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具分布时滞的广义 Cohen-Grossberg 神经网络的指数稳定性*1

华玉春[1]，刘兴桂[2]，李永昆[1]
（1. 云南大学 数学系，云南 昆明　650091；2. 云南农业大学 应用数学系，云南 昆明　650201）

摘要：研究了具分布时滞的广义 Cohen-Grossberg 神经网络的动态性质. 运用非线性测度和构造 Lyapunov 函数，获得了神经网络系统存在唯一平衡点且平衡点全局指数稳定的一个充分条件. 这个条件与时滞无关. 最后，给出了一个实例说明所得结论的有效性.

关键词：广义 Cohen-Grossberg 神经网络；分布时滞；Lyapunov 函数；全局指数稳定性
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近年来，由于神经网络动力系统在图像处理、模式识别、联想记忆以及优化等应用领域的巨大潜能，各种神经网络模型已经引起人们的极大关注. Cohen-Grossberg 模型作为一种常用的神经网络也成为许多学者研究的热门问题. 对于神经网络动力系统，目前多数工作都是集中在稳定性的分析上[1-9].

在文献[5]中，考虑了下列带分布时滞的 Cohen-Grossberg 神经网络模型

$$\frac{\mathrm{d}x_i}{\mathrm{d}t}=-a_i(x_i(t))\left[b_i(x_i(t))-\sum_{j=1}^{n}w_{ij}f_j(x_j(t))-\sum_{j=1}^{n}w_{ij}^{\tau}\int_{-\infty}^{t}k_{ij}(t-s)f_j(x_j(s))\mathrm{d}s+I_i\right],\quad i=1,2,\cdots,n. \tag{1}$$

在一定条件下，运用 Brouwer 不动点定理，得到系统(1)平衡点存在. 本文研究系统(1)的一种推广形式，即带分布时滞的广义的 Cohen-Grossberg 神经网络模型

$$\frac{\mathrm{d}x_i}{\mathrm{d}t}=-a_i(x_i(t))\left[b_i(x_i(t))-\sum_{j=1}^{n}w_{ij}f_j(v_jx_j(t))-\sum_{j=1}^{n}w_{ij}^{\tau}\int_{-\infty}^{t}k_{ij}(t-s)g_j(v_j^{\tau}x_j(s))\mathrm{d}s+I_i\right],\quad i=1,2,\cdots,n. \tag{2}$$

在适当条件下，运用非线性测度和构造 Lyapunov 函数，得到系统(2)平衡点唯一存在且全局指数稳定.

在系统(2)中，$n\geqslant 0$ 是指网络中神经元的个数，$x_i(t)$ 表示神经元的状态向量，a_i 表示放大函数，b_i 表示自信号函数，$W=(w_{ij})_{n\times n}$ 和 $W^{\tau}=(w_{ij}{}^{\tau})_{n\times n}$ 分别表示常规的和带时滞的联系权矩阵，f_j 和 g_j 分别表示常规的和带时滞的作用函数，v_j 和 v_j^{τ} 分别表示常规的和带时滞的放大系数，以及 I_i 恒外部输入. 系统(2)的初值条件如下

$$x_i(s)=\varphi_i(s)\in C((-\infty,0],R),s\in(-\infty,0],i=1,2,\cdots,n.$$

这里 $C((-\infty,0],R)$ 表示 $(-\infty,0]$ 上的连续实值函数空间，其中 $\varphi(s)=(\varphi_1(s),\varphi_2(s),\cdots,\varphi_n(s))^{\mathrm{T}}$. 关于系统(2)作如下假设：

(H_1) $\forall s\in R,i=1,2,\cdots,n,a_i$ 连续以及 $0<\underline{a}_i\leqslant a_i(s)\leqslant\overline{a}_i$.

(H_2) b_i 连续，以及存在 $\lambda_i>0$，使得对所有 $s_1,s_2\in R,(s_1-s_2)[b_i(s_1)-b_i(s_2)]\geqslant\lambda_i(s_1-s_2)^2,i=$

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作者简介：华玉春(1982-　)，女，湖南人，硕士生，主要从事微分方程方面的研究.
通讯作者：李永昆(1961-　)，男，彝族，云南人，博士，教授，博士生导师，主要从事微分方程方面的研究.

