No-common-cause EPR-like funny business in branching space-times

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Abstract

There is "no EPR-like funny business" if (contrary to apparent fact) our world is as indeterministic as you wish, but is free from the EPR-like quantum-mechanical phenomena such as is sometimes described in terms of superluminal causation or correlation between distant events. The theory of branching space-times can be used to sharpen the theoretical dichotomy between "EPR-like funny business" and "no EPR-like funny business." Belnap 2002 offered two analyses of the dichotomy, and proved them equivalent. This essay adds two more, both connected with Reichenbach's "principle of the common cause," the principle that sends us hunting for a common-causal explanation of distant correlations. The two previous ideas of funny business and the two ideas introduced in this essay are proved to be all equivalent, which increases one's confidence in the stability of (and helpfulness of) the BST analysis of the dichotomy between EPR-like funny business and its absence.

1 Background: Two ideas of EPR-like funny business

The vast philosophical literature on quantum mechanics is filled with

(a) accounts of EPR-like or Bell-like correlations between space-like related (I write SLR) events, and also with

(b) discussions of the same phenomena under the heading of superluminal causation.

Belnap 2002 used the austere language of branching space-times (BST) in order to define the following two sharp concepts corresponding respectively to these rough concepts:

> Primary SLR modal-correlation funny business (see Definition 1-2). [1]

> Some-cause-like-locus-not-in-past funny business (see Definition 1-3). [2]

That essay then proved the equivalence of [1] and [2], and suggested that this fact strengthened the case for taking the BST versions as cutting at a joint between indeterminism on the one hand, and on the other a peculiar feature of quantum mechanics that goes strangely beyond mere indeterminism.

Also figuring in philosophical discussions of quantum-mechanical funny business is the oft-cited common-cause principle of Reichenbach 1956; it seems as if this plausible principle is violated by the same phenomena that are sometimes described in terms of (a) and sometimes in terms of (b). The purpose of this paper is to state a BST version of the Reichenbach principle, and then prove that its violation, which we might call "no-common-cause funny business," is equivalent to both the existence of primary SLR modal-correlation funny business and to the existence of some-cause-like-locus-not-in-past funny business. In fact I describe *two* (equivalent) BST versions, which I call more specifically as follows:

> No-prior-screener-off funny business (see Definition 3-3). [3]

> No-prior-common-cause-like-locus funny business (see Definition 4-2). [4]

Thus, to the extent that the two previous ideas [1] and [2] and the two new ideas [3] and [4] are given modal versions in BST, they come to the same thing.

BST is laid out as an exact theory in Belnap 1992 and again in Belnap 2002, to which I must refer the reader for notation, postulates and definitions, and above all for much-needed motivation. Here I list just a few key items as reminders, including the definitions of [1] and [2].

1-1 DEFINITION. *(Key ancillary concepts)*

- The primitives of BST are two: *Our World*, whose members are defined as *point events*, and $<$, the "causal order" on *Our World*. It is assumed that $<$

is a dense strict partial order on *Our World* with no maximal elements. e is a point event, and h is a *history*, i.e., a maximal directed set, where a set is *directed* if it contains an upper bound for each pair of its members. H is a set of histories (also called a *proposition*) and $\mathbf{H}$ is a set of sets of histories (hence a set of propositions).

O is an *outcome chain* (nonempty and lower bounded chain, where a set is a *chain* if each two of its members are comparable by $<$); provably $O \subseteq h$ for some h. It is assumed that O has always a unique infimum $inf(O)$, and it is provable that given $e \in h$, there is an O such that $e < O$ and $e = inf(O)$.

An *initial event* $\mathbf{I}$ is a set of point events all of which are members of some one history, and an *outcome event* $\mathbf{O}$ is a set of outcome chains all of which overlap some one history.

- $H_{(e)} = \{h\colon e \in h\}$, which is the *proposition* saying that e *occurs*. $H_{[\mathbf{I}]} = \{h\colon I \subseteq h\}$, which is the *proposition* that says that $\mathbf{I}$ *occurs*. $H_{\langle O\rangle} = \{h\colon h \cap O \neq \varnothing\}$, which is the *proposition* that O *occurs*. $H_{\langle \mathbf{O}\rangle} = \bigcap\{H_{\langle O\rangle}\colon O \in \mathbf{O}\} \neq \varnothing$, which is the *proposition* that says that $\mathbf{O}$ *occurs*.

  A proposition H is *consistent* iff $H \neq \varnothing$. An *event* of a specified type (e, O, $\mathbf{O}$, or $\mathbf{I}$) is *consistent* iff its listed "occurrence proposition" is consistent. Use of the notations e, O, $\mathbf{O}$, and $\mathbf{I}$ guarantees consistency. A set of propositions $\mathbf{H}$ is *consistent* iff $\bigcap \mathbf{H} \neq \varnothing$. A set of events of various specified types is *consistent* iff the set of their occurrence propositions are consistent.

- $h_1 \equiv_e h_2$ means that h_1 and h_2 are undivided at e ($e \in h_1 \cap h_2$, but e is not maximal therein) and $h_1 \equiv_{\mathbf{I}} h_2$ means $h_1 \equiv_e h_2$ for every $e \in \mathbf{I}$.

  Much-used fact: undividedness-at-e is an equivalence relation on $H_{(e)}$, and accordingly undividedness-at-$\mathbf{I}$ is an equivalence relation on $H_{[I]}$.

