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Existence Theorems of Solutions for the System of Generalized Vector Cone-Properly Quasi-Convex Quasi-Equilibrium Problems*

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Abstract: The system of generalized vector cone-properly quasi-convex quasi-equilibrium problems is considered. As its applications. The existence results are derived for weakly Pareto-Nash equilibrium points for multiobjective generalized game problems and multiobjective game problems in real locally convex Hausdorff topological spaces.

Key words: system of generalized vector cone-properly quasi-convex quasi-equilibrium problems; weakly Pareto-Nash equilibrium point
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1 Introduction

In order to describe the real world and economic behavior better, recently, much attention has been attracted to multicriteria equilibrium models. Motivated by ref. [1－5], we study the following system of generalized vector quasi-equilibrium problems.

Let $I=\{1,2,\cdots,n\}$, for each $i\in I$. Let X_i and Y_i be Hausdorff topological vector spaces. K_i is a nonempty subset of X_i, and $C_i\subset Y_i$ is a closed convex pointed cone with int $C_i\neq\emptyset$, where int C_i denotes the interior of C_i. And let $K=\prod_{i=1}^{n}K_i$. For every $i\in I$, let $f_i: K\times K_i\rightarrow Y_i$ be a vector-valued mapping and $S_i: K\rightarrow 2^{K_i}$ a set-valued mapping, where 2^{K_i} denotes the family of all nonempty subsets of K_i. The generalized vector quasi-equilibrium problem consists in finding $\bar{x}\in K$ such that for each $i\in I$, $\bar{x}_i\in S_i(\bar{x})$, $f_i(\bar{x}, y_i)\notin -\text{int } C_i$, $\forall y_i\in S_i(\bar{x})$, where $\bar{x}_i$ denotes the ith component of $\bar{x}$. For convenience, the generalized vector quasi-equilibrium problem is called the generalized symmetric vector quasi-equilibrium problem (briefly, GSVQEP), and $\bar{x}$ is called a solution of the GSVQEP.

Ref. [5] has studied the generalized vector quasi-equilibrium problems without constraint (briefly, SGVEP), where, for each $i\in I$, f_i is a set-valued mapping. Ref. [2] studied the existence of solutions for the system of generalized vector quasi-equilibrium problems with constraint (briefly, SGQVEP), where for each i, f_i is a set-valued mapping. It is easy to see that SGQVEP includes SGVEP as a special case.

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Although SGQVEP of ref. [2] includes GSVQEP as a special case, the investigation of ref. [2] depends on the advantage of Banach spaces and $S_i(x)\neq\emptyset$. It is clear that if S_i is a singled-mapping, then int $S_i=\emptyset$. In this paper, by using new methods, we derive the existence results for weakly Pareto-Nash equilibrium points for multiobjective generalized game problems and multiobjective game problems in real locally convex Hausdorff topological spaces. As its corollary, one of the open problems proposed in ref. [1] is solved.

For other existence results related to equilibriumproblems, we refer the reader to ref. [1-13] and references therein.

2 Preliminaries

Definition 1 Let Y be a topological vector space, C be a closed convex point cone in Y, D be a nonempty subset of Y, and then a point $a\in D$ is called a minimal points of D if $D\cap(a-C)=\{a\}$. If int $C\neq\emptyset$, a point $a\in D$ is called a weak minimal point of D if $D\cap(a-\text{int } C)=\emptyset$. Min$(D,C)$ and $\min_W(D,C)$ will denote the sets of all minimal points and all weak minimal points of D, respectively.

The following definition can be found in ref. [14].

Definition 2 Let X be a Hausdorff topological space and Y be a Hausdorff topological vector space with a convex cone C. Let $f:X\to Y$ be a vector-valued function.

(i) f is said to be C-continuous at $x_0\in X$ if, for any open neighborhood V of the zero element θ in Y, there is an open neighborhood $N(x_0)$ of x_0 in X such that $f(x)\in f(x_0)+V+C$, $\forall x\in N(x_0)$; f is said to be C-continuous on X if it is C-continuous at every element of X.

