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由混合序列生成的线性过程加权和的极限定理

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摘要: 假设线性过程 $X_t = \sum_{j=0}^{\infty} a_j\xi_{t-j}$, $t\geqslant 1$, 其中 $\{\xi_t, t\in Z\}$ 为一零均值的混合序列, $\{a_j, j\geqslant 0\}$ 为一实数序列, 满足 $\sum_{j=0}^{\infty} j|a_j| < \infty$, $\{a_{ni}, 1\leqslant i\leqslant n, n\geqslant 1\}$ 为一实值的三角阵列, 在适当的假设条件下, 利用混合序列的中心极限定理及相应的概率不等式, 证明了由混合序列生成线性过程加权和的极限定理.

关键词: 混合序列; 线性过程; 加权和; 极限定理
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Limit Theorem for Weighted Sums of Linear Processes Generated by Mixing Sequences

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Abstract: Let $\{X_t, t\geqslant 1\}$ be a linear process of the form $X_t = \sum_{j=0}^{\infty} a_j\xi_{t-j}$, where $\{\xi_t, t\in Z\}$ is a mixing sequence with zero means, $\{a_j, j\geqslant 0\}$ is a sequnce of real numbers with $\sum_{j=0}^{\infty} j|a_j| < \infty$ and $\{a_{ni}, 1\leqslant i\leqslant n, n\geqslant 1\}$ is a triangular array of real numbers, then under some suitable conditions, by means of the central limit theory and corresponding probability inequalities of mixing sequence, we have obtained the limit theorem for weighted sums of linear processes generated by mixing sequences.

Key words: mixing sequences; linear processes; weighted sums; limit theorem

令 $\{X_t, t\in Z\}$ 为定义在概率空间 $(\Omega,\mathscr{F},P)$ 上的线性过程, 其形式和定义如下:

$$X_t = \sum_{j=0}^{\infty} a_j\xi_{t-j}, \qquad t\geqslant 1, \tag{1}$$

其中: $\{a_j, j\geqslant 0\}$ 为一实数序列, 满足

$$A = \sum_{j=0}^{\infty} a_j \neq 0, \qquad \sum_{j=0}^{\infty} j|a_j| < \infty; \tag{2}$$

$\{\xi_t, t\in Z\}$ 为一零均值的随机变量序列.

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线性过程的极限理论对于刻划各种统计推断问题中导出的检验统计量极限分布至关重要. 文献[1-12]分别讨论了不同条件下线性过程的极限定理, 而关于由混合序列生成的线性过程加权和的极限定理的研究目前文献报道较少, 本文对该问题进行探讨.

定义1[13] 设$\mathscr{A}$和$\mathscr{B}$为两个σ-代数, 定义

$$\varphi(\mathscr{A},\mathscr{B}) = \sup_{A\in\mathscr{A},B\in\mathscr{B},P(A)\neq 0} |P(B|A) - P(B)|,$$
$$\rho(\mathscr{A},\mathscr{B}) = \sup_{f\in L_2(\mathscr{A}),g\in L_2(\mathscr{B})} |\mathrm{corr}(f,g)|,$$
$$\alpha(\mathscr{A},\mathscr{B}) = \sup_{A\in\mathscr{A},B\in\mathscr{B}} |P(AB) - P(A)P(B)|,$$

其中

$$\mathrm{corr}(f,g) = \frac{\mathrm{Cov}(f,g)}{\mathrm{Var}(f)\mathrm{Var}(g)}.$$

定义2[14] 假设$\{X_i,i\geqslant 1\}$为一随机序列, 记$\mathscr{F}_n^m=\sigma(X_i,n\leqslant i\leqslant m)$, $n<m$.

1) 如果$\varphi(n)\to 0$, 则称随机序列$\{X_i,i\geqslant 1\}$是φ-混合的, 其中$\varphi(n)=\sup_k\varphi(\mathscr{F}_1^k,\mathscr{F}_{k+n}^\infty)$.

2) 如果$\rho(n)\to 0$, 则称随机序列$\{X_i,i\geqslant 1\}$是ρ-混合的, 其中$\rho(n)=\sup_k\rho(\mathscr{F}_1^k,\mathscr{F}_{k+n}^\infty)$.

3) 如果$\alpha(n)\to 0$, 则称随机序列$\{X_i,i\geqslant 1\}$是α-混合(强混合)的, 其中$\alpha(n)=\sup_k\alpha(\mathscr{F}_1^k,\mathscr{F}_{k+n}^\infty)$.

本文的主要结果如下.