$1,2,\cdots,n$.

(H_3) f_j 和 g_j 都是 Lipschitz 连续的,

$$m_j = \sup_{s_1,s_2\in R, s_1\neq s_2} \frac{|f_j(s_1)-f_j(s_2)|}{|s_1-s_2|}, \quad m_j^{\tau} = \sup_{s_1,s_2\in R, s_1\neq s_2} \frac{|g_j(s_1)-g_j(s_2)|}{|s_1-s_2|}$$

是 f_j 和 g_j 最小的 Lipschitz 常数,$j=1,2,\cdots,n$.

(H_4) $k_{ij}(t)>0$,$i,j=1,2,\cdots,n$ 是连续的且满足

$$\int_0^{+\infty} e^{\theta t} k_{ij}(t)\,\mathrm{d}t < +\infty, \quad \int_0^{+\infty} k_{ij}(t)\,\mathrm{d}t = 1.$$

其中 θ 为正数.

令 R^n 代表赋予欧几里得范数 $|\cdot|_2$ 的 n 维实向量空间,即对每个 $x=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}\in R^n$,$\|x\|_2^2 = \sum_{i=1}^{n} x_i^2$. $\langle\cdot,\cdot\rangle$ 代表 R^n 上任意 2 个向量的内积.

下面在欧几里得范数意义的基础上介绍非线性测度.

定义 1[10]　假设 Ω 是 R^n 的一个开子集,F 是 $\Omega\to R^n$ 的一个算子. 常数

$$m_{\Omega}(F) = \sup_{x,y\in\Omega, x\neq y} \frac{\langle F(x)-F(y), x-y\rangle}{\|x-y\|_2^2}$$

称为是 F 在 Ω 上的非线性测度.

引理 1[10]　如果 $m_{\Omega}(F)<0$,则 $F:\Omega\to R^n$ 是一个一一映射. 此外,若 $\Omega=R^n$,则 F 是 R^n 上的同胚映射.

下面,首先运用非线性测度以及一些非线性微分分析技巧来证明系统(2)平衡点的存在性和唯一性,然后构造 Lyapunov 函数得到系统(2)平衡点全局稳定性的充分条件.

定理 1　假设 $(H_1)\sim(H_4)$ 都成立,并且存在实数 $c_i>0$,$e_i>0$,k_i,d_i,$\tilde{k}_i$,$\tilde{d}_i$,使得

$$m_i v_i \sum_{j=1}^{n} \frac{e_j}{e_i} |w_{ji}|^{\frac{k_j}{d_i}} + \sum_{j=1}^{n} m_j v_j \frac{e_i c_j}{e_j c_i} |w_{ij}|^{2-\frac{k_i}{d_j}} + m_i^{\tau} v_i^{\tau} \sum_{j=1}^{n} \frac{e_j}{e_i} |w_{ji}^{\tau}|^{\frac{\tilde{k}_j}{\tilde{d}_i}} + \sum_{j=1}^{n} m_j^{\tau} v_j^{\tau} \frac{e_i c_j}{e_j c_i} |w_{ij}^{\tau}|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}} < 2\lambda_i,$$
$$i=1,2,\cdots,n, \tag{3}$$

则对于每个外输入 I_i,系统(2)有唯一的平衡点 x^*.

证明　定义算子 $F:R^n\to R^n$,使它们满足

$$F_i(x) = -\left[b_i(x_i(t)) - \sum_{j=1}^{n} w_{ij} f_j(v_j x_j(t)) - \sum_{j=1}^{n} w_{ij}^{\tau} \int_{-\infty}^{t} k_{ij}(t-s) g_j(v_j^{\tau} x_j(s))\,\mathrm{d}s + I_i\right],$$

其中 $x=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}\in R^n$,$F(x)=(F_1(x),F_2(x),\cdots,F_n(x))^{\mathrm{T}}$.