(ii) f is said to be $(-C)$-continuous at $x_0\in X$ if, for any open neighborhood V of θ in Y, there exists an open neighborhood $N(x_0)$of x_0 in X such that $f(x)\in f(x_0)+V-C$, $\forall x\in N(x_0)$; f is said to be $(-C)$-continuous on X if it is $(-C)$-continuous at any point of X.

Remark 1 A vector-valued mapping may be at the same time C-and $(-C)$-continuous, but not continuous (see ref. [14]). It is easily to see that f is C-and $(-C)$-continuity is equivalent to continuity in the scalar case, i. e. $Y=R$ and $C=[0,+\infty)$.

Definition 3 Let Y be a topological vector space with a closed convex pointed cone C, let K be a nonempty convex subset of a vector space X, and let $f:K\to Y$ be given.

(i) f is said to be C-convex if, for any $x,y\in K$ and $t\in[0,1]$. $tf(x)+(1-t)f(y)-f(tx+(1-t)y)\in C$; f is said to be C-concave if $-f$ is C-convex.

(ii) (See ref. [8]) f is said to be C-properly quasi-convex if, for any $y_1,y_2\in K$ and $t\in[0,1]$, one has either $f(ty_1+(1-t)y_2)\in f(y_1)-C$, or $f(ty_1+(1-t)y_2)\in f(y_2)-C$.

Definition 4 Let X and Y be two Hausdorff topological spaces, $T:X\to 2^Y$ is a set-valued mapping.

(i) T is said to be upper semi-continuous (briefly, u. s. c.) at $x_0\in X$ if for any neighborhood $N(T(x_0))$ of $T(x_0)$, there exists a neighborhood $N(x_0)$of x_0 such that $T(x)\subset N(T(x_0))$, $\forall x\in N(x_0)$. We say that T is said to be upper semi-continuous on X if T is u. s. c. at every point $x\in X$.

(ii) T is said to be lower semi-continuous (briefly, l. s. c.) at $x_0\in X$ if for any $y_0\in N(x_0)$ and any neighborhood $N(y_0)$ of y_0, there exists a neighborhood $N(x_0)$ of x_0 such that $T(x)\cap N(y_0)\neq\emptyset$, $\forall x\in N(x_0)$. We say that T is said to be lower semi-continuous on X if it is lower semi-continuous at every $x_0\in X$.

(iii) T is said to be continuous if it is, at the same time, u. s. c. and l. s. c. on X.

(iv) T is said to be a closed mapping if graph $T=\{(x,y)\in X\times Y: y\in T(x)\}$ is a closed set in $X\times Y$.

Lemma 1[4-5] Let X be a locally convex Hausdorff space. $K\subset X$ is a nonempty convex compact subset. Let $T: K \to 2^K$ be u. s. c. with nonempty closed convex values. Then T has a fixed point in K.

Lemma 2 (Ky Fan's Section Theorem) Let x_0 be a nonempty compact convex subset of a Hausdorff topological vector space and A be a subset of $X_0 \times X_0$ such that:

(1) For each $y \in X_0$, the set $x \in \{X_0 : (x, y) \in A\}$ is closed in X_0;

(2) For each $x \in X_0$, the set $x \in \{X_0 : (x, y) \notin A\}$ is convex or empty;

(3) For each $x \in X_0$, $(x, x) \in A$.

Then there exists a point $x^* \in X_0$ such that $\{x^*\} \times X_0 \subset A$.

