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基于共面两囚禁冷离子的信息读写*

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摘　要: 利用离子阱内共面两囚禁冷离子, 在库仑势的条件下, Schrödinger 方程的精确解(即离散的本征态和本征能量)表明: 径向运动量子数为 1/2 的整数倍变化;费米子和玻色子交替出现; 质心运动磁量子数 m_c 和相对运动磁量子数 m 使量子态产生| 个$m_c\beta + m\theta$ 的随机相位因子. 并以 2 个冷 H^+ 为例, 给出 1 个用量子态作资源, 采用量子编码的方法, 提出了在量子信息处理时, 利用离散的本征态 $\psi_{n_c, m_c, l, m}$, 通过操控相位的变化, 从而实现了对信息读写操作.

关键词:
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Paul 阱[1] 技术是当前量子信息研究的物理系统[1-3] 之一, 关于 Paul 阱中囚禁离子体系的研究已取得了令人瞩目的成果[4-10].

对于两离子共线系统, 文献[5] 用将库仑势转化为谐振子势方法进行了分析, 并给出了两离子纠缠检测方法; 文献[11- 14] 在考虑谐振子势和库仑势的基础上, 得出 Schrödinger 方程的无穷级数解; 笔者前期工作中, 也已得到两离子共线及两离子共面系统量子力学精确数值解及其描述的各种性质[15-19].

笔者及其所在的小组运用量子理论研究共面两离子体系精确的量子运动问题[16], 笔者在前期研究的基础上, 分析了基于共面两囚禁冷离子系统精确的离散征态, 以 2 个冷 H^+ 为例, 给出一个用量子态作资源, 量子编码方法, 提出了在量子信息处理时, 从而实现了信息读写.

1　理论模型

考虑 2 个全同离子被囚禁在 Paul 阱[6-7, 15-16] 中的一个平面上, 在赝势近似下, 该系统是一个谐振- 库仑势系统. 设两离子具有相同的质量 M_0, 每个离子所带电量为 e, 第 i 个粒子的坐标为 $\boldsymbol{r}_i = x_i\boldsymbol{e}_x + y_i\boldsymbol{e}_y$, 外加射频场频率 υ(谐振频率), 则系统的 Hamiltonian 量为

$$H = \sum_{i=1}^{2}\left[-\frac{1}{2M_0}(p_{xi}^2 + p_{yi}^2) + \frac{1}{2}M_0 v^2(x_i^2 + y_i^2)\right] + \frac{e^2}{4\pi\varepsilon_0 |\boldsymbol{r}_2 - \boldsymbol{r}_1|}, \tag{1}$$

考虑质心坐标 $\boldsymbol{r}_c = \dfrac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}$ 和相对坐标 $\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, 则

$$\boldsymbol{r}_c = x_c\boldsymbol{e}_x + y_c\boldsymbol{e}^y = \frac{x_1 + x_2}{2}\boldsymbol{e}_x + \frac{y_1 + y_2}{2}\boldsymbol{e}_y, \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 = (x_2 - x_1)\boldsymbol{e}_x + (y_2 - y_1)\boldsymbol{e}_y, \tag{2}$$

其中: $M_c = 2M_0$; $\mu = M_0/2$; $\boldsymbol{e}_x$ 和 $\boldsymbol{e}_y$ 是坐标 x 轴和 y 轴方向的单位矢. 据文献[15] 得系统的精确的本征态和能量本征值表达式为

$$\psi_{n_c m_c lm}(\boldsymbol{r}_c, \boldsymbol{r}) = \psi_c\psi_r e^{-i\frac{Et}{\hbar}} = N_{n_c m_c lm}\boldsymbol{r}_c^{|m_c|}F(-n_c, \gamma, \alpha_c^2|\boldsymbol{r}_c|^2)e^{i(m_c\theta + m\beta)}e^{\left(-\frac{\alpha_c^2|\boldsymbol{r}_c|^2}{2} - \frac{\alpha_{rl}^2|\boldsymbol{r}|^2}{2}\right)}(\alpha_r l\boldsymbol{r})^{|m|}\sum_{i=0}^{l}c_i(\alpha_{rl}\boldsymbol{r})^i e^{-\frac{iEt}{\hbar}} =$$

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$$N_{n_c m_c l m} \boldsymbol{r}_c^{|m_c|} F(-n_c, \gamma, \alpha_c^2 |\boldsymbol{r}_c|^2) e^{\frac{(-\alpha_c^2 |r_c|^2}{2} - \frac{\alpha_{rl}^2 |r|^2}{2}} (\alpha_{rl} \boldsymbol{r})^{|m|} \sum_{i=0}^{l} c_i (\alpha_{rl} \boldsymbol{r})^i \cdot$$

$$[\cos(m_c\theta + m\beta - \frac{Et}{\hbar}) + i\sin(m_c\theta + m\beta - \frac{Et}{\hbar})], \tag{3}$$

$$E_{n_c m_c l m} = E_r + E_c = (2n_c + |m_c| + l + |m| + 2)\hbar v, \tag{4}$$

其中常数 $N_{n_c m_c l m} = A_{n_c m_c} A_{lm}$ 由波函数的归一化条件决定, $F(-n_c, \gamma, \alpha_c^2 r_c^2)$ 是合流超几何函数, $l = 1, 2, \cdots$ 为相对运动主量子数, $m = -l, -l+1, \cdots, l-1, l$ 是相对运动磁量子数.