  $\mathbf{I} \rightarrowtail \Pi_{\mathbf{I}}$ is a *primary propositional spread*, that is, an ordered pair of the initial event $\mathbf{I}$ and the partition $\Pi_{\mathbf{I}}$ of $H_{[\mathbf{I}]}$ that is induced by undividedness at $\mathbf{I}$. For $\mathbf{I} \subseteq h$, $\Pi_{\mathbf{I}}\langle h\rangle$ is the member of $\Pi_{\mathbf{I}}$ to which h belongs. When $\mathbf{I} = \{e\}$, I write $e \rightarrowtail \Pi_e$ and $\Pi_e\langle h\rangle$.

- The idea of $e \rightarrowtail \Pi_e$ is basic, and I call it a *basic primary propositional spread*.[1]

[1]There are two ideas: (1) primary propositional spread and (2) *basic* primary propositional spread. In Belnap 2002 the terminology for exactly the same pair of ideas was (1) *generalized* primary propositional spread and (2) primary propositional spread. I think the change in terminology

Point events are *space-like-related* iff they are distinct, not causally ordered and share a history. $\mathbf{I}_1$ SLR $\mathbf{I}_2$ means that every point event in $\mathbf{I}_1$ is space-like related to every point event in $\mathbf{I}_2$.

- h_1 is separated from h_2 at e, written $h_1 \perp_e h_2$, $\leftrightarrow_{df}$ e is maximal in $h_1 \cap h_2$.

  h_1 *is separated from* H *at* $\mathbf{I}$, written $h_1 \perp_{\mathbf{I}} H$, $\leftrightarrow_{df}$ $\forall h_2[h_2 \in H \rightarrow \exists e[e \in \mathbf{I}$ and $h_1 \perp_e h_2]]$. When $\mathbf{I} = \{e\}$, I write $h \perp_e H$, and also use $H_1 \perp_e H_2$ when every history in H_1 is separated at e from every history in H_2.

  h *is relevantly separated from* H *at* $\mathbf{I}$, written $h \underline{\perp}_{\mathbf{I}} H$, $\leftrightarrow_{df}$ $h \perp_{\mathbf{I}} H$ and $\forall e[e \in \mathbf{I} \rightarrow \exists h_1[h_1 \in H$ and $h \perp_e h_1]]$.

  $\mathbf{I}$ *is a cause-like locus for* $\mathbf{O}$ *with respect to* h $\leftrightarrow_{df}$ $h \underline{\perp}_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$.

1-2 DEFINITION. *(Primary* SLR *modal-correlation funny business)* Two primary propositional spreads $\mathbf{I}_1 \rightarrowtail \Pi_{\mathbf{I}_1}$ and $\mathbf{I}_2 \rightarrowtail \Pi_{\mathbf{I}_2}$ together with two outcome-determining histories h_1 and h_2 such that $\mathbf{I}_1 \subseteq h_1$ and $\mathbf{I}_2 \subseteq h_2$ constitute a case of *primary* SLR *modal-correlation funny business* $\leftrightarrow_{df}$ $\mathbf{I}_1$ SLR $\mathbf{I}_2$ and $\Pi_{\mathbf{I}_1}\langle h_1 \rangle \cap \Pi_{\mathbf{I}_2}\langle h_2 \rangle = \varnothing$.[2]

1-3 DEFINITION. *(Some-cause-like-locus-not-in-past funny business.)* $\mathbf{I}$, h, and $\mathbf{O}$ constitute a case of *some-cause-like-locus-not-in-past funny business* $\leftrightarrow_{df}$ $\mathbf{I}$ *is a cause-like locus for* $\mathbf{O}$ *with respect to* h, but no member of $\mathbf{I}$ lies in the causal past of any member of $\mathbf{O}$.

It is these last two concepts that Belnap 2002 proves equivalent in the sense that there exists a case of one iff there exists a case of the other. Let us go on to the two ideas of funny business introduced above as [3] and [4].

2 Background: Reichenbach's common cause principle

The phrases "common cause" and "screening off" come from Reichenbach 1956. In the words of Arntzenius 1999, the Reichenbach principle comes to this: "Si-

is a small improvement.

[2]The adjective "primary" is important. When outcome events are distant from initials as contemplated in Definition 3-1, then SLR modal correlation is not enough for funny business, since the correlation can be due to perfectly "ordinary" circumstances such as a "common cause." That is what we investigate below. In the primary case, however, there is no "room" for additional causal "influences" from the past.

multaneous correlated events have a prior common cause that screens off the correlation." It seems unrecognized that the idea (but neither the words nor the Reichenbach analysis) of "screening off" and its relation to causality comes first—as far as amateur research has discovered—from Kendall and Lazarsfeld 1950. It is restated in Lazarsfeld 1958 in evident independence of Reichenbach, and reworked by Lazarsfeld's Columbia colleague in the textbook Nagel 1961. Here is a simple abstract example that is a *very* special case of what they have in mind. Let A be dichotomous, having just two values A_1 and A_2, and similarly with B. Suppose $pr(A_1B_1)=0$, and that all other AB combinations have positive probability. A quick cross-multiplication indicates correlation, since one diagonal of the AB matrix gives 0 and the other does not. Or, to use another check, we find that $pr(A_1B_1)\neq pr(A_1)\times pr(B_1)$. Now introduce C with say three values C_1, C_2, and C_3. Suppose for example that $pr(A_1C_1)=0$, $pr(B_1C_2)=0$, and $pr(A_1C_3)=0$. Then a consideration of the three "partial" AB matrices for C_1, C_2, and C_3 shows that the correlation disappears in each case, since all diagonals come out 0. Or, to use another check, we find that the multiplicative relation is restored in each case; that is, for each *i*, $pr(A_1B_1/C_i)$ is in fact equal to $pr(A_1/C_i)\times pr(B_1/C_i)$. Thus, the correlation between A and B is "due to" their interactions with C, and is thus "explained." C is the "common cause." Well, as indicated in the Arntzenius quote, there is in addition a spatio-temporal requirement: Events A and B must be simultaneous and their common cause C must be prior to each. We ask for a *prior* common cause.