令 $P=\mathrm{diag}(c_1,c_2,\cdots,c_n)$,$Q=\mathrm{diag}(e_1,e_2,\cdots,e_n)$. 对任意 $y,z\in R^n$,有

$$\begin{aligned}
&\langle QFQ^{-1}Py - QFQ^{-1}Pz, y-z\rangle = \\
&\quad \sum_{i=1}^{n} \Big\{-e_i\Big\{b_i(e_i^{-1}c_iy_i) - b_i(e_i^{-1}c_iz_i) - \sum_{j=1}^{n} w_{ij}\left[f_j(v_je_j^{-1}c_jy_j) - f_j(v_je_j^{-1}c_jz_j)\right] - \\
&\quad \sum_{j=1}^{n} w_{ij}^{\tau}\int_{-\infty}^{t} k_{ij}(t-s)\left[g_j(v_j^{\tau}e_j^{-1}c_jy_j) - g_j(v_j^{\tau}e_j^{-1}c_jz_j)\right]\mathrm{d}s\Big\}(y_i-z_i)\Big\} \leqslant \\
&\quad \sum_{i=1}^{n} \{-e_i\left[b_i(e_i^{-1}c_iy_i) - b_i(e_i^{-1}c_iz_i)\right](y_i-z_i) + \\
&\quad e_i\sum_{j=1}^{n} (|w_{ij}||f_j(v_je_j^{-1}c_jy_j) - f_j(v_je_j^{-1}c_jz_j)||y_i-z_i|) + \\
&\quad e_i\sum_{j=1}^{n} (|w_{ij}^{\tau}|\int_{-\infty}^{t} k_{ij}(t-s)|g_j(v_j^{\tau}e_j^{-1}c_jy_j) - g_j(v_j^{\tau}e_j^{-1}c_jz_j)|\,\mathrm{d}s)|y_i-z_i|\} \leqslant \\
&\quad -\sum_{i=1}^{n}\lambda_ic_i(y_i-z_i)^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\{e_im_jv_je_j^{-1}c_j|w_{ij}||y_j-z_j||y_i-z_i|\} +
\end{aligned}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\{e_i m_j^\tau v_j^\tau e_j^{-1}c_j|w_{ij}^\tau||y_j-z_j||y_i-z_i|\}\leqslant$$

$$-\sum_{i=1}^{n}\lambda_i c_i(y_i-z_i)^2+\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{e_i m_j v_j e_j^{-1}c_j\frac{1}{2}[|w_{ij}|^{\frac{k_i}{d_j}}(y_j-z_j)^2+|w_{ij}|^{2-\frac{k_i}{d_j}}(y_i-z_i)^2]\right\}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{e_i m_j^\tau v_j^\tau e_j^{-1}c_j\frac{1}{2}[|w_{ij}^\tau|^{\frac{\tilde{k}_i}{\tilde{d}_j}}(y_j-z_j)^2+|w_{ij}^\tau|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}(y_i-z_i)^2]\right\}=$$

$$-\sum_{i=1}^{n}\frac{c_i}{2}\Big\{2\lambda_i-m_i v_i\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{k_j}{d_i}}-\sum_{j=1}^{n}\frac{m_j v_j e_i c_j}{e_j c_i}|w_{ij}|^{2-\frac{k_i}{d_j}}-$$

$$m_i^\tau v_i^\tau\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}^\tau|^{\frac{\tilde{k}_j}{\tilde{d}_i}}-\sum_{j=1}^{n}\frac{m_j^\tau v_j^\tau e_i c_j}{e_j c_i}|w_{ij}^\tau|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}\Big\}(y_i-z_i)^2.$$

根据(3)式,可得$m_{R^n}(QFQ^{-1}P)<0$. 由引理1,得出$QFQ^{-1}P$是R^n的一个同胚映射,这表明在R^n上有唯一一点x^*,使得$QFQ^{-1}P(x^*)=0$. 因为Q和P都是可逆的,所以$F(x)=0$有唯一解x^*,即系统(2)有唯一平衡点x^*,证毕.

