Article ID: 1007- 2985(2004) 02- 0015- 06

Damage Identification of Actual Bridge Based on Subspace Rotation Algorithm

ZHONG Wei-qiu, GONG Jin-xin, LIU Yi
(Department of Civil Engineering, Dalian University of Technology, Dalian 116023, Liaoning China)

Abstract: Structural damage identification is critical to the reliability evaluation of structures. Now damage identification method based on parameters of structural model is one of the research hotspots. Subspace Rotation Algorithm introduced in this paper belongs to the damage identification method based on parameters of structural model. Subspace Rotation Algorithm is based on finite element method of structures, use the matrix transform method and separates the damage location and damage extent problems and is computationally inexpensive. Subspace Rotation Algorithm is used to detect damage of an actual bridge in this paper and practice testifies that Subspace Rotation Algorithm only needs the first order frequency and shape mode to identify the main damage location and damage extent of the actual bridge so that it is simple in calculation and feasible.

Key words: damage identification; Subspace Rotation Algorithm; bridge
CLC number: TU311. 3　　**Document code**: A

1　Subspace Rotation Algorithm

In 1992 Zimmerman and Kaouk[1] presented Subspace Rotation Algorithm in which they separated the damage location and damage extent problems. Both of these sub- problems required very simple mathematical manipulations; therefore, the Subspace Rotation Algorithm was computationally inexpensive. Kahl and Sirkis[2] demonstrated that this Algorithm could detect the spatial location and extent of a damage event or damage events in a truss structure using perfect eigenvalue and eigenvector data. In this paper Subspace Rotation Algorithm is used for detecting the location and extent of damage events in a continuous beam structure. In this paper there is an example in which Subspace Rotation Algorithm is used for detecting the damage of an actual bridge.

1. 1 Damage Location

The damage location Algorithm, as developed by Zimmerman and Kaouk, starts with an n- degree- of- freedom finite element model of the undamaged structure:

$$\boldsymbol{M}\boldsymbol{x} + \boldsymbol{C}\boldsymbol{x} + \boldsymbol{K}\boldsymbol{x} = 0, \tag{1}$$

where $\boldsymbol{M}$ is the $n \quad n$ ideal mass matrix, $\boldsymbol{C}$ is the $n \quad n$ ideal damping matrix, $\boldsymbol{K}$ is the $n \quad n$ ideal stiffness matrix, $\boldsymbol{x}$ is the $n \quad 1$ displacement vector. By assuming that the displacement vector is a harmonic solution of the form $\boldsymbol{x} = \; e^{-j \; t}$, an associated eigen- problem is obtained:

$$(- \;{}^{2}_{hi}\boldsymbol{M} - j \;{}_{hi}\boldsymbol{C} + \boldsymbol{K}) \;{}_{hi} = 0, \tag{2}$$

Received date: 2003- 06- 29
Foundation item: Supported by National Natural Science Foundation of China(50078009) ; Provincial Natural Science Foundation of Liaoning Province of China(001077)
Biography: ZHONG Wei-qiu(1974-) , male, was born in Changchun City, Jilin Province, PhD, lecturer of the Department of Civil Engineering, Dalian University of Technology, majoring in structural engineering.

© 1994-2012 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

where $_i$ is the ith natural frequency and $_i$ is the corresponding mode shape ($i = 1, \;, n$). The subscript h indicates a healthy pre- damaged structure in which the natural frequencies and mode shapes satisfy the ideal eigen-problem.