Lemma 3 For each $i \in I$, let X_i be locally Hausdorff topological vector space and Y_i be Hausdorff topological vector space K_i be nonempty compact convex subset of X_i, $K = \prod_{i=1}^{n} K_i$. Let $f_i: K \times K_i \to Y_i$ be a vector valued mapping and $S_i: K \to 2^{K_1}$ a set-valued mapping, suppose that $Y_1 \subset Y_2 \subset \cdots \subset Y_n$ and for each $i \in I$, $C_i = C_n \cap Y_i$ is a closed convex pointed cone with int $C_i \neq \emptyset$. For each $i \in I$, assume that:

(i) S_i is continuous on K with nonempty convex compact values;

(ii) For each $x \in K$, $f_i(x, x_i) = \theta$, where x_i is the ith component of x;

(iii) For each $(x, y_i) \in K \times K_i$, the $f_i(\cdot, \cdot)$ is $(-C_i)$-continuous on $K \times K_i$;

(iv) For each fixed $x \in K$, $f_i(x, \cdot)$ is C_i-convex.

Then GSVQEP has a solution.

3 Main Results

For any $i \in I = \{1, \cdots, n\}$, let X_i and Y_i be two Hausdorff topological vector spaces and K_i a nonempty subset of X_i, and C_i a closed convex pointed cone of Y_i with int $C_i \neq \emptyset$. Write $K = \prod_{i=1}^{n} K_i$ and for each $i \in I$. Let $K_{\hat{i}} = \prod_{i=1, i \neq \hat{i}} K_i$. Thus we can write $x = (x_i, x_{\hat{i}})$, for each $x \in K$, and for each $i \in I$. Let $g_i: K \to Y_i$ be a vector-valued mapping. The multiobjective game problem consists in finding $\bar{x} \in K$ such that, for any $i \in I$, $g_i(y_i, x_{\hat{i}}) - g_i(\bar{x}_i, \bar{x}_{\hat{i}}) \notin -\text{int } C_i$, $\forall y_i \in K_i$. A multiobjective game problem is often denoted by $\{K_i, g_i\}_{i \in I}$.

For each $i \in I$, let $G_i: K_{\hat{i}} \to 2^{K_i}$ be a feasible strategy correspondence. The multiobjective generalized game problem consists in finding $\bar{x} \in K$ such that for each $i \in I$, $\bar{x}_i \in G_i(\bar{x}_{\hat{i}})$, and $g_i(y_i, \bar{x}_{\hat{i}}) - g_i(\bar{x}_i, \bar{x}_{\hat{i}}) \notin -\text{int } C_i$, $\forall y_i \in G_i(\bar{x}_{\hat{i}}) \notin -\text{int } C_i$, $\forall y_i \in G_i(\bar{x}_{\hat{i}})$. A multiobjective generalized game problem is usually denoted by $\{K_i, G_i, g_i\}_{i \in I}$.

In above two cases, $\bar{x}$ is said to be a weakly Pareto-Nash equilibrium point.

Corollary 1 For each $i \in I$, let X_i, Y_i, K_i, C_i, K as stated in lemma 3, and let $G_i: K_i \to 2^{K_i}$ and $g_i: K \to Y_i$ be vector valued mapping. For each $i \in I$, assume that:

(i) G_i is continuous with nonempty convex compact values;

(ii) $g_i(x)$ is at the same time C_i-and $(-C_i)$-continuous on K;

(iii) For each $x_{\hat{i}} \in K_{\hat{i}}$, $g_i(\cdot, x_{\hat{i}})$ is C_i-convex.

Then the multiobjective generalized game problem $\Gamma = \{K_i, G_i, f_i\}_{i \in I}$ has a weakly Pareto-Nash equilibrium point.

Proof For any $i \in I$, let $f_i(x, y_i) = g_i(y_i, x_{\hat{i}}) - g_i(x_i, x_{\hat{i}})$ and $S_i(x) = G_i(x_{\hat{i}})$, for any $x \in K$. It is easy to see that the conditions of lemma 3 hold. Hence the result follows.

Remark 2 Corollary 1 is a new existence result of weakly Pareto-Nash equilibrium points for the multiobjective generalized game problem in real locally convex Hausdorff topological spaces.

In some sense, it improves on corollary 2 of ref. [2].

Corollary 2　For each $i\in I$, let X_i, Y_i, K_i, C_i, K and g_i, as stated in corollary 1. Then the multiobjective game problem $\Gamma=\{K_i, g_i\}_{i\in I}$ has a weakly Pareto-Nash equilibrium point.