定理2 假设$(H_1)\sim(H_4)$以及(4)式都成立,则对于每个外输入I_i,系统(1)有唯一平衡点x^*,且x^*是全局稳定的,即存在常数$\sigma>0$,使得系统(1)任意解的指数衰减估计满足

$$\|x(t)-x^*\|_2\leqslant\sqrt{c}e^{-\sigma t}\sup_{s\in(-\infty,0]}|\Psi(s)-x^*\|_2,\quad t>0,\tag{4}$$

其中$c=\max\limits_{1\leqslant i\leqslant n}\left\{\frac{e_i^2}{\underline{a}_i c_i}+\frac{e_i}{2\sigma}\sum_{j=1}^{n}m_j^\tau v_j^\tau\frac{e_j}{c_j}|w_{ij}^\tau|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\left(1-\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}\mathrm{d}s\right)\right\}\Big/\min\limits_{1\leqslant i\leqslant n}\left\{\frac{e_i^2}{\bar{a}_i c_i}\right\}$.

证明 根据$(H_1)\sim(H_4)$以及(3)式,由定理1,可得系统(2)有唯一平衡点$x^*=(x_1^*,x_2^*,\cdots,x_n^*)^{\mathrm{T}}$. 令

$$u_i(t)=\frac{e_i}{\sqrt{c_i}}(x_i(t)-x_i^*),i=1,2,\cdots,n.$$

把$x_i(t)=\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*$代入系统(2),得

$$\frac{\mathrm{d}u_i}{\mathrm{d}t}=-\frac{e_i}{\sqrt{c_i}}a_i\left(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*\right)\Big\{b_i\left(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*\right)-b_i(x_i(t))-$$

$$\sum_{j=1}^{n}w_{ij}[f_j(v_j(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*))-f_j(v_jx_j(t))]-$$

$$\sum_{j=1}^{n}w_{ij}^\tau\int_{-\infty}^{t}k_{ij}(t-s)[g_j(v_j^\tau(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*))-g_j(v_j^\tau x_j(s))\mathrm{d}s]\Big\},$$

$$i=1,2,\cdots,n.\tag{5}$$

记

$$p_i(u_i(t))=a_i\left(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*\right),$$

$$s_j(u_i(t))=f_j\left(v_j\left(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*\right)\right)-f_j(v_jx_j(t)),$$

$$q_i(u_i(t))=b_i\left(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*\right)-b_i(x_i(t)),$$

$$s_j^\tau(u_i(t))=g_j\left(v_j^\tau\left(\frac{\sqrt{c_i}}{e_i}u_i(t)+x_i^*\right)\right)-g_j(v_j^\tau x_j(s)),$$

系统(5)转化成

$$\frac{\mathrm{d}u_i}{\mathrm{d}t}=-\frac{e_i}{\sqrt{c_i}}p_i\left(u_i(t)\right)\left\{q_i\left(u_i(t)\right)-\sum_{j=1}^{n}w_{ij}s_j\left(u_i(t)\right)-\sum_{j=1}^{n}w_{ij}^{\tau}\int_{-\infty}^{t}k_{ij}(t-s)s_j^{\tau}\left(u_i(t)\right)\mathrm{d}s\right\},$$
$$i=1,2,\cdots,n. \tag{6}$$

显然 0 是系统(6)的平衡点.

根据(3)式,找到一个常数 $\sigma>0$,对所有的 $i=1,2,\cdots,n$,有

$$2\lambda_i-m_iv_i\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{k_j}{d_i}}-\sum_{j=1}^{n}m_jv_j\frac{e_ic_j}{e_jc_i}|w_{ij}|^{2-\frac{k_i}{d_j}}-$$
$$m_i^{\tau}v_i^{\tau}\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{\tilde{k}_j}{\tilde{d}_i}}-\frac{2\sigma}{\underline{a}_i}-\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_ic_j}{e_jc_i}|w_{ij}|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)e^{2\sigma s}\mathrm{d}s>0.$$

