{c'\theta_a(1-2\nu)}{2G(1-\nu)} \frac{1}{x^2} \left\{ \alpha \int_1^x \tilde{x}^2 \left(\frac{p(\tilde{x},\tilde{t})}{c'\theta_a} \right) d\tilde{x} + \right. \\ & \left. \left(\frac{K}{c'\theta_a} \right) (\alpha_s \theta_a) \int_1^x \tilde{x}^2 \left(\frac{\theta_{ave}(\tilde{x},\tilde{t})}{\theta_a} \right) d\tilde{x} \right\}, \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\sigma_r(x,\tilde{t})}{c'\theta_a} = & -\frac{2(1-2\nu)}{(1-\nu)} \frac{1}{x^3} \left\{ \alpha \int_1^x \tilde{x}^2 \left(\frac{p(\tilde{x},\tilde{t})}{c'\theta_a} \right) d\tilde{x} + \right. \\ & \left. \left(\frac{K}{c'\theta_a} \right) (\alpha_s \theta_a) \int_1^x \tilde{x}^2 \left(\frac{\theta_{ave}(\tilde{x},\tilde{t})}{\theta_a} \right) d\tilde{x} \right\}, \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{\sigma_\varphi(x,\tilde{t})}{c'\theta_a} = & \frac{(1-2\nu)}{(1-\nu)} \frac{1}{x^3} \left\{ \alpha \int_1^x \tilde{x}^2 \left(\frac{p(\tilde{x},\tilde{t})}{c'\theta_a} \right) d\tilde{x} + \right. \\ & \left. \left(\frac{K}{c'\theta_a} \right) (\alpha_s \theta_a) \int_1^x \tilde{x}^2 \left(\frac{\theta_{ave}(\tilde{x},\tilde{t})}{\theta_a} \right) d\tilde{x} \right\} - \frac{(1-2\nu)}{(1-\nu)} \cdot \\ & \left\{ \alpha \left(\frac{p(x,\tilde{t})}{c'\theta_a} \right) + \left(\frac{K}{c'\theta_a} \right) (\alpha_s \theta_a) \left(\frac{\theta_{ave}(x,\tilde{t})}{\theta_a} \right) \right\}. \end{aligned} \quad (22)$$

对于热局部平衡情况,同样也可获得固相位移以及径向应力、切向正应力的无量纲形式解.

4 数值结果和分析

对于温度场和孔隙压力,本工作直接采用文献[20]的方法数值反演式(12),(14),(16),(19).设 $F(s)$ 为函数 $f(x)$ 的 Laplace 变换,则函数 $f(x)$ 可由下式^[20]获得:

$$\begin{aligned} f(t) = & \frac{e^{at}}{T^*} \left\{ \frac{1}{2} F(a) + \sum_{k=1}^{\infty} \left[\operatorname{Re} \left[F \left(a + \frac{k\pi i}{T^*} \right) \right] \cos \frac{k\pi t}{T^*} - \right. \right. \\ & \left. \left. \operatorname{Im} \left[F \left(a + \frac{k\pi i}{T^*} \right) \right] \sin \frac{k\pi t}{T^*} \right] \right\}. \end{aligned} \quad (23)$$

考虑计算的精度和收敛性,一般要求 $T^* > t/2$, $2sT^* = 5 \sim 10$,且对式(23)所取的有限项数 $N = 500 \sim 1\,000$.具体材料(粘土)参数参见文献[4,14],即 $\phi = 0.375, G = 1.2 \text{ MPa}, \nu = 0.2, K = 1.6 \text{ MPa}, k = 5.0 \times 10^{-17} \text{ m}^2, \alpha = 1.0, B = 1.0, \rho_s = 2\,600 \text{ kg/m}^3, \rho_f = 1\,000 \text{ kg/m}^3, c_s = 937 \text{ J/(kg}\cdot\text{C)}, c_f = 4\,200 \text{ J/(kg}\cdot\text{C)}, k_{Ts} = 3.29 \text{ W/(m}\cdot\text{C)}, k_{Tf} = 0.6 \text{ W/(m}\cdot\text{C)}, \alpha_s = 3.3 \times 10^{-5} \text{ /C}, \alpha_f = 3.0 \times 10^{-4} \text{ /C}, a = 0.5 \text{ m}$.

由式(6)和(17)以及文献[11],计算可得 $c = 1.6 \times 10^{-7} \text{ m}^2/\text{s}$, $c' = 3.72 \times 10^2 \text{ Pa}/^\circ\text{C}$, $b_1/b_2 = 9.4820$, $b_2 = 0.1091$, $b_3 = 0.3553$. 在数值计算中,假设 $b_5 = b_{4f}$ (即假设热局部平衡情况下多孔介质的 Biot 数与热局部非平衡情况下孔隙流体的 Biot 数相等),并取 $\tilde{h} = 1.0$ (参数 $S_p \approx \tilde{h} b_3 = 0.3553$). 由于在短时间内热局部非平衡影响效应是相似的,下面仅给出无量纲时间 $\tilde{t} = \kappa t/a^2 = 0.1$ 时的结果图.

