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广义 Tortoise 坐标变换与动态 Kerr－Newman－de sitter 黑洞的热辐射*1

李国平，蒋青权，冯中文，邓　娟
（西华师范大学 理论物理研究所，四川 南充　637009）

摘要：根据广义乌龟坐标变换法（GTCT），对动态 Kerr－Newman－de sitter 黑洞的热辐射机制进行了研究．首先，利用在弯曲时空中描述自旋为0，质量为 m 的标量粒子的动力学方程（Klein－Gordon 方程），再通过2种不同广义 Tortoise 坐标变换，最终得到了动态 Kerr－Newman－de sitter 黑洞在这2种不同情况下的热辐射谱，以及各自在事件视界处的霍金温度．通过对比研究发现，在2种不同 Tortoise 坐标变换下得出的热辐射谱，其形式是相同的，但是其值却是有所差异的．其根本原因在于所用的坐标变换不同导致了值存在一定的差别．同时，还分析了新广义 Tortoise 坐标在量纲上的合理性等问题．

关键词：新 Tortoise 坐标变换；Kerr－Newman－de sitter 黑洞；热辐射
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关于黑洞的 Hawking 辐射，早在1973年，霍金就已经证明了在黑洞的事件视界附近存在热辐射[1]．紧接着，人们对热辐射相关问题做了一系列研究[2-8]．黑洞热效应的发现不但解决了黑洞热力学当时存在的矛盾，而且深入地揭示了量子力学、热力学及引力之间的内在联系，所以考察各种类型黑洞的热性质成为黑洞物理学中的重要课题之一．在1976年，Damour 和 Ruffini 介绍了一种不使用二次量子化证明 Hawking 辐射存在的方法，此方法是使用弯曲时空背景上的相对论量子力学证明黑洞存在辐射，而且此方法不仅可以用于讨论表面处各点温度不同的黑洞的热辐射，还可以用于讨论动态黑洞的热辐射．后来，Sannan 用量子场论和量子统计的思想对 Damour 和 Ruffini 的工作进行了一定的改进，并将这种证明热辐射存在的方法的应用范围加以了推广．到2000年，Parikh 和 Wilczek 提出了一种计算黑洞 Hawking 辐射修正谱的半经典方法——隧穿方法[2-10]，人们将这一方法推广到计算不同时空中的黑洞的 Hawking 辐射谱的修正谱[11-19]，所得的结果均满足幺正性原理，并且支持信息守恒．在这种方法的基础上，人们运用旧 Tortoise 变换研究了各种黑洞的热辐射问题，并得出了辐射谱及黑洞的表面引力系数和事件视界出的温度．

然而，最近提出了一种有别于旧 Tortoise 坐标变换的新 Tortoise 坐标变换，本文的目的就是要运用上述证明 Hawking 辐射的 Damour－Ruffini 方法研究新 Tortoise 坐标变换下黑洞的热辐射机制，并将得到的辐射谱其与旧 Tortoise 坐标变换下的辐射谱加以对比．从而进一步认识新 Tortoise 坐标变换的一些特征．

1　动态的 Kerr－Newman－de sitter 黑洞

用超前爱丁顿坐标表示的动态 Kerr－Newman－de sitter 黑洞线元为[20]：

$$\mathrm{d}s^2 = A\frac{1}{\sum}[\Delta_\lambda - \Delta_\theta a^2\sin^2\theta]\mathrm{d}v^2 - 2\sqrt{A}[\mathrm{d}v - a\sin^2\theta\mathrm{d}\varphi]\mathrm{d}r -$$

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作者简介：李国平（1986－），男，四川人，硕士生，主要从事广义相对论与黑洞物理方向的相关研究．E－mail：jli_gp2009@163.com．
通迅作者：蒋青权（1979－），男，四川人，副教授，主要从事黑洞物理与宇宙学相关研究．E－mail：jiangqq@iopp.ccnu.edu.cn．

$$\frac{\sum}{\Delta_\theta}\mathrm{d}\theta^2 + A\frac{2a}{\sum}[\Delta_\theta(r^2+a^2)-\Delta_\lambda]\sin^2\theta\mathrm{d}\upsilon\mathrm{d}\varphi -$$

$$A\frac{1}{\sum}[\Delta_\theta(r^2+a^2)^2-\Delta_\lambda a^2\sin^2\theta]\sin^2\theta\mathrm{d}\varphi^2, \tag{1}$$