下面构造下列 Lyapunov 函数

$$V\left(u(t)\right)=\sum_{i=1}^{n}\left\{2e^{2\sigma t}\int_0^{u_i(t)}\frac{s}{p_i(s)}\mathrm{d}s+\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)\int_{t-s}^{t}e^{2\sigma z}u_j^2(z)\mathrm{d}z\mathrm{d}s\right\}. \tag{7}$$

对 V 求导,有

$$\begin{aligned}
\frac{\mathrm{d}V\left(u(t)\right)}{\mathrm{d}t}=&-\sum_{i=1}^{n}2e^{2\sigma t}u_i(t)\frac{e_i}{\sqrt{c_i}}\\
&\left\{q_i\left(u_i(t)\right)-\sum_{j=1}^{n}w_{ij}s_j\left(u_i(t)\right)-\sum_{j=1}^{n}w_{ij}^{\tau}\int_{-\infty}^{t}k_{ij}(t-s)s_j^{\tau}\left(u_i(t)\right)\mathrm{d}s\right\}+\\
&\sum_{i=1}^{n}4\sigma e^{2\sigma t}\int_0^{u_i(t)}\frac{s}{p_i(s)}\mathrm{d}s+\sum_{i=1}^{n}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)e^{2\sigma t}u_j^2(t)\mathrm{d}s-\\
&\sum_{i=1}^{n}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)e^{2\sigma(t-s)}u_j^2(t-s)\mathrm{d}s\leqslant\\
&e^{2\sigma t}\left\{-2\sum_{i=1}^{n}\lambda_iu_i^2(t)+2\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{e_i}{\sqrt{c_i}}m_jv_j\frac{\sqrt{c_j}}{e_j}|w_{ij}||\left(u_i(t)\right)||u_j(t)|+\right.\\
&2\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{e_i}{\sqrt{c_i}}m_j^{\tau}v_j^{\tau}|w_{ij}^{\tau}||u_i(t)|\int_{-\infty}^{t}k_{ij}(t-s)|u_j(s)|\mathrm{d}s+\\
&\sum_{i=1}^{n}\frac{2\sigma}{\underline{a}_i}u_i^2(t)+\sum_{i=1}^{n}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}u_j^2(t)-\\
&\left.\sum_{i=1}^{n}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}u_j^2(t-s)\mathrm{d}s\right\}\leqslant\\
&e^{2\sigma t}\left\{-2\sum_{i=1}^{n}\lambda_iu_i^2(t)+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{e_i}{e_j}m_jv_j\left[\frac{c_j}{c_i}|w_{ij}|^{2-\frac{k_i}{d_j}}\left(u_i^2(t)\right)+|w_{ij}|^{\frac{k_i}{d_j}}u_j^2(t)\right]+\right.\\
&\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{e_i}{e_j}m_j^{\tau}v_j^{\tau}\left[\frac{c_j}{c_i}|w_{ij}^{\tau}|^{2-\frac{k_i}{d_j}}\int_{-\infty}^{t}k_{ij}(t-s)e^{2\sigma(t-s)}u_i^2(t)\mathrm{d}s+\right.\\
&\left.|w_{ij}^{\tau}|^{\frac{k_i}{d_j}}\int_{-\infty}^{t}k_{ij}(t-s)e^{-2\sigma(t-s)}u_j^2(s)\mathrm{d}s\right]+\\
&\sum_{i=1}^{n}\frac{2\sigma}{\underline{a}_i}u_i^2(t)+\sum_{i=1}^{n}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}u_j^2(t)-\\
&\left.\sum_{i=1}^{n}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}u_j^2(t-s)\mathrm{d}s\right\}=\\
&-e^{2\sigma t}\sum_{i=1}^{n}\left\{2\lambda_i-\sum_{j=1}^{n}m_jv_j\frac{e_ic_j}{e_jc_i}|w_{ij}|^{2-\frac{k_i}{d_j}}-m_iv_i\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{k_j}{d_i}}-\right.
\end{aligned}$$

$$\frac{2\sigma}{\underline{a}_i}-\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_ic_j}{e_jc_i}|w_{ij}|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}e^{2\sigma\tau_{ij}}-m_i^{\tau}v_i^{\tau}\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{\tilde{k}_j}{\tilde{d}_i}}\Big\}u_i^2(t)\leqslant$$