It is worth noticing that whereas commentators are generally thoroughly rigorous about the calculus-of-probabilities aspect of the principle, invariably including elaborate mathematical calculations, the spatio-temporal aspect is often left to marginal comment, with no theory offered that could support rigorous deduction. This is not a new thought; for example, Uffink 1999 urges and makes plain the necessity of being explicit about the spatio-temporal aspects of the situation: It is not enough to formulate the principle in the language of the probability calculus: "Reichenbach's PCC [principle of common cause] and its variants are crippled because they lack any explicit reference to space-time structure." In a nice phrase, Uffink suggests that "the natural habitat for the PCC is an application for localized events in space-time, rather than in formal phase spaces," though in a tellingly honest remark, he also explicitly leaves "aside the question of how to interpret the required probabilities in this problem."

Not everyone neglects rigor in treating the spatio-temporal aspects of Reichenbach's principle. Uffink 1999 points out that Penrose and Percival 1962 deals explicitly with space-time in their formulation of a kind of common-cause principle.

Letting a, b, and c range respectively over states (they say *histories*) of 4-regions A, B, and C, they write

> If A and B are two disjoint 4-regions, and C is any 4-region which divides the union of the pasts of A and B into two parts, one containing A and the other containing B, then A and B are conditionally independent given c. That is,
>
> $p(a\&b/c) = p(a/c) \times p(b/c)$ for all a, b (p. 611).

There is, for better or worse, little resemblance between their ideas and those of BST. For one thing, they work with a single-space time endowed with fixed fields and particle trajectories; that is, their underlying structure is deterministic. Their notion of "the probability that region A has history a" comes from counting up the 4-regions A' into which A can be translated, and considering the proportion of these that have the history a. Although useful epistemologically, this attempt at a frequentist approach in the context of indeterminism seems to me unacceptable as an account of objective probability. Nor, as far as I can see, does their account help with single-case transitional probabilities. (Cartwright 1983 suggests that "all real probabilities in quantum mechanics are transitional probabilities.")

Arntzenius 1999 (with reference to Arntzenius 1997) suggests understanding the principal of the common cause by supposing that "in nature there are transition chances from values of quantities at earlier times to values of quantities at later times." His idea is then to state the following as a common cause principle:

> Conditional upon the values of all the quantities upon which the transition chances to quantities X and Y depend, X and Y will be probabilistically independent.

The idea of "transition chances" is in the spirit of BST, well worth making rigorous by providing, for example, a theory of "quantities," of "values of quantities at times," and of the "dependence" of one quantity on another.

Hofer-Szabó, Rédei and Szabó 1999 make the distinction between a "common cause" and a "common common cause," an idea implicit but not explicit in Szabo and Belnap 1996. In this essay I am concurring with the opinion that a violation of a the weaker *plain* common-cause principle is sufficient and necessary for no-common-cause funny business. The distinction is made further use of in Hofer-Szabó, Rédei and Szabó 2000 and Szabo 2000. Rédei 1996, Rédei 1997, Rédei 2002 give Reichenbach's principle an explicit spatio-temporal reading by reference to relativistic quantum field theory (RQFT). I confess (if that is the right word) that my training does not fit me to understand RQFT, but one thing seems clear enough: There is in the cited essays no requirement that the common cause be *prior*, something upon which I shall insist. Szabo and Belnap 1996 give a definition of "common cause" in the context of BST theory that puts the common cause explicitly in the causal past. The actual analysis of the GHZ theorem of that essay, however, makes no use of this requirement; which is to say, as far as that analysis goes, the causal-past requirement remains idle.

Another line of research that takes space-time seriously is that founded on the Kowalski and Placek 1999 analysis of outcomes in branching space-time, including Placek 2000a, Placek 2000b, Müller and Placek 2001, and Müller 2002, Placek 2002**??**. The BST structure of those essays is richer than the BST theory employed here, explicitly giving a Minkowski structure to each of the branching space-times. I note that the idea of "outcome" in those papers is a clear-cut and interesting alternative to the notion of "outcome" that I employ in this essay. The work of comparing the two analyses has not been done. It needs noting, however, that Müller 2002 proves that any BST structure such as is considered here can be embedded in the richer branching structure that is there defined.

Upshot: There is some work on Reichenbach's idea that is not fully rigorous and there is some that is. In both cases the theories contemplated are significantly more complicated than the BST theory, employed here, that grows out of Belnap 1992: The only primitives are the set of point events and a binary causal ordering upon that set. The postulates governing these primitives are simple; and everything else is introduced by fully rigorous definition. I am not at all suggesting that simplicity is of itself a virtue (Whitehead: Seek simplicity and distrust it). I do suggest that the simplicity of BST theory has its place; namely its simplicity helps BST theory in delineating some key structural features of quantum-mechanical funny business, features that can otherwise become lost by attending to more complicated structures.