Proof　For each $i\in I$, let $f_i(x, y_i)=g_i(y_i, x_{\hat{i}})-g_i(x_i, x_{\hat{i}})$ and $G_i(x_i)=K_i$ for each $x_{\hat{i}}\in K_i$. It is easily seen that the conditions of corollary 1 hold. Hence the result follows.

Lemma 4　let Y be a topological vector space, and $C\subset Y$ a closed convex pointed cone. Let K be a nonempty compact subset of a topological space X and $f: K\to Y$ is C-continuous. The $\min(f(K), C)\neq\varnothing$.

Proof　If $f: K\to Y$ is C-continuous, we can see that for each $y\in Y$, $\{x\in K: f(x)\in y-C\}$ is closed. $f(K)$ is a C-semicompact set and $\min(f(K), C)\neq\varnothing$. The proof is completed.

Theorem 1　For each $i\in I$, let X_i be a real locally convex Hausdorff topological vector space, Y_i a real Hausdorff topological vector space, K_i a nonempty convex compact subset of X_i, and C_i a closed convex pointed cone of Y_i with $\operatorname{int} C_i\neq\varnothing$. Write $K=\prod_{i=1}^{n} K_i$ and $K_{x_{\hat{i}}}=\prod_{j=1, j\neq i}^{n} K_j$. For each $i\in I$, let $f_i: K\to Y_i$ be a vector valued mapping and $S_i: K\to 2^{k_i}$ a set-valued mapping, for each $i\in I$. Assume that:

(1) For each $i\in I$, S_i is continuous on K with nonempty convex compact values;

(2) For each $i\in I$, f_i is C_i-continous and $-C_i$-continous on K at the same time;

(3) For any fixed $f_i(\cdot, x_{\hat{i}})$ is C_i-properly quasi-convex.

Then there exists $\bar{x}\in K$ such that, for each $i=I$, $\bar{x}_i\in S_i(\bar{x})$, $f_i(y_i, \bar{x}_{\hat{i}})-f_i(\bar{x}_i, \bar{x}_{\hat{i}})\notin-\operatorname{int} C_i$, $\forall y_i\in S_i(\bar{x})$.

Proof　Define $A_i: K\to 2^{K_i}$ by $A_i(x)=\{v\in S_i(x): f_i(v, x_{\hat{i}})\in\min_w(f_i(S_i(x), x_{\hat{i}}), C_i), \forall x\in K$.

Step I　$\forall x\in K, i\in I, A_i(x)$ is a nonempty convex closed subset of K_i.

In fact, since f_i is C_i-continuous and $S_i(x)$ is a nonempty convex compact subset of K_i, by lemma 4. $\min(f_i(S_i(x), x_{\hat{i}}), C_i)\neq\varnothing$ and hence $\min_w(f_i(S_i(x), x_{\hat{i}}), C_i)\neq\varnothing$. Thus, $A_i(x)\neq\varnothing$. Let $v_1, v_2\in A_i(x)$, $t\in[0,1]$, $v=tv_1+(1-t)v_2$. We need to show that $v\in A_i(x)$. It follows from v_1, $v_2\in S_i(x)$ and $f_i(v_j, x_{\hat{i}})\in\min_w(f_i(S_i(x), x_{\hat{i}}), C_i)$, $j=1,2$, that

$$f_i(y_i, x_{\hat{i}})-f_i(v_j, x_{\hat{i}})\notin-\operatorname{int} C_i \qquad \forall y_i\in S_i(x), j=1,2. \tag{1}$$

Since $f_i(., x_{\hat{i}})$ is C_i-properly quasi-convex, we have either

$$f_i(v, x_{\hat{i}})\in f_i(v_1, x_{\hat{i}})-C_i, \tag{2}$$

or

$$f_i(v, x_{\hat{i}})\in f_i(v_2, x_{\hat{i}})-C_i. \tag{3}$$

By (1), (2), (3),

$$f_i(y_i, x_{\hat{i}})-f_i(v, x_{\hat{i}})\notin-\operatorname{int} C_i \qquad \forall y_i\in S_i(x),. \text{ i. e.},$$

$$f_i(v, x_{\hat{i}})\in\min_w(f_i(S_i(x), x_{\hat{i}}), C_i).$$

Hence $v\in A_i(x)$.