$$-e^{2\sigma t}\min_{1\leqslant i\leqslant n}\Big\{2\lambda_i-\sum_{j=1}^{n}m_jv_j\frac{e_ic_j}{e_jc_i}|w_{ij}|^{2-\frac{k_i}{d_j}}-m_iv_i\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{k_j}{d_i}}-$$

$$\frac{2\sigma}{\underline{a}_i}-\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_ic_j}{e_jc_i}|w_{ij}|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)e^{2\sigma s}\mathrm{d}s-m_i^{\tau}v_i^{\tau}\sum_{j=1}^{n}\frac{e_j}{e_i}|w_{ji}|^{\frac{\tilde{k}_j}{\tilde{d}_i}}\Big\}\|u(t)\|_2^2<0.$$

因此对所有 $t\geqslant0$,$V(u(t))\leqslant V(u(0))$.

根据(7)式和 $u_i(t)=\frac{e_i}{\sqrt{c_i}}(x_i(t)-x_i^*)$,有

$$V(u(0))=\sum_{i=1}^{n}\Big\{2\int_0^{u_i(0)}\frac{s}{p_i(s)}\mathrm{d}s+\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\int_0^{+\infty}k_{ij}(s)\int_{-s}^{0}e^{2\sigma z}u_j^2(z)\,\mathrm{d}z\mathrm{d}s\Big\}\leqslant$$

$$\sum_{i=1}^{n}\Big\{\frac{1}{\underline{a}_i}\frac{e_i^2}{c_i}(x_i(0)-x_i^*)^2+$$

$$\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_i}{e_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\sup_{s\in(-\infty,0]}\frac{e_j^2}{c_j}(x_j(s)-x^j)^2\int_0^{+\infty}k_{ij}(s)\int_{-s}^{0}e^{2\sigma z}\,\mathrm{d}z\mathrm{d}s\Big\}\leqslant$$

$$\sum_{i=1}^{n}\Big\{\frac{1}{\underline{a}_i}\frac{e_i^2}{c_i}(x_i(0)-x_i^*)^2+$$

$$\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_ie_j}{c_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\sup_{s\in(-\infty,0]}(x_j(s)-x_j^*)^2\frac{1}{2\sigma}\Big(1-\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}\mathrm{d}s\Big)\Big\}\leqslant$$

$$\max_{1\leqslant i\leqslant n}\Big\{\frac{1}{\underline{a}_i}\frac{e_i^2}{c_i}+\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_ie_j}{c_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\frac{1}{2\sigma}\Big(1-\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}\mathrm{d}s\Big)\Big\}\sup_{s\in(-\infty,0]}\|\varphi_j(s)-x^*\|_2^2=$$

$$\max_{1\leqslant i\leqslant n}\Big\{\frac{e_i^2}{\underline{a}_ic_i}+\frac{e_i}{2\sigma}\sum_{j=1}^{n}m_j^{\tau}v_j^{\tau}\frac{e_j}{c_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\Big(1-\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}\mathrm{d}s\Big)\Big\}\sup_{s\in(-\infty,0]}\|\varphi_j(s)-x^*\|_2^2$$

和

$$\min_{1\leqslant i\leqslant n}\Big\{\frac{1}{\overline{a}_i}\Big(\frac{e_i}{\sqrt{c_i}}\Big)^2\Big\}e^{2\sigma t}\sum_{i=1}^{n}(x_i(t)-x_i^*)^2\leqslant(Vu)(t).$$

进一步得到

$$\|x(t)-x^*\|_2^2\leqslant ce^{-2\sigma t}\sup_{s\in(-\infty,0]}\|\Psi(s)-x^*\|_2^2,\quad t>0.$$

证毕.

对系统(1)的作用函数作如下假设:

$(H_3)'$:f_j 是 Lipschitz 连续的,令 m_j 为 f_j 的最小 Lipschitz 常数.