$$\alpha_c = (\frac{M_c v}{\hbar})^{1/2}, \gamma = \frac{1}{2}(2|m_c| + 1), \tag{5}$$

且其质心量子数变化约束条件为

$$N = 2n_c + |m_c| = 0, 1, 2, \cdots, \tag{6}$$

其中: $m_c = 0, \pm 1, \pm 2, \cdots, \pm N$ 是质心运动的磁量子数; $n_c = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots, \frac{N}{2}$ 或 $\frac{N-1}{2}$ 是质心运动的径向量子数. $N = 1, 2, 3$ 时, m_c, n_c, γ 的对应取值见表 1.

表 1 质心量子数的取值

N	0	1	2			3				…
m_c	0	0	0	±1	±2	0	±1	±2	±3	…
n_c	0	$\frac{1}{2}$	$1\ \frac{1}{2}$	$0\ \frac{1}{}$	0	$\frac{3}{2}$	1	$\frac{1}{2}$	0	…
γ	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	…

从表 1 知 n_c 的取值为 $\frac{1}{2}$ 的整数倍变化, 当 n_c 是 $\frac{1}{2}$ 的偶数倍时, 该质心运动径向量子数取值类似于玻色子量子数变化特征; 当 n_c 是 $\frac{1}{2}$ 的奇数倍时, 该质心运动径向量子数取值类似于费米子量子数变化特征.

(3) 式中的系数由下式决定[14]:

$$(l-j)(l-j+2|m|)(C_{l-j} - 2\sigma_l C_{l-j-1} + 2(j+2)C_{l-j-2} = 0 \qquad l = 1, 2, 3, \cdots, j = 0, 1, 2, \cdots, l-1, \tag{7}$$

$$\frac{2E_r}{\hbar \upsilon} - 2|m| - 2 - 2l = 0, \tag{8}$$

$$\upsilon = \upsilon, \alpha_{rl} = (\frac{\pi\upsilon}{\hbar})^{1/2} = \frac{1}{2}\alpha_c, \sigma_l = \frac{e^2}{4\pi\varepsilon_0}(\frac{\mu}{\hbar^3 \upsilon})^{1/2}. \tag{9}$$

考虑两超冷离子, 取 = 1, 2, 取最大 σ 时[15], 利用(7) 式计算, 得到(3) 式中相对运动波函数的展开系数系数 c_i 如表 2.

表 2 相对运动中, $l = 1, 2$ 时的展开系数 c_j 和常数 σ_l

l	m	c_0	c_1	c_2	σ_l
1	0	1	$\sqrt{2}$	0	$1/\sqrt{2}$
1	±1	1	$\sqrt{\frac{2}{3}}$	0	$\sqrt{\frac{3}{2}}$
2	0	1	$2\sqrt{3}$	2.000	$\sqrt{3}$
2	±1	1	1.764	0.667	2.646
2	±2	1	1.327	0.400	3.317

2 系统量子态

以 2 个 H^+ 为例作如下讨论. 用 $|N\rangle$ 表示系统在外加射频场作用下的质心量子态, 当 $N = 0, 1$ 时, 则

系统质心量子态有 $|0\rangle_c$ 和 $|1\rangle_c$. 当外场作用时间满足 t 远小于每个能态的生存寿命 τ 时, 以每个离子作为 1 个量子位, 定义第 i 个离子的量子位[16] 为

$$|0\rangle_i = \sqrt{\frac{\sqrt{2}\alpha_r}{\sqrt{\pi}}}\exp(-\alpha_{rl}^2 r_i^2),\ |1\rangle_i = \alpha_{rl}\boldsymbol{r}_i|\rangle_i,\ |2\rangle_i = \alpha_{rl}\boldsymbol{r}_i|1\rangle_i,\ |n\rangle_i = \alpha_{rl}\boldsymbol{r}_i|n-1\rangle_i \quad i=1,2, n=0,1,2,\cdots, \tag{10}$$

其中:

$$|\boldsymbol{r}_c|^2 = \boldsymbol{r}_c\cdot\boldsymbol{r}_c = \frac{1}{4}(\boldsymbol{r}_1\cdot\boldsymbol{r}_1+\boldsymbol{r}_2\cdot\boldsymbol{r}_2+2\boldsymbol{r}_1\cdot\boldsymbol{r}_2), \tag{11}$$

$$|\boldsymbol{r}|^2 = \boldsymbol{r}\cdot\boldsymbol{r} = \boldsymbol{r}_1\cdot\boldsymbol{r}_1+\boldsymbol{r}_2\cdot\boldsymbol{r}_2-2\boldsymbol{r}_1\cdot\boldsymbol{r}_2. \tag{12}$$