定理1 设$\{X_t,t\geqslant 1\}$为满足式(1)和(2)的线性过程, $\{a_{ni},1\leqslant i\leqslant n, n\geqslant 1\}$为一实值的三角阵列, 且满足如下条件:

1) $\sup_n\sum_{i=1}^n a_{ni}^2 < \infty$, $\limsup_{n\to\infty} n\cdot\max_{1\leqslant i\leqslant n} a_{ni}^2 \leqslant C$;

2) $\mathrm{Var}\left(\sum_{i=1}^n a_{ni}\xi_i\right)\to 1$, $\{\xi_i^2,i\in Z\}$ 一致可积.

并且如果下述条件之一成立:

(i) $\{\xi_i,i\in Z\}$ 是φ-混合的, 且满足$\sum_{n=1}^\infty \varphi^{1/2}(2^n) < \infty$;

(ii) $\{\xi_i,i\in Z\}$ 是ρ-混合的, 且满足$\sum_{n=1}^\infty \rho(2^n) < \infty$;

(iii) 对某个$\delta>0$, $\{\xi_i,i\in Z\}$是强混合的, $\{|\xi_i|^{2+\delta},i\in Z\}$一致可积,

$$\inf_i \mathrm{Var}(\xi_i) > 0, \qquad \sum_{n=1}^\infty n^{2/\delta}\alpha(n) < \infty.$$

则有

$$\sum_{i=1}^n a_{ni}X_i \xrightarrow{\mathrm{d}} \mathscr{N}(0,A^2). \tag{3}$$

推论1 设$\{\xi_i,i\in Z\}$是一独立同分布零均值的随机变量序列, 满足定理1中的条件1)和2), 则式(3)成立.

注1 显然满足定理1中假设条件1)的$\{a_{ni}\}$有很多, 如常见的等权, 即$a_{ni}=\dfrac{1}{\sqrt{n}}$. 事实上, 对于$a_{ni}=\dfrac{1}{n^\beta}$, $\beta\geqslant\dfrac{1}{2}$都满足条件1). 此外, 如$a_{ni}=\dfrac{i}{n^{3/2}}$, $a_{ni}=\dfrac{n+1-i}{n^{3/2}}$等也都满足定理1的假设条件.

注2 由定理1的证明过程可知, 如果取$a_0=1$, $a_j=0$, $j\geqslant 1$, 条件1)中的$\limsup_{n\to\infty} n\cdot\max_{1\leqslant i\leqslant n} a_{ni}^2\leqslant C$可减弱为$\max_{1\leqslant i\leqslant n}|a_{ni}|\to 0$, 则可得到引理2.

设C表示正常数, 不同之处可表示不同的值.

引理1[15] 假设$A_1,A_2,\cdots,A_n$和$B_1,B_2,\cdots,B_n(B_1\geqslant B_2\geqslant\cdots\geqslant B_n\geqslant 0)$为两个实数序列, 令

$$S_k = \sum_{i=1}^{k} A_i, \quad M_1 = \min_{1\leqslant k\leqslant n} S_k, \quad M_2 = \max_{1\leqslant k\leqslant n} S_k.$$

则有

$$B_1 M_1 \leqslant \sum_{i=1}^{n} A_i B_i \leqslant B_1 M_2.$$

引理 2[2] 假设$\{\xi_i, i\geqslant 1\}$为一零均值的随机序列, $\{a_{ni}, 1\leqslant i\leqslant n, n\geqslant 1\}$为一实值的三角阵列, 如果定理 1 中的条件 1) 被如下条件替代:

1)′ $\sup\limits_n \sum\limits_{i=1}^{n} a_{ni}^2 < \infty$, $\max\limits_{1\leqslant i\leqslant n} |a_{ni}| \to 0$;

其他假设条件不变, 则有

$$\sum_{i=1}^{n} a_{ni}\xi_i \xrightarrow{\mathrm{d}} \mathscr{N}(0,1). \tag{4}$$

下面证明定理 1. 由线性过程的分解易知,

$$X_k = A\xi_k + \tilde{\xi}_{k-1} - \tilde{\xi}_k,$$

其中

$$\tilde{\xi}_k = \sum_{j=0}^{\infty} \tilde{a}_j \xi_{k-j}, \qquad \tilde{a}_j = \sum_{i=j+1}^{\infty} a_i.$$

因此

$$\sum_{i=1}^{n} a_{ni} X_i = A\sum_{i=1}^{n} a_{ni}\xi_i + \sum_{i=1}^{n} a_{ni}(\tilde{\xi}_{i-1} - \tilde{\xi}_i) =: I_n + J_n. \tag{5}$$