3 From coincidence to no-prior-screener-off funny business

In common-cause discussions based on Reichenbach's account, there is generally talk of a surprising *coincidence* of kinds of outcomes, for example, everyone at a picnic becomes sick, although from an earlier perspective such an outcome was in each case contingent. Given this coincidence, one looks for a prior event that "screens off" the coincidence in something like the sense explained in §2. (In the story, the common food of the picnickers is poisoned, whereas from an early enough vantage point, the introduction of the poison was a contingent matter—the food might not have been poisoned.) Such a description requires *similarity*, something that I try to do without insofar as possible. That is the reason that I structure the common-cause idea in terms of impossibilities instead of coincidences. First I give definitions, based on Belnap 2002 and Szabo and Belnap 1996, and then work out how they apply to the picnic example.

3-1 DEFINITION. *(Modal correlation of spreads)*

- $\mathbf{I} \rightarrowtail H_{\langle \mathbf{O} \rangle}$ is a *propositional transition* $\leftrightarrow_{df}$ for all $O \in \mathbf{O}$, $\mathbf{I} < O$.[3]
- $\mathbf{I} \rightarrowtail \mathbf{H}$ is a *propositional spread* $\leftrightarrow_{df}$ for each $H \in \mathbf{H}$ there is an $\mathbf{O}$ such that $H = H_{\langle \mathbf{O} \rangle}$ and $\mathbf{I} \rightarrowtail H_{\langle \mathbf{O} \rangle}$ is a propositional transition and $\mathbf{H}$ partitions $H_{[\mathbf{I}]}$.

  Suppose we have two propositional spreads $\mathbf{I}_1 \rightarrowtail \mathbf{H}_1$ and $\mathbf{I}_2 \rightarrowtail \mathbf{H}_2$, with $\mathbf{I}_1$ and $\mathbf{I}_2$ each consistent.
- $\mathbf{I}_1 \rightarrowtail \mathbf{H}_1$ and $\mathbf{I}_2 \rightarrowtail \mathbf{H}_2$ are *modally correlated* $\leftrightarrow_{df}$ $H_1 \cap H_2 = \varnothing$ for some $H_1 \in \mathbf{H}_1$ and $H_2 \in \mathbf{H}_2$.
- If we specify both the two propositional spreads $\mathbf{I}_1 \rightarrowtail \mathbf{H}_1$ and $\mathbf{I}_2 \rightarrowtail \mathbf{H}_2$ and an inconsistent pair of propositional outcomes $H_1 \in \mathbf{H}_1$ and $H_2 \in \mathbf{H}_2$ (i.e., $H_1 \cap H_2 = \varnothing$), we say that we have a *modal correlation between spreads*. If furthermore every member of $\mathbf{I}_1$ is space-like related to every member of $\mathbf{I}_2$, then we call it a *space-like-related modal correlation between spreads*.
- For many purposes it suffices to consider modal correlation as holding between outcome events instead of between spreads. If $H_{\langle \mathbf{O}_1 \rangle}$ and $H_{\langle \mathbf{O}_2 \rangle}$ are individually consistent, but $H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle} = \varnothing$, then we have a *modal correlation between outcome events*.

The definition of modal correlation between propositional spreads is closely tied to its probabilistic ancestor. Two "variables" are correlated, according to a

[3]There is no thought that the transition should be "primary" or immediate. That is the difference between this set-up and that of Definition 1-2. Observe that each of $\mathbf{I}$ and $\mathbf{O}$ may be spread out both time-like and space-like.

simple concept, when for each of the two variables there is a (separate) probability distribution for the occurrence of its values, but for some value of the one and some value of the other, you cannot get the probability of the joint occurrence by multiplying. A special case of this is when there is a value of the one with positive probability, and a value of the other also with positive probability, but with zero probability for their joint occurrence. (You cannot get zero by multiplying two positive numbers.) That is the probabilistic version of "modal correlation," with the following adjustments.

- That the values have positive probability is replaced by saying that each outcome event, taken individually, is possible or consistent. That a combination has zero probability is replaced by saying that the combination is impossible or inconsistent. This corresponds to the "special case" indicated above.
- A much deeper point: I am not speaking of "variables" in some abstract and perhaps unexplained fashion. Rather, it is spreads—which have concrete locations on *Our World*—that play the role of "variables," and concrete outcome events that play the role of "values."

The present concepts must be able to apply to cases such as the poisoned picnickers. Here is how such an application might go.

EXAMPLE. *(The poison case)* Pick an early $\mathbf{I}_1$ and $\mathbf{I}_2$, respectively in the life of person 1 and person 2. From these earlier perspectives, it is not settled whether or not, after the evening meal, person 1 gets sick and similarly for person 2. Let us introduce an outcome event $\mathbf{O}_{1+}$ to represent a completely specific "sick" outcome for person 1, an event that is part of person 1's later life as sick in that particular way. Also, let $\mathbf{O}_{1-}$ represent a completely specific "nonsick" outcome for person 1. Let us symmetrically introduce $\mathbf{O}_{2+}$ and $\mathbf{O}_{2-}$ as outcome events in two fully specific possible later lives of person 2, in one of which person 2 becomes sick and in one of which he doesn't. Represent the lack of causal influence between the two persons by choosing $\mathbf{I}_1$ and $\mathbf{I}_2$ as space-like related.