其中:

$$\Delta_\lambda = r^2+a^2-2mr+Q^2-\frac{1}{3}\Lambda r^2(r^2+a^2), A = \left[1+\frac{1}{3}\Lambda a^2\right]^{-2}, \tag{2}$$

$$\Delta_\theta = 1+\frac{1}{3}\Lambda a^2\cos^2\theta, \sum = r^2+a^2\cos^2\theta. \tag{3}$$

在上述参数中,Λ 是宇宙学常数,而 A 则是由方程 $A=\left[1+\frac{1}{3}\Lambda a^2\right]^{-2}$ 决定. m 和 Q 则分别是动态 Kerr – Newman – de sitter 黑洞的质量和电荷,并且都随 υ 变化. 从方程(1)可以很容易得到动态 Kerr – Newman – de sitter 黑洞的逆变度规为:

$$g^{00} = -\frac{1}{A}\frac{1}{\sum\Delta_\theta}a^2\sin^2\theta, g^{01} = g^{10} = -\frac{1}{\sqrt{A}}\frac{1}{\sum}(r^2+a^2), g^{03} = g^{30} = -\frac{1}{A}\frac{a}{\sum\Delta_\theta}, \tag{4}$$

$$g^{11} = -\frac{\Delta_\lambda}{\sum}, g^{13} = g^{31} = -\frac{1}{\sqrt{A}}\frac{a}{\sum}, g^{22} = -\frac{\Delta_\theta}{\sum}, g^{33} = -\frac{1}{A\sum\Delta_\theta\sin^2\theta}, \tag{5}$$

其中:$g = -A^2\sum{}^2\sin^2\theta.$ (6)

相关的电磁四矢表示为:

$$A_\mu = \left[\frac{Qr\sqrt{A}}{\sum}, 0, 0, -\frac{Qr\sqrt{A}a\sin^2\theta}{\sum}\right]. \tag{7}$$

我们根据零超曲面方程:

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu} = 0, \tag{8}$$

得到:

$$g^{00}\dot{r}_H^2 - 2g^{01}\dot{r}_H + g^{11} + g^{22}r_H'^2 = 0. \tag{9}$$

将逆变度规带入到方程(9)中得:

$$\frac{a^2\sin^2\theta\dot{r}_H^2}{A\sum\Delta_\theta} - \frac{2(r^2+a^2)\dot{r}_H}{\sqrt{A\sum}} + \frac{\Delta_\lambda}{\sum} + \frac{\Delta_\theta r_H'^2}{\sum} = 0, \tag{10}$$

在这里:$\dot{r} = \frac{\partial r}{\partial \upsilon}, r' = \frac{\partial r}{\partial \theta}$,所以方程(10)式就是决定黑洞视界面 r_H 的方程.

2　动态 Kerr – Newman – de sitter 黑洞在新 Tortoise 坐标变换下的热辐射

弯曲时空中,质量为 μ 的粒子动力学方程用 Klein – Gordon 方程表示为:

$$\frac{1}{\sqrt{g}}\left(\frac{\partial}{\partial x^\mu} - \mathrm{i}eA_\mu\right)\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^\nu} - \mathrm{i}eA^\nu\right)\right]\Phi - \mu^2\Phi = 0, \tag{11}$$