Zimmerman and Kaouk assumed that damage to the structural system is manifested by changes of the property matrices from ($\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$) to ($\boldsymbol{M} - \; \boldsymbol{M}_d, \boldsymbol{C} - \; \boldsymbol{C}_d, \boldsymbol{K} - \; \boldsymbol{K}_d$) with associated changes of the natural frequencies and mode shapes from ($_{hi}, \; _{hi}$) to ($_{di}, \; _{di}$), where the subscript d denotes a damaged structure. The damaged strueture then satisfies the following equation:

$$[- \; ^2_{di}(\boldsymbol{M} - \; \boldsymbol{M}_d) - \mathrm{j} \; _{di}(\boldsymbol{C} - \; \boldsymbol{C}_d) + (\boldsymbol{K} - \; \boldsymbol{K}_d)] \; _{di} = 0. \quad (3)$$

Because the perturbation matrices ($\boldsymbol{M}_d, \; \boldsymbol{C}_d, \; \boldsymbol{K}_d$) are assumed to be exact, eq. (3) holds for any set of measured natural frequencies and mode shapes. This assumption is important because it may be impractical or impossible to measure every mode with complex structural components. The Algorithm must produce reliable results with only p measured modes, where $p \; n$. From this point forward in the discussion i denotes the measured modes ($i = 1, \;, p$). Equation (3) can be rearranged leaving the original matrices on one side and moving the perturbation matrices to the other side:

$$(- \; ^2_{di}\boldsymbol{M} - \mathrm{j} \; _{di}\boldsymbol{C} + \boldsymbol{K}) \; _{di} = (- \; ^2_{di} \; \boldsymbol{M}_d - \mathrm{j} \; _{di} \; \boldsymbol{C}_d + \; \boldsymbol{K}_d) \; _{di}, \quad (4)$$

which can be simplified to

$$d^1_i = d^2_i, \quad (5)$$

where

$$d^1_i = (- \; ^2_{di}\boldsymbol{M} - \mathrm{j} \; _{di}\boldsymbol{C} + \boldsymbol{K}) \; _{di} \quad (6)$$

and

$$d^2_i = (- \; ^2_{di} \; _d - \mathrm{j} \; _{di} \; \boldsymbol{C}_d + \; \boldsymbol{K}_d) \; _{di}. \quad (7)$$

In the case of no damage, the perturbation matrices are all zero so that by eq. (7), $d^2_i = 0$, which in turn means that $d^1_i = 0$ because of eq. (5). If, on the other hand, some damage has occurred, values in certain rows of the associated perturbation matrix will be nonzero, which in turn will cause zero nonzero values in d^2_i and d^1_i corresponding to the rows of the perturbation matrices. This provides an indication of the damage location because the row of the damage vector affected by damage corresponds to the degree of freedom of the finite element model that is affected by damage. Notice that even though the preceding discussion was based on the behavior of the perturbation matrices, it is not necessary to actually know these perturbation matrices to implement the concept, instead, d^2_i can be found with knowledge of the original property matrices and the damaged mode shapes and natural frequencies because, according to eq. (5). Therefore, only simple matrix multiplications are required to locate eamage in structural systems.

1.2 Damage Extent

In looking at the finite element model of the structure, the extent of damage, as developed by Zimmerman and Kaouk, can be determined by the perturbation matrices ($\boldsymbol{M}_d, \; \boldsymbol{C}_d, \; \boldsymbol{K}_d$) such that eq. (3) is satisfied. It is assumed that damping may be neglected and that damage initiation does not alter the mass of the structure. Making this assumption simplifies the problem by reducing the number of required matrix permutations by one third. As a result, the extent of damage is embodied in the change in the stiffness matrix ($\boldsymbol{K}_d$). With these assumptions, eq. (3) can then be reduced and rewritten:

$$(- \; ^2_{di}\boldsymbol{M} + \boldsymbol{K}) \; _{di} = \; \boldsymbol{K}_d \; _{di}. \quad (8)$$

Zimmerman and Kaouk found the following result for the change in the stiffness matrix $\boldsymbol{K}_d$:

$$\boldsymbol{K}_d = \boldsymbol{d}[\boldsymbol{d}^{\mathrm{T}} \; _d]^{-1}\boldsymbol{d}^{\mathrm{T}}, \quad (9)$$

where $\boldsymbol{d}$ is the matrix of damage vectors [$d_1 \; d_2 \; \; d_p$] and $_d$ is the matrix of mode shapes [$_{d1} \; _{d2} \; _{dp}$]. Thus, the extent of the damage can be determined.