It is altogether natural to think of the situation in terms of "positive" coincidence: In the circumstances, person 1 gets sick if and only if person 2 gets sick. But any "iff" story can be told instead in terms of impossibility, and such a telling gives us more control over the details of the situation. In the present case, the story is such that in the circumstances it cannot happen that person 1 gets sick while person 2 doesn't. (It is a separate fact that it cannot happen that person 2 gets sick while person 1 doesn't.) Therefore, as a specific consequence of this, $H_{\langle\mathbf{O}_{1+}\rangle}\cap H_{\langle\mathbf{O}_{2-}\rangle}=\varnothing$: There is no history in which both person 1 becomes sick in the detailed way represented by $\mathbf{O}_{1+}$ and person 2 fails to become sick in the specific way represented by $\mathbf{O}_{2-}$. That is, we have a space-like-related modal correlation.

I continue the example in order to motivate the BST screening-off version of Reichenbach's idea. We have a common-cause explanation for this modal correlation: There is an initial e_3 *in the past of both* $\mathbf{O}_{1+}$ and $\mathbf{O}_{2-}$ at which it is not yet fixed whether or not the food for the evening meal is poisonous. Immediately after e_3, however, it is a settled matter whether or not the food is poisonous, and we can imagine that there are several outcomes of e_3 representing types of poisoning and also several representing types of non-poisoning. Gather these into a set of immediate outcomes of e_3, so that $e_3 \rightarrowtail \Pi_{e_3}$ is a basic primary propositional spread. When e_3 issues in a poisoning sort of outcome, both person 1 and person 2 become sick. And when e_3 issues in a non-poisoning sort, neither person 1 nor person 2 becomes sick. So each and every immediate outcome $\Pi_{e_3}\langle h\rangle$ of e_3 is either inconsistent with the occurrence of $\mathbf{O}_{1+}$ or with the occurrence of $\mathbf{O}_{2-}$. The structure of the poisoning spread $e_3 \rightarrowtail \Pi_{e_3}$ therefore gives a common-cause or screening-off explanation of the modal correlation between $H_{\langle\mathbf{O}_{1+}\rangle}$ and $H_{\langle\mathbf{O}_{2-}\rangle}$ with which we started.

In the story, the single basic primary spread $e_3 \rightarrowtail \Pi_{e_3}$ provides a further explanation, namely the same spread that screens off the correlation of $H_{\langle\mathbf{O}_{1+}\rangle}$ and $H_{\langle\mathbf{O}_{2-}\rangle}$ also screens off the correlation between $H_{\langle\mathbf{O}_{1-}\rangle}$ and $H_{\langle\mathbf{O}_{2+}\rangle}$. Our story thus contains what Hofer-Szabó *et al*. 1999, as I remarked in §2, call a "common common cause." Let us, however, concentrate on just the one correlation, that between $H_{\langle\mathbf{O}_{1+}\rangle}$ and $H_{\langle\mathbf{O}_{2-}\rangle}$. Whence the language of "screening off"? The idea from Lazersfeld on has been that a third variable "explains" a so-called spurious correlation between two given variables when the correlation disappears for (is screened off by) each possible value of the third variable, provided the third variable is "earlier" than the others. In BST we can, as before, replace "variable" by "spread," and we can in addition give real causal bite to the idea that the explaining variable should be *earlier*. In fact, the causal relation of priority plays such a heavy role that we can for technical purposes of definition simply *omit* references to modal correlation of spreads, substituting the much simpler idea of correlation of outcome events. For the third or "explaining" spread, we invoke only basic primary propositional spreads. Here is the definition.

3-2 DEFINITION. *(Screening off)* Let $\mathbf{O}_1$ and $\mathbf{O}_2$ be two outcome events, each individually consistent, that are modally correlated in the sense than $H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle} = \varnothing$. A basic primary spread $e \rightarrowtail \Pi_e$ is a *prior screener-off*[4] of that correlation iff

- **Causal priority.** $e < O_1$ for some $O_1 \in \mathbf{O}_1$ and $e < O_2$ for some $O_2 \in \mathbf{O}_2$.
- **Screening off.** $\forall h[e \in h \rightarrow$ (either $\Pi_e\langle h \rangle \cap H_{\langle \mathbf{O}_1 \rangle} = \varnothing$ or $\Pi_e\langle h \rangle \cap H_{\langle \mathbf{O}_2 \rangle} = \varnothing$)]. That is, no matter which immediate outcome of e you consider, that outcome will be inconsistent with the occurrence of at least one of $\mathbf{O}_1$ and $\mathbf{O}_2$.

Comments are in order. First, the version of "causal priority" stated is deliberately weak. The reason is that by making it weak we make it easier to find a prior screener-off, so that to say that we cannot find a screener-off that is prior in even that weak sense is a strong statement of funny business. Second, the modal version of screening-off is exactly what one is led to if one starts with probabilities. If you identify impossibility and zero probability, then screening-off here is a special case of "for each *i*, *pr*(A_1B_1/C_i) is equal to *pr*(A_1/C_i)$\times$*pr*(B_1/C_i)" as discussed in §2; namely, both sides evaluate to zero. In other words, the correlation between the two outcome events $H_{\langle \mathbf{O}_1 \rangle}$ and $H_{\langle \mathbf{O}_2 \rangle}$ is "explained away" by means of their individual interactions with the *causally prior* basic primary spread $e \rightarrowtail \Pi_e$. Who could ask for anything more? We are accordingly led to the following definition, which is based on Szabo and Belnap 1996 and Belnap 2002.