由(4),(5)和(11)式可得:

$$-\frac{a^2\sin^2\theta}{A\sum\Delta_\theta}\frac{\partial^2\Phi}{\partial\upsilon^2} - \frac{2(r^2+a^2)}{\sqrt{A}\sum}\frac{\partial^2\Phi}{\partial\upsilon\partial r} - \frac{2a}{A\sum\Delta_\theta}\frac{\partial^2\Phi}{\partial\upsilon\partial\varphi} - \frac{2a}{\sqrt{A}\sum}\frac{\partial^2\Phi}{\partial r\partial\varphi} -$$

$$\frac{\Delta_\theta}{\sum}\frac{\partial^2\Phi}{\partial\theta^2} - \frac{\Delta_\lambda}{\sum}\frac{\partial^2\Phi}{\partial r^2} - \frac{1}{A\sum\Delta_\theta\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2} - \frac{2r}{\sqrt{A}\sum}\frac{\partial\Phi}{\partial\upsilon} -$$

$$\left[\frac{2\mathrm{ie}Qr}{\sum}+\frac{2\left(r-m-\frac{2}{3}\Lambda r^{3}-\frac{1}{3}\Lambda a^{2}r\right)}{\sum}\right]\frac{\partial\Phi}{\partial r}+$$

$$\frac{\frac{1}{3}\Lambda a^{2}\sin2\theta\sin\theta-\cos\theta\Delta_{\theta}}{\sum\sin\theta}\frac{\partial\Phi}{\partial\theta}-\mu^{2}\Phi=0. \tag{12}$$

引入新 Tortoise 变换:

$$r_{*}=r+\frac{1}{2\kappa(\upsilon_{0},\theta_{0})}\ln\left[\frac{r-r_{\mathrm{H}}(\upsilon,\theta)}{r_{\mathrm{H}}(\upsilon,\theta)}\right],\upsilon_{*}=\upsilon-\upsilon_{0},\theta=\theta_{*}-\theta_{0}, \tag{13}$$

其中υ_0和θ_0在新 Tortoise 坐标变换下都是常数,并且υ_0表示粒子从黑洞的事件视界逃离的时刻及描述黑洞的演化,θ_0则表示粒子从事件视界逃离出去的位置和描述黑洞的形状.

将(13)式带入 Klein - Gordon 方程(12)得:

$$\left\{-\frac{\Delta_{\lambda}}{\sum}\left[1+\frac{1}{2\kappa(r-r_{\mathrm{H}})}\right]^{2}-\frac{\Delta_{\theta}}{\sum}\frac{r^{2}r_{\mathrm{H}}'^{2}}{4\kappa^{2}r_{\mathrm{H}}^{2}(r-r_{\mathrm{H}})^{2}}-\frac{a^{2}\sin^{2}\theta}{A\sum\Delta_{\theta}}\frac{r^{2}\dot{r}_{\mathrm{H}}^{2}}{4\kappa^{2}r_{\mathrm{H}}^{2}(r-r_{\mathrm{H}})^{2}}+\right.$$

$$\left.\frac{2(r^{2}+a^{2})}{\sqrt{A}\sum}\frac{r\dot{r}_{\mathrm{H}}}{2\kappa r_{\mathrm{H}}(r-r_{\mathrm{H}})}\left[1+\frac{1}{2\kappa(r-r_{\mathrm{H}})}\right]\right\}\frac{\partial^{2}\Phi}{\partial r_{*}^{2}}+\left\{\frac{a^{2}\sin^{2}\theta}{A\sum\Delta_{\theta}}\frac{2r\dot{r}_{\mathrm{H}}}{2\kappa r_{\mathrm{H}}(r-r_{\mathrm{H}})}-\right.$$

$$\left.\frac{2(r^{2}+a^{2})}{\sqrt{A}\sum}\left[1+\frac{1}{2\kappa(r-r_{\mathrm{H}})}\right]\right\}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\upsilon_{*}}+\left\{-\left[\frac{2\left(r-m-\frac{2}{3}\Lambda r^{3}-\frac{1}{3}\Lambda a^{2}r\right)}{\sum}+\frac{2\mathrm{ie}Qr}{\sum}\right]\cdot\right.$$