© 1994-2012 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

2 Application of Subspace Rotation Algorithm on an Actual Bridge

2. 1 Structural Model

The testing actual bridge is a continuous beam structure located in Dalian city. Finite element modeling of this structure is shown in fig. 1. The structure is divided into 32 elements evenly. That is to say, each element is 2. 5 meters. Except for nodes at the support, there are 3 degrees of freedom at each node, one for horizontal translation u and one for vertical translation v and one for rotation . There is only one degrees of freedom of the node at the first support, viz. rotation . There are two degrees of freedom of the node at other supports, viz. horizontal translation u and rotation . Cross section of the bridge is shown in fig. 2 and fig. 3, in which digits denote reference points and coordinates of these reference points are shown in table 1.

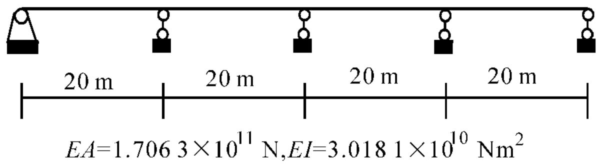

Fig. 1　Finite Element Modeling of Ninety- Seven Bridge

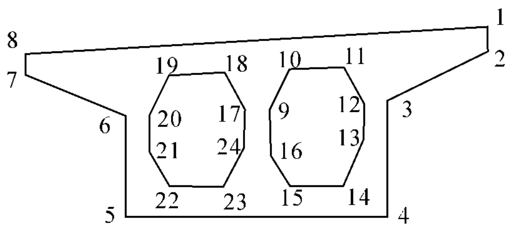

Fig. 2　Cross Section Eastern Box Type Beam

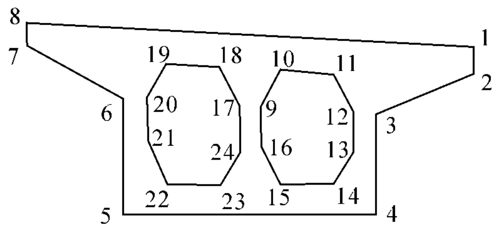

Fig. 3　Cross Section Western Box Type Beam

Table 1　Coordinates of Reference Points

digit	coordinate of eastern box type beam		coordinate of western box type beam	
	X/ cm	Y/ cm	X/ cm	Y/ cm
1	575	0	– 575	0
2	575	– 15	– 575	– 15
3	325	– 48.75	– 325	– 48.75
4	325	– 124	– 325	– 124
5	– 325	– 124	325	– 124
6	– 325	– 58.53	25	– 58. 5
7	– 575	– 32.35	575	– 32.35
8	– 575	– 17.25	575	– 17.25
9	20	– 53.325	– 20	– 53.325
10	70	– 27.575	– 70	– 27.575
11	235	– 25.1	– 235	– 25. 1
12	285	– 49.35	– 285	– 49.35
13	285	– 74.35	– 285	– 74.35
14	235	– 104	– 235	– 104
15	70	– 104	– 70	– 104
16	20	– 78.325	– 20	– 78.325

© 1994-2012 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

To Continue

digit	coordinate of eastern box type beam		coordinate of western box type beam	
	X / cm	Y/ cm	X / cm	Y/ cm
17	− 20	− 53.925	20	− 53.925
18	− 70	− 29.675	70	− 29.675
19	− 235	− 32.15	235	− 32.15
20	− 285	− 57.6	285	− 57.6
21	− 285	− 82.6	285	− 82.6
22	− 235	− 104	235	− 104
23	− 70	− 104	70	− 104
24	− 20	− 78.925	20	− 78.925

2.2 Numerical Results

Seventeen measuring points are distributed evenly along midline of the bridge. Natural frequency and mode shape of the structure are measured by pulse method. The natural frequency of the original structure is 6.13, and the natural frequency of the damaged structure is 5.94. The loss of the natural frequency is 3.1% . The measured mode shape values and ideal mode shape values of the structure are shown in table 2. The measured mode shape curve and ideal mode shape curve of the structure are shown in fig. 4 and fig. 5. Comparing natural frequency and mode shape of original structure with natural frequency and mode shape of damaged structure, it can be decided preliminarily that the structure may be damaged.