3-3 DEFINITION. *(No prior-screener-off funny business)* A pair of outcome events (nonempty sets of outcome chains) $\mathbf{O}_1$ and $\mathbf{O}_2$ constitute a case of *no-prior-screener-off funny business* $\leftrightarrow_{df}$

1. Each of $H_{\langle \mathbf{O}_1 \rangle}$ and $H_{\langle \mathbf{O}_2 \rangle}$ is individually consistent; i.e., $H_{\langle \mathbf{O}_1 \rangle} \neq \varnothing$ and $H_{\langle \mathbf{O}_2 \rangle} \neq \varnothing$. (This is part of the definition of "outcome event.")
2. $H_{\langle \mathbf{O}_1 \rangle}$ is inconsistent with $H_{\langle \mathbf{O}_2 \rangle}$; i.e., $(H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle}) = \varnothing$.
3. $\sim\exists e \exists O_1 \exists O_2[O_1 \in \mathbf{O}_1$ and $O_2 \in \mathbf{O}_2$ and $e < O_1$ and $e < O_2$ and $\forall h[e \in h \rightarrow (\Pi_e\langle h \rangle \cap H_{\langle \mathbf{O}_1 \rangle} = \varnothing)$ or $(\Pi_e\langle h \rangle \cap H_{\langle \mathbf{O}_2 \rangle} = \varnothing)]]$. (This is the no-prior-screener-off condition, with "prior" given its weakest reading.)

[4]By analogy one should say "prior *modal* screener-off," but let us tolerate the potential ambiguity. One will need the disambiguation, however, in any discussion that includes reference to *probabilistic* screening-off.

4 From prior choice postulate to no-prior-common-cause-like-locus funny business

"No-prior-common-cause-like-locus funny business" is the version of "no common cause funny business" which, although appearing to be more distant from Reichenbach, arises most naturally from BST. It comes about in this way. The key "prior choice postulate" of BST theory says that

> for any outcome chain O, if $O \subseteq h_1 - h_2$, then there is a point event e in the past of O such that $h_1 \perp_e h_2$. [5]

It is an easy consequence of [5] that

> if O is inconsistent with h (i.e., if $H_{\langle O \rangle} \cap h = \varnothing$), then the same prior point event e will work for any history in which O occurs: $\exists e[e < O$ and $h \perp_e H_{\langle O \rangle}]$. [6]

We may use the definition of "cause-like locus" in order to put [6] into something like an English statement of the prior choice postulate:

> If O is inconsistent with h, then there is a point event e such that e is a cause-like locus for O with respect to h that lies in the causal past of O. [7]

Two uses of [7] then yield the following:

> If O_1 is inconsistent with O_2 (i.e., if $H_{\langle O_1 \rangle} \cap H_{\langle O_2 \rangle} = \varnothing$), then for any $h_1 \in H_{\langle O_1 \rangle}$ and any $h_2 \in H_{\langle O_2 \rangle}$,
>
> 1. you can find point event e_1 in the past of O_1 that is cause-like in separating h_2 from O_1 ($e_1 < O_1$ and $h_2 \perp_{e_1} H_{\langle O_1 \rangle}$), and
> 2. you can also find a point event e_2 in the past of O_2 that is cause-like in separating h_1 from O_2 ($e_2 < O_2$ and $h_1 \perp_{e_2} H_{\langle O_2 \rangle}$).

It is striking, however, that it is *not* guaranteed by the prior choice postulate that there is a *single* point event that will serve simultaneously in both capacities; it is not guaranteed that there is a single cause-like locus in the common past of O_1 and O_2 that can serve both to separate O_1 from h_2 and O_2 from h_1. This failure is exactly what happens in many cases of EPR-like funny business, an observation I convert into a definition.

4-1 DEFINITION. *(No-prior-common-cause-like-locus funny business, simplest kind)* A pair of outcome chains O_1 and O_2 together with a pair of histories h_1 and h_2 constitute a case of *no-prior-common-cause-like-locus funny business of the simplest kind* $\leftrightarrow_{df}$

1. $h_1 \in H_{\langle O_1 \rangle}$ and $h_2 \in H_{\langle O_2 \rangle}$ and
2. $H_{\langle O_1 \rangle} \cap H_{\langle O_2 \rangle} = \varnothing$ and
3. $\sim \exists e[(e < O_1$ and $h_2 \perp_e H_{\langle O_1 \rangle})$ and $(e < O_2$ and $h_2 \perp_{e_1} H_{\langle O_\rangle \rangle}]$.

In BST theory the last clause (3) has two fully equivalent formulations each of which is somewhat simpler in appearance:

$$\sim \exists e[e < O_1 \text{ and } e < O_2 \text{ and } h_1 \perp_e h_2]. \tag{8}$$

$$\sim \exists e[e < O_1 \text{ and } e < O_2 \text{ and } H_{\langle O_1 \rangle} \perp_e H_{\langle O_2 \rangle}]. \tag{9}$$

We might use the term "no-prior-history-splitter funny business" for the principle that arises by substituting [8] for (3) in Definition 4-1, and "no-prior-outcome-splitter funny business" for the variant using [9]. Instead, however, I just use whichever form seems convenient under the one heading, "no-prior-common-cause-like-locus funny business." (Observe that since [9] does not mention the particular histories h_1 and h_2, if we use that variation we could drop (1).)