图2中分别给出了 Biot 数 $Bi = 1/b_{4f} = 1.0$ 和 $Bi = 10.0$ 时热局部非平衡情况下的加权平均温度、固相和流相以及热局部平衡情况下的温度沿径向的分布情况. 可以看出,固相温度总高于流相温度,这是由于在球洞边界处固相温度为给定温度,而流相温度为对流条件;同时,固相的热扩散速度快于流相的热扩散速度. 比较图2(a)和图2(b),可以看出,当 Bi 逐渐增大时,热局部非平衡下的加权平均温度与热局部平衡下的多孔介质温度之间的差异变小,但是热局部非平衡下的加权平均温度总是高于热局部平衡下的多孔介质温度.

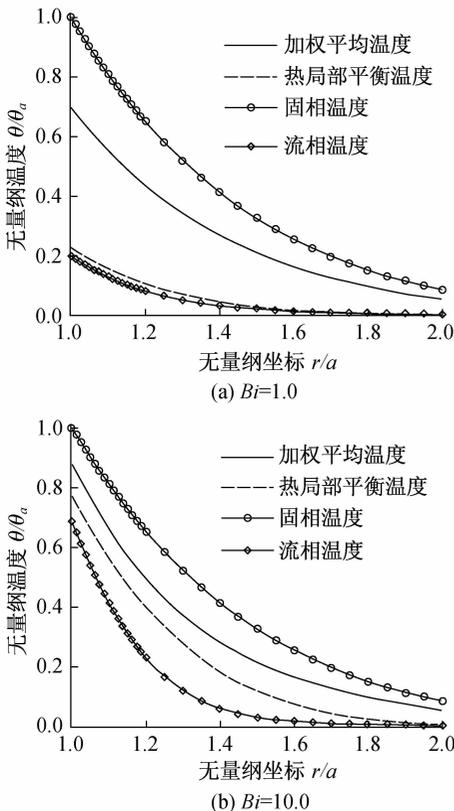


图2 无量纲温度沿径向的分布

Fig.2 Normalized temperature distributions along the radial direction

时无量纲孔隙压力沿径向的分布. 可以看出,由温度所引起的孔隙压力首先随径向距离的增加而单调减小,其最大值发生在球洞边界处. 如 $Bi = 1.0$ 时(见图3(a)),热局部非平衡下无量纲孔隙压力的最大值为42,约为热局部平衡下无量纲孔隙压力最大值(12)的3.5倍. 对于较大的 Biot 数 $Bi = 10.0$ 时(见图3(b)),热局部非平衡与平衡下无量纲孔隙压力的最大值之间的差异变小,但其差异仍是十分明显的.

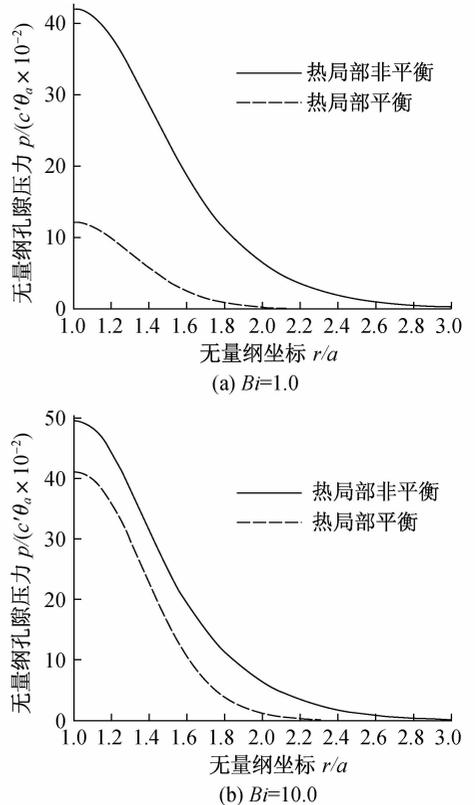


图3 无量纲孔隙压力沿径向的分布

Fig.3 Normalized pore pressure distributions along the radial direction

图4中分别给出了 Biot 数 $Bi = 1.0$ 和 $Bi = 10.0$ 时无量纲径向应力沿径向的分布. 可以看出,径向应力为压应力,径向应力绝对值开始时随径向距离的增加迅速达到峰值,然后,随径向距离的进一步增加而逐步衰减. 热局部非平衡下无量纲径向应力绝对值的峰值明显高于热局部平衡下无量纲径向应力绝对值的峰值. 特别是当 $Bi = 1.0$ 时,热局部非平衡下无量纲径向应力绝对值的峰值为13.6,约为热局部平衡下无量纲径向应力绝对值峰值的3.8倍.