Table 2 Mode Shape Values of the Structure

position	mode shape of eastern box type beam		mode shape of western box type beam	
	measured	ideal	measured	ideal
0	0.00	0.000 00E+ 00	0.00	0.000 00E+ 00
5	− 0.39	− 0.698 43E+ 00	− 0.57	− 0.701 52E+ 00
10	− 0.56	− 0.100 00E+ 01	− 0.81	− 0.100 00E+ 01
15	− 0.42	− 0.698 43E+ 00	− 0.57	− 0.701 52E+ 00
20	0.00	0.000 00E+ 00	0.00	0.000 00E+ 00
25	0.49	0.698 43E+ 00	0.60	0.701 52E+ 00
30	0.70	0.100 00E+ 01	0.85	0.100 00E+ 01
35	0.55	0.698 43E+ 00	0.65	0.701 52E+ 00
40	0.00	0.000 00E+ 00	0.00	0.000 00E+ 00
45	− 0.67	− 0.698 43E+ 00	− 0.68	− 0.701 52E+ 00
50	− 1.00	− 0.100 00E+ 01	− 1.00	− 0.100 00E+ 01
55	− 0.73	− 0.698 43E+ 00	− 0.75	− 0.701 52E+ 00
60	0.00	0.000 00E+ 00	0.00	0.000 00E+ 00
65	0.63	0.698 43E+ 00	0.71	0.701 52E+ 00
70	0.85	0.100 00E+ 01	0.97	0.100 00E+ 01
75	0.59	0.698 43E+ 00	0.67	0.701 52E+ 00
80	0.00	0.000 00E+ 00	0.00	0.000 00E+ 00

© 1994-2012 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

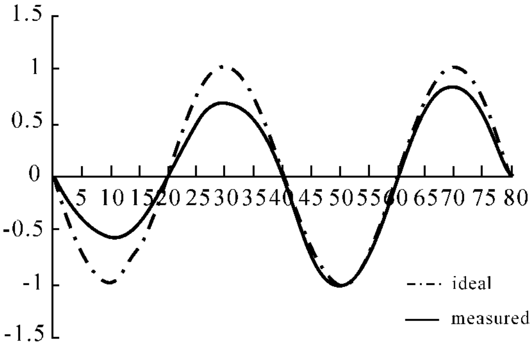

Fig. 4 **Mode Shape of Eastern Box Type Beam**

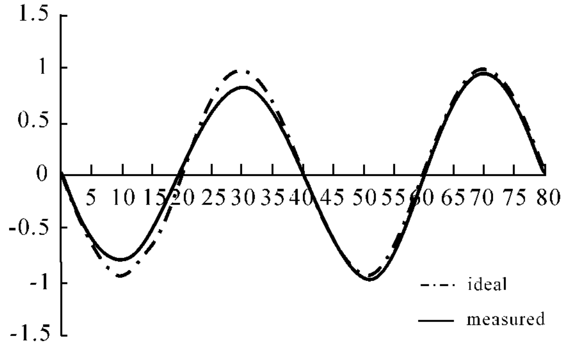

Fig. 5 **Mode Shape of Western Box Type Beam**

In practice rotation and some translation degrees of freedom of the structure cannot be measured. That is to say, damage identification will be performed in the condition of incomplete information measured. One way of overcoming this difficulty is eliminating these degrees of freedom. One of the simplest and most common methods used for eliminating these degrees of freedom is a static condensation method known as Guyan reduction[3]. This method partitions the structural model into master m degrees of freedom, which are to be retained, and slave s degrees of freedom, which are to be condensed out as follows:

$$\left\{\begin{bmatrix} \boldsymbol{K}_{mm} & \boldsymbol{K}_{ms} \\ \boldsymbol{K}_{sm} & \boldsymbol{K}_{ss} \end{bmatrix} - {}^{2}\begin{bmatrix} \boldsymbol{M}_{mm} & \boldsymbol{M}_{ms} \\ \boldsymbol{M}_{sm} & \boldsymbol{M}_{ss} \end{bmatrix}\right\}\begin{Bmatrix} {}_{m} \\ {}_{s} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \tag{10}$$

The reduced mass and stiffness matrices are $\boldsymbol{K}^{*} = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{T}$ and $\boldsymbol{M}^{*} = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{T}$, where $\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{K}_{ss}^{-1}\boldsymbol{K}_{sm} \end{bmatrix}$, $\boldsymbol{I}$ denotes unit matrix.