$$\left[1+\frac{1}{2\kappa(r-r_{\mathrm{H}})}\right]+\frac{2r}{\sqrt{A}\sum}\frac{r\dot{r}_{\mathrm{H}}}{2\kappa r_{\mathrm{H}}(r-r_{\mathrm{H}})}-\frac{rr_{\mathrm{H}}'}{2\kappa r_{\mathrm{H}}(r-r_{\mathrm{H}})}\left[\frac{\frac{1}{3}\Lambda a^{2}\sin2\theta\sin\theta-\cos\theta\Delta_{\theta}}{\sum\sin\theta}\right]+$$

$$\frac{1}{2\kappa(r-r_{\mathrm{H}})^{2}}\frac{\Delta_{\lambda}}{\sum}+\frac{\Delta_{\theta}}{\sum}\frac{rr_{\mathrm{H}}''r_{\mathrm{H}}(r-r_{\mathrm{H}})-rr_{\mathrm{H}}'(r-2r_{\mathrm{H}})}{2\kappa r_{\mathrm{H}}^{2}(r-r_{\mathrm{H}})^{2}}+\frac{r\ddot{r}_{\mathrm{H}}r_{\mathrm{H}}(r-r_{\mathrm{H}})-r\dot{r}_{\mathrm{H}}^{2}(r-2r_{\mathrm{H}})}{2\kappa r_{\mathrm{H}}^{2}(r-r_{\mathrm{H}})^{2}}\cdot$$

$$\left.\frac{a^{2}\sin^{2}\theta}{A\sum\Delta_{\theta}}-\frac{\dot{r}_{\mathrm{H}}}{2\kappa(r-r_{\mathrm{H}})^{2}}\frac{2(r^{2}+a^{2})}{\sqrt{A}\sum}\right\}\frac{\partial\Phi}{\partial r_{*}}+\left\{\frac{r\dot{r}_{\mathrm{H}}}{2\kappa r_{\mathrm{H}}(r-r_{\mathrm{H}})}\frac{2a}{A\sum\Delta_{\theta}}+\right.$$

$$\left.\left[1+\frac{1}{2\kappa(r-r_{\mathrm{H}})}\right]+\frac{2a}{\sqrt{A}\sum}\right\}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\varphi}+\left\{\frac{rr_{\mathrm{H}}'}{2\kappa r_{\mathrm{H}}(r-r_{\mathrm{H}})}\frac{\Delta_{\theta}}{\sum}\right\}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\theta_{*}}+$$

$$\left\{\frac{a^{2}\sin^{2}\theta}{A\sum\Delta_{\theta}}\frac{\partial^{2}\Phi}{\partial\upsilon_{*}^{2}}-\frac{\Delta_{\theta}}{\sum}\frac{\partial^{2}\Phi}{\partial\theta_{*}^{2}}-\frac{1}{A\sum\Delta_{\theta}\sin^{2}\theta}\frac{\partial^{2}\Phi}{\partial\varphi^{2}}-\frac{2r}{\sqrt{A}\sum}\frac{\partial\Phi}{\partial\upsilon_{*}}-\right.$$

$$\left.\frac{2a}{A\sum\Delta_{\theta}}\frac{\partial^{2}\Phi}{\partial\upsilon_{*}\partial\varphi}-\mu^{2}\Phi+\frac{\frac{1}{3}\Lambda a^{2}\sin2\theta\sin\theta-\cos\theta\Delta_{\theta}}{\sum\sin\theta}\frac{\partial\Phi}{\partial\theta_{*}}\right\}=0, \tag{14}$$

在黑洞的视界面附近,有$r\to r_{\mathrm{H}},\upsilon\to\upsilon_0,\theta\to\theta_0$,整理方程(14)可得:

$$\alpha\frac{\partial^{2}\Phi}{\partial r_{*}^{2}}+\frac{\partial^{2}\Phi}{\partial r_{*}\partial\upsilon_{*}}+2\Omega_{\mathrm{H}}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\varphi}+2C_{2}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\theta_{*}}+2(C_{1}+\mathrm{ie}\Phi_{\mathrm{H}})\frac{\partial\Phi}{\partial r_{*}}=0, \tag{15}$$