Step II　Now we need to show that $A_i(x)$ is closed. Indeed, let a net $\{v_\alpha\}\subset A_i(x)$ with $v_\alpha\to v$. We need to show $v\in A_i(x)$. By the closeness of $S_i(x)$ and $v_\alpha\in S_i(x)$, $v\in S_i(x)$. Since $f_i(v_\alpha, x_{\hat{i}})\in\min_w(f_i(S_i(x), x_{\hat{i}}), C_i)$, we have $f_i(y_i, x_{\hat{i}})-f_i(v_\alpha, x_{\hat{i}})\notin-\operatorname{int} C_i$, $\forall y_i\in S_i(x)$.

Suppose that $v\notin A_i(x)$. Then there exists $y_i\in S_i(x)$ such that $f_i(y_i, x_{\hat{i}})-f_i(v, x_{\hat{i}})\in-\operatorname{int} C_i$. Since $-\operatorname{int} C_i$ is an open set, there exists a symmetric open neighborhood O of the zero element in Y_i such that $f_i(y_i, x_{\hat{i}})-f_i(v, x_{\hat{i}})+O\subset-\operatorname{int} C_i$. Thus $f_i(y_i, x_i)-f_i(v_\alpha, x_{\hat{i}})+O-C_i\subset-\operatorname{int} C_i$. By the C_i-continuity of f_i, there exists α_0. Such that

$$f_i(y_i, x_{\hat{i}})-f_i(v_\alpha, x_{\hat{i}})\in f_i(y_i, x_{\hat{i}})-f_i(v, x_{\hat{i}})+O-C_i\subset-\operatorname{int} C_i \qquad \text{for all } \alpha\geqslant\alpha_0,$$

which contradicts that

$$f_i(y_i, x_{\hat{i}})\in\min_w(f_i(S_i(x), x_{\hat{i}}), C_i). \tag{4}$$

If (4) is not true, then there exists $\bar{y}_i\in S_i(x)$ such that $f_i(\bar{y}_i, x_{\hat{i}})-f_i(v, x_{\hat{i}})\in-\operatorname{int} C_i$. Since

$-\text{int } C_i$ is open, there exists a symmetric open neighborhood U of the zero element in Y_i such that $f_i(\bar{y}_i, x_{\hat{i}}) - f_i(v, x_{\hat{i}}) \in -\text{int } C_i$. Since C_i is convex, we have

$$f_i(\bar{y}_i, x_{\hat{i}}) + U - (f_i(v, x_i) + U + C_i) \subset -\text{int } C_i. \tag{5}$$

Since S_i is lower semi-continuous and $x_\alpha \to x$, for above $\bar{y}_i \in S_i(x)$, there exists a net $\{y_i^\alpha\}$ with $y_i^\alpha \in S(x_\alpha)$ such that $y_i^\alpha \to \bar{y}_i$. Since f_i is C_i-continuous and $(-C_i)$-continuous on K and $y_i^\alpha \to \bar{y}_i$, $v_\alpha \to v$ and $x_{\hat{i}}^\alpha \to x_{\hat{i}}$, there exists α_0, such that

$$f_i(y_i^\alpha, x_{\hat{i}}^\alpha) \in f_i(\bar{y}_i, x_{\hat{i}}) + U - C_i \qquad \text{for all } \alpha \geqslant \alpha_0$$

and

$$f_i(v_\alpha, x_{x_{\hat{i}}}^\alpha) \in f_i(v, x_{\hat{i}}) + U + C_i \qquad \text{for all } \alpha \geqslant \alpha_0.$$

By (5), we have $f_i(y_i^\alpha, x_{\hat{i}}^\alpha) - f_i(v_\alpha, x_{\hat{i}}^\alpha) \in -\text{int } C_i$. This contradicts that $v_\alpha \in A_i(x_\alpha)$, since $y_i^\alpha \in S_i(x_\alpha)$.