推论 1 假设(H_1),(H_2),$(H_3)'$以及(H_4)成立,并且存在实数 $l_i>0$,$e_i>0$,k_i,d_i,$\tilde{k}_i$,$\tilde{d}_i$,使得

$$\sum_{j=1}^{n}\frac{e_j}{e_i}\{|w_{ji}|^{\frac{k_j}{d_i}}+|w_{ji}^{\tau}|^{\frac{\tilde{k}_j}{\tilde{d}_i}}\}+\sum_{j=1}^{n}\frac{l_j}{l_i}\{|w_{ij}|^{2-\frac{k_i}{d_j}}+|w_{ij}^{\tau}|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}\}<2\frac{\lambda_i}{m_i},\quad i=1,2,\cdots,n,\tag{8}$$

则对于每个外输入 I_i,系统(1)有唯一平衡点 x^*,且 x^* 全局指数稳定,即存在常数 $\sigma>0$,使得系统(1)任意解的指数衰减估计满足

$$\|x(t)-x^*\|_2\leqslant\sqrt{c}e^{-\sigma t}\sup_{s\in(-\infty,0]}\|\Psi(s)-x^*\|_2,\quad t>0,$$

其中

$$c=\max_{1\leqslant i\leqslant n}\Big\{\frac{m_ie_i}{\underline{a}_il_i}+\frac{e_i}{2\sigma}\sum_{j=1}^{n}\frac{m_j^2}{l_j}|w_{ij}^{\tau}|^{\frac{\tilde{k}_i}{\tilde{d}_j}}\Big(1-\int_0^{+\infty}k_{ij}(s)e^{-2\sigma s}\mathrm{d}s\Big)\Big\}\Big/\min_{1\leqslant i\leqslant n}\Big\{\frac{m_ie_i}{\overline{a}_il_i}\Big\}.$$

证明　因为 $m_j^\tau = m_j, v_j^\tau = v_j = 1$,令 $c_i = \dfrac{e_i l_i}{m_i}, i = 1,2,\cdots,n$,则条件(7)式表明条件(3)式成立.因此由定理 2 直接得出推论 1 的结论,证毕.

注 1　在作用函数满足 Lipschitz 条件下,定理 2 阐述了系统(2)全局稳定性的一个充分条件,这个条件与时滞无关.但由(5)式决定的收敛率 σ 却与时滞有关.

注 2　需注意条件(3)式中所有参数的适当选择,能增加定理的灵活性和有效性.通过对参数的具体选择,可得到一系列新定理.在相应条件下,定理 2 使文献[11 – 13]的定理得到了推广.

我们还需注意,文献[5]在(H_1),(H_2),(H_4)及 b_i^- 连续,且 f_j Lipschitz 连续有界的条件下,得到系统平衡点存在.另外增加条件

$$\max_{1\leqslant i\leqslant n}\Big\{\frac{1}{\lambda_i d_i}\Big(\frac{2\sigma d_i}{\underline{a}_i} + d_i\sum_{j=1}^{n}\big(m_j^{2r_1}|w_{ij}| + m_j^{2r_2}|w_{ij}^\tau|\int_0^{+\infty}k_{ij}(s)e^{2\sigma s}\mathrm{d}s\big) + d_j\sum_{j=1}^{n}\big(m_i^{2(1-r_1)}|w_{ij}| + m_i^{2(1-r_2)}|w_{ij}^\tau|\big)\Big)\Big\}, \tag{9}$$

其中 $d_i > 0, i = 1,2,\cdots,n, \sigma > 0, r_1, r_2 \in [0,1]$ 下,证明了系统(1)全局指数稳定.本文在(H_1),(H_2),(H_4)及 f_j Lipschitz 连续,且不等式(8)

$$\sum_{j=1}^{n}\frac{e_j}{e_i}\{|w_{ji}|^{\frac{k_j}{d_i}} + |w_{ji}^\tau|^{\frac{\tilde{k}_j}{\tilde{d}_i}}\} + \sum_{j=1}^{n}\frac{l_j}{l_i}\{|w_{ij}|^{2-\frac{k_i}{d_j}} + |w_{ij}^\tau|^{2-\frac{\tilde{k}_i}{\tilde{d}_j}}\} < 2\frac{\lambda_i}{m_i},\quad i = 1,2,\cdots,n,$$

成立的条件下,得到系统(1)存在唯一平衡点且平衡点全局指数稳定.显然,在不要求 b_i^- 连续和 f_j 有界的情形下,本文所给推论 1 是文献[5]中定理在某种程度上的推广.条件(8)与条件(9)的强弱关系有待以后研究.