由引理 2 可知

$$I_n \xrightarrow{\mathrm{d}} \mathscr{N}(0, A^2). \tag{6}$$

由 Slutsky 定理, 只需证明 $J_n \xrightarrow{\mathrm{P}} 0$ 即可. 不失一般性, 假设 $a_{n1}\geqslant a_{n2}\geqslant\cdots\geqslant a_{nn}$. 记 $B_s = a_{ns} - a_{nn}$, $1\leqslant s\leqslant n-1$, $B_n = 0$. 则由引理 1 可知,

$$\begin{aligned}|J_n| &\leqslant \Big|\sum_{i=1}^{n}(a_{ni} - a_{nn})(\tilde{\xi}_{i-1} - \tilde{\xi}_i)\Big| + \Big|\sum_{i=1}^{n} a_{nn}(\tilde{\xi}_{i-1} - \tilde{\xi}_i)\Big| \leqslant \\ &(a_{n1} - a_{nn})\max_{1\leqslant m\leqslant n}\Big|\sum_{i=1}^{m}(\tilde{\xi}_{i-1} - \tilde{\xi}_i)\Big| + |a_{nn}||\tilde{\xi}_0 - \tilde{\xi}_n| \leqslant \\ &3\max_{1\leqslant i\leqslant n}|a_{ni}|(|\tilde{\xi}_0| + \max_{1\leqslant m\leqslant n}|\tilde{\xi}_m|).\end{aligned} \tag{7}$$

由 $\sum\limits_{j=0}^{\infty} j|a_j| < \infty$, 易得 $\sum\limits_{j=0}^{\infty}|\tilde{a}_j| < \infty$, 则对于任意的 $\epsilon > 0$, 有

$$P(\max_{1\leqslant i\leqslant n}|a_{ni}||\tilde{\xi}_0| > \epsilon) \leqslant \frac{1}{\epsilon}\max_{1\leqslant i\leqslant n}|a_{ni}|E|\tilde{\xi}_0| \leqslant \frac{C}{n^{1/2}}\sum_{j=0}^{\infty}|\tilde{a}_j|E|\xi_{-j}| \to 0, \qquad n\to\infty. \tag{8}$$

另一方面, 注意到

$$|\tilde{\xi}_m| \leqslant \sum_{j=0}^{m}|\tilde{a}_j||\xi_{m-j}| + \sum_{j=1}^{\infty}|\tilde{a}_{m+j}||\xi_{-j}| \leqslant \sum_{j=0}^{\infty}|\tilde{a}_j|\max_{0\leqslant j\leqslant m}|\xi_j| + \sum_{j=1}^{\infty}|\tilde{a}_{m+j}||\xi_{-j}|. \tag{9}$$

类似对式(8)的讨论, 可知

$$P\Big(\max_{1\leqslant i\leqslant n}|a_{ni}|\max_{1\leqslant m\leqslant n}\sum_{j=1}^{\infty}|\tilde{a}_{m+j}||\xi_{-j}| > \epsilon\Big) \leqslant \frac{C}{n^{1/2}}\sum_{j=0}^{\infty}|\tilde{a}_j|E|\xi_{-j}| \to 0, \qquad n\to\infty. \tag{10}$$

从而由式(7), (8)可知, 要证明 $J_n \xrightarrow{\mathrm{P}} 0$, 只需证明下式成立即可:

$$\max_{1\leqslant i\leqslant n}|a_{ni}|\max_{0\leqslant m\leqslant n}|\xi_m| \xrightarrow{\mathrm{P}} 0. \tag{11}$$

由文献[16]知, 式(11)等价于下式成立:

$$\max_{1\leqslant i\leqslant n}|a_{ni}|^2\sum_{m=0}^{n}\xi_m^2 I\{\max_{1\leqslant i\leqslant n}|a_{ni}||\xi_m| > \epsilon\} \xrightarrow{\mathrm{P}} 0, \qquad \forall\epsilon > 0. \tag{12}$$

最后由定理1的假设条件1)和2),有

$$\max_{1\leqslant i\leqslant n}|a_{ni}|^2\sum_{m=0}^{n}E\xi_m^2 I\{\max_{1\leqslant i\leqslant n}|a_{ni}||\xi_m|>\epsilon\}\leqslant\frac{C}{n}\sum_{m=0}^{n}E\xi_m^2 I\{|\xi_m|>Cn^{1/2}\epsilon\}\leqslant$$

$$C\sup_m E\xi_m^2 I\{|\xi_m|>Cn^{1/2}\epsilon\}\to 0,\quad n\to\infty. \tag{13}$$

证毕.

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