其中:

$$\alpha=\frac{2A\Delta_{\theta}^{2}r_{\mathrm{H}}'^{2}+2a^{2}\sin^{2}\theta_{0}\dot{r}_{\mathrm{H}}^{2}+A\Delta_{\lambda}'\Delta_{\theta}r_{\mathrm{H}}-2\sqrt{A}\Delta_{\theta}\dot{r}_{\mathrm{H}}(3r_{\mathrm{H}}^{2}+a^{2})}{2\kappa[(r_{\mathrm{H}}^{2}+a^{2})\Delta_{\theta}r_{\mathrm{H}}\sqrt{A}-a^{2}\sin^{2}\theta_{0}r_{\mathrm{H}}\dot{r}_{\mathrm{H}}]-4\kappa\Delta_{\lambda}\Delta_{\theta}Ar_{\mathrm{H}}+4\kappa\sqrt{A}\Delta_{\theta}r_{\mathrm{H}}\dot{r}_{\mathrm{H}}(r_{\mathrm{H}}^{2}+a^{2})}, \tag{16}$$

$$\Omega_{\mathrm{H}}=\frac{a\Delta_{\theta}\sqrt{A}-a\dot{r}_{\mathrm{H}}}{(r_{\mathrm{H}}^{2}+a^{2})\Delta_{\theta}\sqrt{A}-a\sin^{2}\theta_{0}\dot{r}_{\mathrm{H}}}, \tag{17}$$

$$C_2 = \frac{-A\Delta_\theta^2 \dot{r}_H}{(r_H^2 + a^2)\Delta_\theta\sqrt{A} - a^2\sin^2\theta_0\dot{r}_H}, \tag{18}$$

$$C_1 = \frac{A\Delta_\theta r'_H\left(\frac{1}{3}\right)\Lambda a\sin 2\theta_0 - 2r_H\dot{r}_H\sqrt{A}\Delta_\theta + 4r_H\dot{r}_H\Delta_\theta\sqrt{A} - a^2\sin^2\theta_0\ddot{r}_H - A\Delta_\theta^2 r''_H - \frac{\cot\theta_0\Delta_\theta}{r_H}}{2[(r_H^2 + a^2)\Delta_\theta\sqrt{A} - a^2\sin^2\theta_0\dot{r}_H]}, \tag{19}$$

$$\Phi_H = \frac{QA\Delta_\theta r_H}{[(r_H^2 + a^2)\Delta_\theta\sqrt{A} - a^2\sin^2\theta_0\dot{r}_H]}, \tag{20}$$

取 α 的值为 1,则可得:

$$\kappa = \frac{2A\Delta_\theta^2 r_H'^2 + 2a^2\sin^2\theta_0\dot{r}_H^2 + A\Delta'_\lambda\Delta_\theta r_H - 2\sqrt{A}\Delta_\theta\dot{r}_H(3r_H^2 + a^2)}{2[(r_H^2 + a^2)\Delta_\theta r_H\sqrt{A} - a^2\sin^2\theta_0 r_H\dot{r}_H] - 4\Delta_\lambda\Delta_\theta A r_H + 4\sqrt{A}\Delta_\theta r_H\dot{r}_H(r_H^2 + a^2)}, \tag{21}$$

$$\kappa = \frac{\left(\frac{A\Delta'_\lambda\Delta_\theta}{2}r_H - 2\sqrt{A}\Delta_\theta r_H\dot{r}_H\right) + 2\sqrt{A}\Delta_\theta r_H\dot{r}_H - A\Delta_\theta\Delta_\lambda + \sqrt{A}\Delta_\theta\dot{r}_H(r_H^2 + a^2)}{\sqrt{A}\Delta_\theta(r_H^2 + a^2)r_H - \frac{r_H a^2\sin^2\theta_0}{2(r_H^2 + a^2)}\left(\frac{a^2\dot{r}_H\sin^2\theta_0}{\sqrt{A}\Delta_\theta} + \sqrt{A}\Delta_\lambda + \sqrt{A}\Delta_\theta r_H'^2\right) - 2\Delta_\theta\Delta_\lambda r_H + 2\sqrt{A}\Delta_\theta\dot{r}_H(r_H^2 + a^2)}. \tag{22}$$