Using subspace rotation Algorithm the essential damages of the actual bridge are detected. The identification of essential damages location is absolute. Identification of damages extent is relative. That is to say, loss percentage of element stiffness is a relative value, which denotes relative degree of each element s damage. Concrete results of damage identification are shown in fig. 6 and fig. 7($y-$ coordinate denotes loss percentage of element stiffness; $x-$ coordinate denotes damage location) .

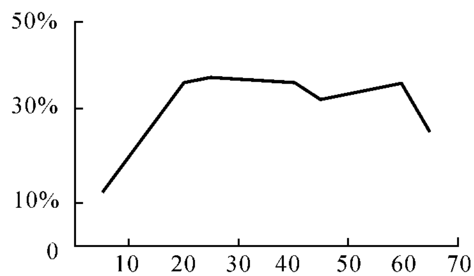

Fig. 6　**Damage Identification Result of Eastern Box Type Beam**

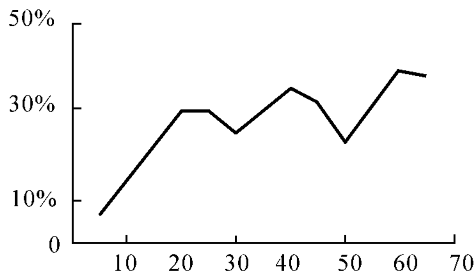

Fig. 7　**Damage Identification Result of Western Box Type Beam**

3　Conclusion

Subspace Rotation Algorithm separates the damage location and damage extent problems and is computationally inexpensive. Subspace Rotation Algorithm is used for detecting damage of an actual bridge in this paper and practice testifies that Subspace Rotation Algorithm is feasible and simple for damage identification of actual bridges.

© 1994-2012 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

References:

[1] ZIMMERMAN D C, KAOUK M. Structural Damage Detection Using a Subspace Rotation Algorithm [R]. Proceedings of the AIAA 33rd Structures. Structural Dynamics and Materials Conference (Dallas, TX), AIAA, Washington DC, 1992. 2 341- 2 350.

[2] KAHL K, SIRKIS J S. Damage Detection in Beam Structures Using Subspace Rotation Algorithm With Strain Data [J]. AIAA Journal, 1996, 34: 2 609- 2 614.

[3] COOK R D, MALKUS D S, PLESHA M E. Concepts and Applications on Finite Element Analysis (4rd) [M]. New York: Wiley, 2001.

基于子空间旋转算法的实桥损伤识别

仲伟秋, 贡金鑫, 刘 毅

(大连理工大学土木工程系, 辽宁 大连 116023)

摘 要: 子空间旋转算法就是基于结构模型参数的损伤识别方法之一. 子空间旋转算法基于结构的有限元模型, 利用矩阵变换的方法, 将损伤位置和损伤程度问题区分开来, 实际应用表明, 只需利用一阶频率和振型, 就可以识别桥的主要损伤位置和损伤程度.

关键词: 损伤识别; 子空间旋转算法; 桥梁

中图分类号: TU311.3 文献标识码: A

(上接第2页)

Euler 数的整除性

乐茂华

(湛江师范学院数学系, 广东 湛江 524048)

摘 要: 设 p 是奇素数, 给出了 E_t 0(mod p) 成立的充要条件, 其中 $t = 2[p\ 4]$, E_t 是第 t 个Euler 数. 特别是当 p 5(mod 8) 时, E_t 0(mod p).

关键词: Euler 数; 整除性; 同余

中图分类号: O156 文献标识码: A

© 1994-2012 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net