Step III Define $\varphi: K \to 2^K$ by $\varphi(x) = (A_1(x), \cdots, A_n(x))$, $\forall x \in K$. Then, for each $x \in K$, $\varphi(x)$ is a nonempty convex closed subset of K, and φ is u. s. c.. By lemma 1, here is a point $\bar{x} \in K$ such that $\bar{x} \in \varphi(\bar{x})$. That is, for any $i \in I$, $x_i \in A_i(\bar{x})$. By the definition of $A_i(\bar{x})$, for each $i \in I$, $\bar{x} \in S_i(\bar{x})$, $f_i(y_i, \bar{\hat{i}}) - f_i(x_i, \bar{\hat{i}}) \notin -\text{int } C_i$, $\forall y \in S_i(\bar{x})$.

The proof is completed.

Remark 3 It is easy to see that lemma 3 is an existence theorem for the GSVQEP.

Corollary 3 For each $i \in I$, let $X_i, Y_i, C_i, K_i, K_{\hat{i}}, K$ as stated in lemma 3. For each $i \in I$, let $g_i: K \to Y_i$ be a vector-valued mapping and $G_i: K_i \to 2^{K_i}$ a set-valued mapping. For each $i \in I$, assume that:

(i) G_i is continuous with convex compact values;

(ii) $g_i(x)$ is at the same time C_i-and $(-C_i)$-continuous;

(iii) For each $x_{\hat{i}} \in K_{\hat{i}}$, $g(\cdot, x_{\hat{i}})$ is C_i-properly quasi-convex.

Then the multiobjective generalized game problem $\Gamma = \{K_i, G_i, f_i\}_{i \in I}$ has a weakly Pareto-Nash equilibrium point.

Proof For each $i \in I$, let $S_i(x) = G_i(x_{\hat{i}})$ for any $x \in K$ and $f_i = g_i$. It is easy to see that the conditions of lemma 4 hold. Hence the result follows.

Remark 4 Corollary 3 is a new existence result of weakly Pareto-Nash equilibrium points for the multiobjective generalized problem in real locally Hausdorff topological spaces. It improves the corollary 2 of ref. [2].

Crollary 4 For each $i \in I$, let $X_i, Y_i, C_i, K_i, K_{\hat{i}}, K, g_i$ as stated in corollary 3. For each $i \in I$, assume that:

(i) g_i is at the same time C_i-and $(-C_i)$-continuous;

(ii) For each $x_{\hat{i}} \in K_{\hat{i}}$, $g(\cdot, x_{\hat{i}})$ is C_i-properly quasi-convex.

Then the multiobjective game problem $\Gamma = \{K_i, g_i\}_{i \in I}$ has a weakly Pareto-Nash equilibrium point.

Proof For each $i \in I$, let $g_i = g_i$ and $G_i(x_{\hat{i}}) = K_i$ for each $x_{\hat{i}} \in K_i$. It is easily seen that the conditions of corollary 3 hold. Hence the result follows.

Remark 5 Corollary 4 is a new existence theorem of weakly Pareto-Nash equilibrium points for the multiobjective game problem in real locally convex Hausdorff topological vector spaces. It is a generalization of corollary 4 of ref. [2].

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广义向量锥拟凸拟平衡系统的存在性定理

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摘　要：在实局部凸 Hausdorff 拓扑空间中证明了广义向量锥拟凸拟平衡系统的存在性定理. 作为它的应用，得到了多目标广义系统问题弱 Pareto-Nash 均衡点的存在性结果.

关键词：广义向量锥拟凸拟平衡系统；存在性定理；弱 Pareto-Nash 均衡点

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