对于方程(15):

$$\frac{\partial^2\Phi}{\partial r_*^2} + 2\frac{\partial^2\Phi}{\partial r_*\partial v_*} + 2\Omega_H\frac{\partial^2\Phi}{\partial r_*\partial\varphi} + 2C_2\frac{\partial^2\Phi}{\partial r_*\partial\theta_*} + 2(C_1 + \mathrm{i}e\Phi_H)\frac{\partial\Phi}{\partial r_*} = 0, \tag{23}$$

采用分离变量法求解,则将波函数 Φ 写成如下形式:

$$\Phi = R(r_*)\Theta(\theta_*)\mathrm{e}^{\mathrm{i}\varepsilon\phi - \mathrm{i}\omega v_*}, \tag{24}$$

其中,ω 是 Klein - Gordon 粒子的能量,而 ε 则是 Klein - Gordon 粒子的角动量在 ϕ 轴上的投影,经过计算,得出方程(23) 的解为:

$$\Theta' = \lambda\Theta,$$

$$R'' + 2(A + \mathrm{i}\varepsilon\Omega_H + \mathrm{i}e\Phi_H - \mathrm{i}\omega)R' = 0, \tag{25}$$

其中 λ 是引入的常数,而:$A = \lambda C_2 + C_1$, (26)

那么,很容易解出方程(25) 式的解为:

$$\Theta = c_1\mathrm{e}^{\lambda\theta_*}, \tag{27}$$

$$R = c_2\mathrm{e}^{2\mathrm{i}(\omega - \zeta\Omega_H - e\Phi_H)r_* - 2Ar_*} + c_3, \tag{28}$$

在(27),(28) 式中,c_2,c_2 和 c_3 都是常数,θ_* 是极角,所以动态 Kerr - Newman - de sitter 黑洞带电粒子的径向波动方程的入射波解和出射波解为:

$$R_{in} = R_{in}^+ = \mathrm{e}^{-\mathrm{i}\omega v_*}, \tag{29}$$

$$R_{out} = R_{out}^+ = \mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega - \varepsilon\Omega_H - e\Phi_H)r_*}\mathrm{e}^{-2Ar_*}, \tag{30}$$

由于 R_{in}^+ 在视界处是解析的,而 R_{out}^+ 则存在奇异点,所以用 Darmour - Ruffini 方法进行解析沿拓到黑洞的视界内部,得:

$$\bar{R}_{out}^+ = \mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega - \varepsilon\Omega_H - e\Phi_H)r_*}\mathrm{e}^{-2Ar_*}\cdot\mathrm{e}^{\frac{\mathrm{i}\pi A}{\kappa}}\mathrm{e}^{\frac{\pi(\omega - \varepsilon\Omega_H - e\Phi_H)}{\kappa}}, \tag{31}$$

再根据 Darmour - Ruffini - Sannan 的思想,就可获得出射波在视界外的辐射谱为:

$$N^+ = \frac{1}{\mathrm{e}^{\frac{(\omega - \varepsilon\Omega_H - e\Phi_H)}{K_B T^+}} \pm 1}, \tag{32}$$

很容易得到其温度为:

$$T^+ = \frac{\kappa}{2\pi}, \tag{33}$$

其中 κ 如(22) 式所示.

同理,可以获得在黑洞宇宙视界附近的辐射谱与温度:

$$N^{c}=\frac{1}{e^{\frac{(\omega-\varepsilon\Omega_{H}-e\Phi_{H})}{K_{B}T^{c}}}\pm 1}, \tag{34}$$

$$T^{c}=\frac{\kappa_{c}}{2\pi}, \tag{35}$$

κ_c 是宇宙视界处的表面引力.