图3中分别给出了 Biot 数 $Bi = 1.0$ 和 $Bi = 10.0$

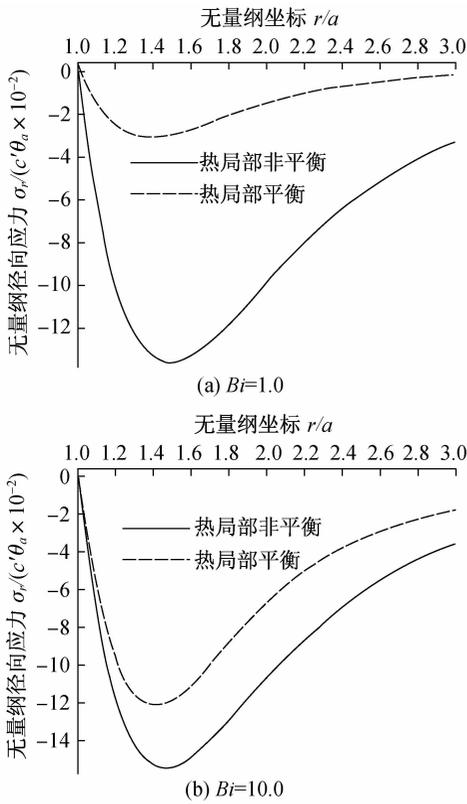


图4 无量纲径向应力沿径向的分布

Fig.4 Normalized radial stress distributions along the radial direction

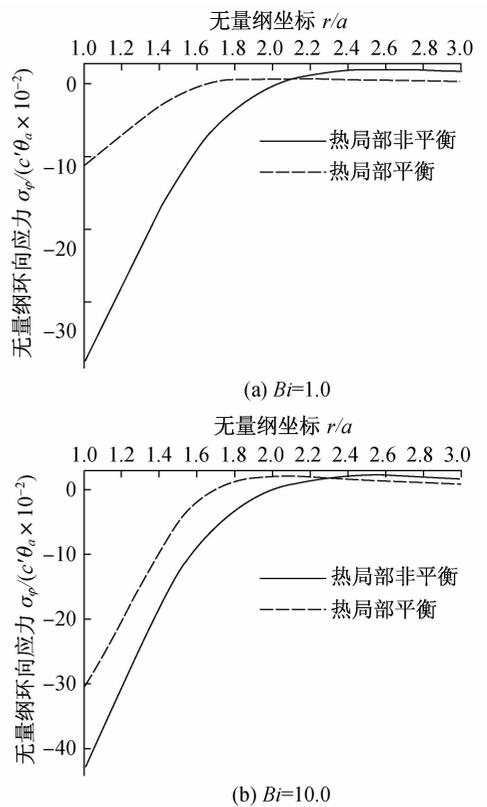


图5 无量纲切向正应力沿径向的分布

Fig.5 Normalized tangential stress distributions along the radial direction

图5中分别给出了Biot数 $Bi = 1.0$ 和 $Bi = 10.0$ 时无量纲切向正应力沿径向的分布。可以看出,在距球洞边附近处切向正应力为压应力,且切向正应力的绝对值随径向距离的增加而单调递减。随着远离球洞边界,切向正应力由压应力逐步变为拉应力,但拉应力的峰值明显远小于压应力绝对值的峰值。对于 $Bi = 1.0$ 时(见图5(a)),热局部非平衡下无量纲切向正应力绝对值的最大值为38,远远超过热局部平衡下无量纲切向正应力绝对值的最大值(11)。

5 结束语

基于热局部非平衡条件下饱和和多孔介质热-弹性分析模型^[11-12],对于含球形空洞流体饱和和多孔弹性介质(无限大体),获得了热对流边界以及完全接触型(固定)边界条件情况下饱和多孔弹性介质温度、孔隙压力在Laplace变换域中的解。结合具体粘土材料,对于Sparrow参数 $S_p \approx 0.3553$ 以及Biot数 $Bi = 1.0$ 和 10.0 的情况,应用Laplace数值反演法分别考察了热局部非平衡条件对温度、孔隙压力以及

固相热应力的影响。当温度边界条件为热对流(突然升温或冷却)情况时,由于热局部非平衡与平衡下温度场在时空上的不同,孔隙压力以及热应力的热局部非平衡影响效应是非常明显的。特别是Biot数 $Bi = 1.0$ 的情况,热局部非平衡条件下孔隙压力以及径向应力和切向正应力绝对值峰值都高于热局部平衡条件下所对应的值的3倍。

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