下面用一个例子来说明我们所得到的结果.

例 1　考虑下列带分布时滞的 CGNNs 系统:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = -\begin{bmatrix}2+\sin x_1(t) & 0\\ 0 & 2+\cos x_2(t)\end{bmatrix}\times\left\{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2x_1 + |x_1(t)|\\ 2x_2 + |x_2(t)|\end{bmatrix} - \begin{bmatrix}\frac{1}{16} & \frac{1}{12}\\ \frac{7}{48} & \frac{1}{24}\end{bmatrix}\begin{bmatrix}\frac{1}{2}(x_1+\sin x_1)\\ \frac{1}{2}(x_2+\sin x_2)\end{bmatrix} - \begin{bmatrix}\frac{1}{24} & \frac{1}{48}\\ \frac{1}{16} & \frac{1}{6}\end{bmatrix}\int_{-\infty}^{t}\begin{bmatrix}e^{-(t-s)} & e^{-(t-s)}\\ e^{-(t-s)} & e^{-(t-s)}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\mathrm{d}s + \begin{bmatrix}I_1\\ I_2\end{bmatrix}\right\},$$

则有 $m_1 = m_2 = 2, m_1^\tau = m_2^\tau = 1, v_1 = v_2 = \dfrac{1}{2}, v_1^\tau = v_2^\tau = 1, \lambda_1 = \lambda_2 = 2$.当 $i = 1,2$,取 $c_i = 1, e_i = 1, k_i = d_i, \tilde{k}_i = \tilde{d}_i$.通过计算,有

$$|w_{11}| + |w_{21}| + |w_{11}| + |w_{12}| + |w_{11}^\tau| + |w_{21}^\tau| + |w_{11}^\tau| + |w_{12}^\tau| = \frac{25}{48} < 2\lambda_1,$$

$$|w_{12}| + |w_{22}| + |w_{21}| + |w_{22}| + |w_{12}^\tau| + |w_{22}^\tau| + |w_{21}^\tau| + |w_{22}^\tau| = \frac{27}{48} < 2\lambda_2,$$

因此,由定理 2,此系统全局指数稳定.

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(1. Department of Communication Engineering, Yunnan University, Kunming 650091, China;
2. Meizhouwang College of Technology, Putian 351254, China)

Abstract: This paper presented a new random multi – access MAC protocol for WSN, through dividing the $1+a$ of packet transmit, using probability detection and 1 – persistent control, gave the flow chart and part of program code, realized protocol on GAINZ hardware platform. We analyzed multi – channel random multi – access WSN in terms of theory, then gave some import system parameters include throughput of system, delay of packet transmit and analyzed the energy efficiency. Then, we proved theoretical analysis was true through computer simulation experiment.

Key words: Wireless Sensor Networks; random multi – access; probability detection; 1 – persistent CSMA; protocol designing

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Exponential stability of generalized Cohen – Grossberg neural networks with distributed delays

HUA Yu-chun[1], LIU Xin-gui[2], LI Yong-kun[1]
(1. Department of Mathematics, Yunnan University, Kunming 650091, China;
2. Department of Applied Mathematics, Yunnan Agriculture University, Kunming 650201, China)

Abstract: The dynamics is studied for generalized Cohen – Grossberg neural networks with distributed delays. By means of the nonlinear measure approach and constructing a Lyapunov function, a sufficient condition is obtained for the existence anduniqueness of an equilibrium point of this equations, which of the equilibrium point is global exponential stability. This condition is independent of the delays. Finally, an illustrative examples is given to show the effectiveness of the presented criteria.

Key words: generalized Cohen – Grossberg neuralnetworks; distributed; Lyapunov functional; global exponential stability