3 旧 Tortoise 坐标变换下动态 Kerr - Newman - de sitter 黑洞的热辐射

在以前引用旧 Tortoise 坐标变换[20] 同样可以得到黑洞的热辐射,旧 Tortoise 坐标为:

$$r_{*}=\frac{1}{2\kappa(v_{0},\theta_{0})}\ln[r-r_{H}(v,\theta)],v_{*}=v-v_{0},\theta_{*}=\theta-\theta_{0}, \tag{36}$$

将旧 Tortoise 坐标带入 Klein - Gordon 方程中,在趋近视界面附近时,整理化简方程得:

$$\alpha\frac{\partial^{2}\Phi}{\partial r_{*}^{2}}+2\frac{\partial^{2}\Phi}{\partial r_{*}\partial v_{*}}+2\Omega_{H}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\varphi}+2C_{2}\frac{\partial^{2}\Phi}{\partial r_{*}\partial\theta_{*}}+2(C_{1}+\mathrm{i}e\Phi_{H})\frac{\partial\Phi}{\partial r_{*}}=0, \tag{37}$$

其中:

$$\alpha=\frac{-2\sqrt{A}\Delta_{\theta}r_{H}\dot{r}_{H}+A\Delta_{\theta}[r_{H}-m-\frac{2}{3}\Lambda r_{H}^{3}-\frac{1}{3}\Lambda a^{2}r_{H}]}{\kappa[\sqrt{A}\Delta_{\theta}(r_{H}^{2}+a^{2})-a^{2}\sin^{2}\theta_{0}\dot{r}_{H}]}, \tag{38}$$

$$\Omega_{H}=\frac{a\Delta_{\theta}\sqrt{A}-a\dot{r}_{H}}{(r_{H}^{2}+a^{2})\Delta_{\theta}\sqrt{A}-a\sin^{2}\theta_{0}\dot{r}_{H}}, \tag{39}$$

$$C_{2}=\frac{-A\Delta_{\theta}^{2}r'_{H}}{(r_{H}^{2}+a^{2})\Delta_{\theta}\sqrt{A}-a^{2}\sin^{2}\theta_{0}\dot{r}_{H}}, \tag{40}$$

$$\Phi_{H}=\frac{QA\Delta_{\theta}r_{H}}{[(r_{H}^{2}+a^{2})\Delta_{\theta}\sqrt{A}-a^{2}\sin^{2}\theta_{0}\dot{r}_{H}]}, \tag{41}$$

$$C_{1}=-\frac{\Phi_{H}}{2\Delta_{\theta}r_{H}}\left[\frac{a^{2}\sin^{2}\theta_{0}\ddot{r}_{H}}{A\Delta_{\theta}}-\frac{2r_{H}r'_{H}}{\sqrt{A}}+\Delta_{\theta}r_{H}-\frac{\frac{1}{3}\Lambda a^{2}\sin2\theta_{0}\sin\theta_{0}-\cos\theta_{0}\Delta_{\theta}r'_{H}}{\sin\theta_{0}}\right]. \tag{42}$$

κ' 则可以表示为:

$$\kappa'=\frac{2\sqrt{A}\Delta_{\theta}r_{H}\dot{r}_{H}+A\Delta_{\theta}[r_{H}-m-\frac{2}{3}\Lambda r_{H}^{3}-\frac{1}{3}\Lambda a^{2}r_{H}]}{\sqrt{A}\Delta_{\theta}(r_{H}^{2}+a^{2})-\frac{a^{2}\sin^{2}\theta_{0}}{2(r_{H}^{2}+a^{2})}\left(\frac{a^{2}\dot{r}_{H}\sin^{2}\theta_{0}}{\sqrt{A}\Delta_{\theta}}+\sqrt{A}\Delta_{\lambda}+\sqrt{A}\Delta_{\theta}r_{H}'^{2}\right)}. \tag{43}$$

最终,通过计算波动方程(37),得到出射波和入射波的解,并且通过 Darmour - Ruffini - Sunnan 的方法和建议,计算出旧 Tortoise 坐标变换下的辐射谱和温度分别为:

$$N^{+}=\frac{1}{e^{\frac{(\omega-\varepsilon\Omega_{H}-e\Phi_{H})}{K_{B}T^{+}}}\pm 1}, \tag{44}$$

$$T_{+}=\frac{\kappa'}{2\pi}, \tag{45}$$

κ' 如(43) 式所示.