What is it like when there *is* a common cause-like locus of the simplest kind? Given the variant [9], to say that there is no funny business is to say that if you are in one outcome O_1 and consider another outcome O_2 that is inconsistent with yours, then you can find a point event e in the *common* past of both outcome events that is a cause-like locus of their inconsistency: $H_{\langle O_1 \rangle} \perp_e H_{\langle O_2 \rangle}$. In words, before and at e, both outcome events were possible, but immediately after e, no matter what happens, at least one of the outcome events becomes impossible.

While the simplest case, especially in the variant with [9], is easiest to understand, we need a generalization that treats cases in which one or both of the outcome events are spread out instead of localized in a single outcome chain. The generalization simply promotes the outcomes from chains to the more general notion of outcome event, that is, to a set of outcome chains. In stating this generalization, it is technically convenient for us to use an analog to the variant using [8]

4-2 DEFINITION. *(No-prior-common-cause-like-locus funny business)* A pair of outcome events (nonempty sets of outcome chains) $\mathbf{O}_1$ and $\mathbf{O}_2$ together with a pair of histories h_1 and h_2 constitute a case of *no-prior-common-cause-like-locus funny business* $\leftrightarrow_{df}$

1. Each of $H_{\langle \mathbf{O}_1 \rangle}$ and $H_{\langle \mathbf{O}_2 \rangle}$ is individually consistent; i.e., $H_{\langle \mathbf{O}_1 \rangle} \neq \varnothing$ and $H_{\langle \mathbf{O}_2 \rangle} \neq \varnothing$; and in particular, $h_1 \in H_{\langle \mathbf{O}_1 \rangle}$ and $h_2 \in H_{\langle \mathbf{O}_2 \rangle}$,

2. $H_{\langle \mathbf{O}_1 \rangle}$ is inconsistent with $H_{\langle \mathbf{O}_2 \rangle}$; i.e., $(H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle}) = \varnothing$.

3. $\sim \exists e \exists O_1 \exists O_2 [O_1 \in \mathbf{O}_1$ and $O_2 \in \mathbf{O}_2$ and $e < O_1$ and $e < O_2$ and $h_1 \perp_e h_2]$.

Clause (1) picks out two histories for bookkeeping and to witness that each of the two outcome events is consistent in its own right. Clause (2) simply states that the outcome events are inconsistent: In no history do all of the “parts” of both begin to be. Finally, (3) makes the strong claim that the inconsistency between the two outcome events cannot be localized in a single point event in their common past in even the weakest possible sense (“weakest” because it is only required that the point be in the causal past of *some* part of $\mathbf{O}_1$ and also in the causal past of *some* part of $\mathbf{O}_2$).

5 Equivalence of four ideas of funny business

I have defined four ideas of no funny business: Definition 1-2, Definition 1-3, Definition 3-3, and Definition 4-2. In spite of rhetorical differences, they come to the same thing:

5-1 THEOREM. *(Equivalence of four ideas of funny business)*

PROOF is by way of four lemmas that put them into a circle.

5-2 LEMMA. *(Some-cause-like-locus-not-in-past funny business implies primary* SLR *modal-correlation funny business.)* If there is a case of some-cause-like-locus-not-in-past funny business (Definition 1-3) then there is a case of primary SLR modal-correlation funny business (Definition 1-2).

PROOF is given as Lemma 2 of Belnap 2002.

5-3 LEMMA. *(Primary SLR modal-correlation funny business implies no-prior-screener-off funny business)* If there is a case of primary SLR modal-correlation (Definition 1-2) then there is a case of no-prior-screener-off funny business (Definition 3-3).

PROOF. Suppose in accord with Definition 1-2 that there are two primary propositional spreads $\mathbf{I}_1 \rightarrowtail \Pi_{\mathbf{I}_1}$ and $\mathbf{I}_2 \rightarrowtail \Pi_{\mathbf{I}_2}$ together with two outcome-determining histories h_1 and h_2 such that $\mathbf{I}_1 \subseteq h_1$ and $\mathbf{I}_2 \subseteq h_2$ and $\mathbf{I}_1$ SLR $\mathbf{I}_2$ and $\Pi_{\mathbf{I}_1}\langle h_1\rangle \cap \Pi_{\mathbf{I}_2}\langle h_2\rangle = \varnothing$. Define $\mathbf{O}_i = \{O_i\colon O_i \subseteq h_i$ and $\mathit{inf}(O_i) < O_i$ and $\mathit{inf}(O_i) \in \mathbf{I}_i\}$, $i = 1, 2$. It is observed in Belnap 2002 that $\Pi_{\mathbf{I}_i}\langle h_i\rangle = H_{\langle \mathbf{O}_i\rangle}$, so that since $h_i \in H_{\langle \mathbf{O}_i\rangle}$, $i = 1, 2$, each $H_{\langle \mathbf{O}_i\rangle}$ is consistent—and in particular $h_i \in H_{\langle \mathbf{O}_i\rangle}$—whereas $H_{\langle \mathbf{O}_1\rangle} \cap H_{\langle \mathbf{O}_2\rangle} = \varnothing$. For no-prior-screener-off funny business, we need only suppose that $O_1 \in \mathbf{O}_1$ and $O_2 \in \mathbf{O}_2$ and $e < O_1$ and $e < O_2$, and then find a history h such that (z) ($\Pi_e\langle h\rangle \cap H_{\langle \mathbf{O}_1\rangle} \neq \varnothing$) and ($\Pi_e\langle h\rangle \cap H_{\langle \mathbf{O}_2\rangle} \neq \varnothing$). By properties of infima, $e \leqslant \mathit{inf}(O_1)$ and $e \leqslant \mathit{inf}(O_2)$, and since the two infima are space-like-related, it must be that $e < \mathit{inf}(O_1)$ and $e < \mathit{inf}(O_2)$. Let h_3 witness the consistency aspect of the space-like-relatedness of $\mathit{inf}(O_1)$ and $\mathit{inf}(O_2)$. Then $\mathit{inf}(O_1)$ certifies that $h_1 \equiv_e h_3$ and $\mathit{inf}(O_2)$ that $h_3 \equiv_e h_2$. Now choose $h = h_3$ for (z). Evidently $h_1 \in (\Pi_e\langle h_3\rangle \cap H_{\langle \mathbf{O}_1\rangle})$ and $h_2 \in (\Pi_e\langle h_3\rangle \cap H_{\langle \mathbf{O}_2\rangle})$, finishing the proof.
□