将(10) ~ (12), (4) 式、表 1, 2 的数据代入(3) 式, 可得:

$$|\psi_{0,0,0,0}(\boldsymbol{r}_c,\boldsymbol{r}\rangle = |0_1 0_2\rangle, \tag{13}$$

$$|\psi_{0,\pm1,0,0}\rangle = N_{0,\pm1,0,0}|0_1 0_2\rangle e^{\pm i\beta - i3\vartheta}, \tag{14}$$

$$|\psi_{0,\pm1,1,0}\rangle = N_{0,\pm1,1,0}e^{i(\pm\theta-4\vartheta)}(|1_1 0_2\rangle + |0_1 1_2\rangle + \sqrt{2}|2_1 0_2\rangle - \sqrt{2}|0_1 2_2\rangle), \tag{15}$$

$$|\psi_{\frac{1}{2},0,0,0}\rangle = N_{\frac{1}{2},0,0,0}e^{-i2\vartheta}(|0_1 0_2\rangle - |2_1 0_2\rangle - |0_1 2_2\rangle - 2|1_1 1_2\rangle). \tag{16}$$

对于 $|\psi_{\frac{1}{2},0,1,0}\rangle$、$|\psi_{\frac{1}{2},0,1,\pm1}\rangle$ 和 $|\psi_{0,\pm1,1,\pm1}\rangle$ 态存在项 $-\alpha_c^2 r_c^2 c_1\alpha_{r1} r\exp[-\frac{1}{2}(\alpha_c^2 r_c^2+\alpha_r^2 r_2)-i4\vartheta]$, 即离子 1 与离子 2 有 $|0\rangle_i$, $|1\rangle_i$, $|2\rangle_i$, $|3\rangle_i$ 4 个能级, 笔者仅考虑 3 个能级离子, 故对该类状态不作讨论. $N_{0,1,0,0}$、$N_{0,-1,0,0}$、$N_{0,1,1,0}$、$N_{0,-1,1,0}$ 和 $N_{\frac{1}{2},0,0,0}$ 是幅模因子, $e^{\pm i\beta-i\vartheta}$, $e^{i(\pm\theta-2\vartheta)}$ 和 $e^{-i\vartheta}$ 相位因子, (13) 式表明系统处于基态时, 相对稳定. (14) ~ (16) 式的 $e^{\pm i\beta-i3\vartheta}$, $e^{i(\pm\theta-4\vartheta)}$ 和 $e^{-i3\vartheta}$ 项为对应 $\psi_{n,m_c,l,m}$ 的退相, 它表明每个态有一个不确定的相因子, 有一定生存寿命. 我们可通过量子操控, 改变幅模因子、相位因子, 实现进行量子信息处理.

3　信息存取的实现

用 4 个射频场作用, 适当地选择射频场的振幅和相位, 使离子 2 态进行演化,

$$|1\rangle_2 \to \frac{1}{\sqrt{2}}(|0\rangle_2+|1\rangle_2),\ |0\rangle_2 \to \frac{1}{\sqrt{2}}(|0\rangle_2-|1\rangle_2). \tag{17}$$

考虑离子质心初始时刻量子态 $|n\rangle_c$, 设离子 1, 2 的初始时刻的状态制备为

$$|\psi(0)\rangle_{12} = A_0|1\rangle_1\otimes\frac{1}{\sqrt{2}}(|0\rangle_2+|1\rangle_2), \tag{18}$$

两离子通过质心模发生相互作用, 对某定态的两离子, 阱频 ν, 作用时间 t, 由(3) ~ (4) 式知, 当满足

$$m_c\theta + m\beta - (2n_c+|m_c|+l+|m|+2)\vartheta = (2k+1)\pi, \tag{19}$$

$$|\psi(t)\rangle_{12} = -|\psi(0)\rangle_{12} = -A_0|1\rangle_1\otimes\frac{1}{\sqrt{2}}(|0\rangle_2+|1\rangle_2). \tag{20}$$

这样就可把原 $|\psi(0)\rangle_{12}$ 记录的信息用 $-|\psi(0)\rangle_{12}$ 记录备份(数据写入), 当再施加 1 次(20) 式条件的变换, 就可把信息复原(读取数据时). 利用量子态此特征, 在量子信息处理时, 对信息进行存取.

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Information Reading and Writing Based on Two Cold Ions in a Two-Dimensional Trap

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Abstract: When the Coulomb potential is considered, a set of exact solutions of Schr? dinger equation are obtained based on two trapped cold ions in a two-dimensional ion trap. Discrete eigenstates and eigenenergies indicate that the radial motion quantum number is changed in an integral multiple of 1/2, fermions and bosons appear alternately, the motion of mass center magnetic quantum number mc and relative motion magnetic quantum number m make the quantum state generate a stochastic phase factor $m_c\beta + m\theta$. Taking several states with cold two H^+ as examples, a kind of ways of quantun coding by quantum state are obtained. In quantum information processing, utilizing discrete eigenstates $\psi_{n_c, m_c, l, m}$ is generated in order to realize the quantum information reading and writing by phase change.

Key words: ion trap; quantum state; trap frequency; phase change

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