所以黑洞在宇宙视界处的辐射谱和温度为:

$$N^{c}=\frac{1}{e^{\frac{(\omega-\varepsilon\Omega_{H}-e\Phi_{H})}{K_{B}T^{c}}}\pm 1}, \tag{46}$$

$$T_{c}=\frac{\kappa'_{c}}{2\pi}, \tag{47}$$

κ'_c是宇宙视界处的表面引力.

4　讨　论

在以往的研究中,人们用旧 Tortoise 坐标变换对一系列黑洞的热辐射特性进行了研究和讨论,并推动了黑洞物理的发展. 由文中可以看出,在新Tortoise坐标变换下,黑洞视界处的辐射谱 $N^{+}=\frac{1}{e^{\frac{(\omega-\varepsilon\Omega_{H}-e\Phi_{H})}{K_{B}T^{+}}}\pm 1}$,它们的具体形式是没有差别的,但是和辐射谱相关的 κ 值却有一定的微小差别. 同理在宇宙视界处,新旧 Tortoise 坐标变换下的 κ_c 也是有一定的差别. 因此,本文认为新 Tortoise 坐标的量纲性质更加合理,所得出的结果也更具有可靠性.

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(下转第 206 页)

Abstract: A method is established for the determination of notoginsenoside R_1, ginsenoside Rg_1 and ginsenoside Rb_1 in *Panax zingiberensis*. A HPLC method was adopted. The analysis was carried out on all analytical column C18(50 mm × 4.6 mm, luna 5 μm). The flow rate was 1.0 mL · min^{-1} and the detective wavelength was set at 203 mm with column temperature of 30 ℃. The mobile phase consisted of acetonitrile – water(gradient elution). The linear range was 0.315—1.575 μg(r = 0.999 1) and the average recovery was 101.4% with the RSD of 1.79% for notoginsenoside R_1. The linear range was 1.203—6.015 μg(r = 0.999 8) and the average recovery was 98.54% with the RSD of 1.9% for ginsenoside Rg_1. The linear range was 0.276—1.38 μg(r = 0.999 6) and the average recovery was 102.1% with the RSD of 1.53% for ginsenoside Rb_1. The method is simple, reproducible and suitable for determination of notoginsenoside R_1, ginsenoside Rg_1 and ginsenoside Rb_1 in *Panax zingiberensis*.

Key words: *Panax zingiberensis*; notoginsenoside R_1; ginsenoside Rg_1; ginsenoside Rb_1; HPLC

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(上接第201页)

A new tortoise coordination transformation and the thermal radiation of non – stationary Kerr – Newman – de sitter black hole

LI Guo-ping, JIANG Qing-quan, FENG Zhong-wen, DENG Juan

(Institute of Theoretical Physics, China West Normal University, Nanchong 637009, China)

Abstract: Based on the method of Tortoise coordination transformation(GTCT), the thermal radiation of Kerr – Newman – de sitter black hole is investigated. First, We make use of the Klein – Gordon function which depicted by the scalar particle, its mass is μ and spin is zero. Then, the thermal radiation spectrum of the dynamic Kerr – Newman – de sitter black hole is obtained in two different tortoise coordination transformations. Finally, we can attain the temperature at the event horizon. Also We find the form of the thermal radiation spectrum with two different tortoise coordination transformations are same, but the value of the thermal radiation spectrum are different. That different tortoise coordination transformations lead the value is different is the basic reason. Furthermore, we study the rationality and other related issues of the new tortoise coordination transformation.

Key words: new Tortoise coordination transformation; Kerr – Newman – de sitter black hole; thermal radiation