5-4 LEMMA. *(No-prior-screener-off funny business implies no-prior-common-cause-like-locus funny business)* If there is a case of no-prior-screener-off funny business (Definition 3-3), then there is a case of no-prior-common-cause-like-locus funny business (Definition 4-2).

PROOF. In effect it suffices to show that each prior common cause-like locus is itself a prior screener-off. So suppose that $h_1 \in H_{\langle \mathbf{O}_1\rangle}$ and $h_2 \in H_{\langle \mathbf{O}_2\rangle}$, $H_{\langle \mathbf{O}_1\rangle} \cap H_{\langle \mathbf{O}_2\rangle} = \varnothing$, and there is a prior common cause-like locus, namely $O_1 \in \mathbf{O}_1$ and $O_2 \in \mathbf{O}_2$ and $e < O_1$ and $e < O_2$ and $h_1 \perp_e h_2$. To show: We also have a prior screener-off, to wit, $\forall h[e \in h \rightarrow (\Pi_e\langle h\rangle \cap H_{\langle \mathbf{O}_1\rangle} = \varnothing)$ or $(\Pi_e\langle h\rangle \cap H_{\langle \mathbf{O}_2\rangle} = \varnothing)]$. Suppose for *reductio* that $\Pi_e\langle h\rangle \cap H_{\langle \mathbf{O}_1\rangle} \neq \varnothing$ and $\Pi_e\langle h\rangle \cap H_{\langle \mathbf{O}_2\rangle} \neq \varnothing$, with $h_{1'}$ witness to the former and $h_{2'}$ witness to the latter. So $h_{1'} \equiv_e h$ and $h_1 \equiv_e h_{1'}$, hence $h_1 \equiv_e h$ by transitivity of undividedness. Similarly, $h \equiv_e h_{2'}$, $h_{2'} \equiv_e h_2$, and hence $h \equiv_e h_2$. Therefore $h_1 \equiv_e h_2$ by yet a further use of the transitivity of undividedness; which contradicts $h_1 \perp_e h_2$ and finishes the *reductio*. □

5-5 LEMMA. *(No-prior-common-cause-like-locus funny business implies some-cause-like-locus-not-in-past funny business)* Whenever there is a case of no-prior-common-cause-like-locus funny business (Definition 4-2), there is also a case of some-cause-like-locus-not-in-past funny business (Definition 1-3).

PROOF. Assume that $\mathbf{O}_1$ and $\mathbf{O}_2$ and h_1 and h_2 constitute a case of no-prior-common-cause-like-locus funny business (Definition 4-2). Define

$$\mathbf{I}_1 =_{df} \{e_1 : \exists O_1[e_1 < O_1 \text{ and } O_1 \in \mathbf{O}_1 \text{ and } \exists h_{2'}[h_{2'} \in H_{\langle \mathbf{O}_2 \rangle} \text{ and } h_1 \perp_{e_1} h_{2'}]]\}.$$

For some-cause-like-locus-not-in-past funny business (Definition 1-3), we show that (*y*) $h_1 \perp\!\!\!\perp_{\mathbf{I}_1} H_{\langle \mathbf{O}_2 \rangle}$ and that (*z*) no member of $\mathbf{I}_1$ is in the past of any member of $\mathbf{O}_2$. The "relevance" part of (*y*) is built into the definition of $\mathbf{I}_1$, since obviously if $e_1 \in \mathbf{I}_1$ then $\exists h_{2'}[h_{2'} \in H_{\langle \mathbf{O}_2 \rangle}$ and $h_1 \perp_{e_1} h_{2'}]$. For the splitting part of (*y*), assume that $h_{2'} \in H_{\langle \mathbf{O}_2 \rangle}$. So $h_{2'} \notin H_{\langle \mathbf{O}_1 \rangle}$ by Definition 4-2(2), which implies that we may choose O_1 such that $O_1 \in \mathbf{O}_1$ and $O_1 \cap h_{2'} = \varnothing$. Also Definition 4-2(1) implies that $O_1 \cap h_1 \neq \varnothing$, so that by the prior choice postulate, we may choose e_1 such that $h_1 \perp_{e_1} h_{2'}$ and $e_1 < O_1$. Hence $e_1 \in \mathbf{I}_1$ by the definition of $\mathbf{I}_1$. Since $h_{2'}$ was arbitrary, we may conclude that $h_1 \perp_{\mathbf{I}_1} H_{\langle \mathbf{O}_2 \rangle}$, as required.

Finally we may show (*z*) by *reductio*. Suppose for some $e_1 \in \mathbf{I}_1$ and $O_2 \in \mathbf{O}_2$ that $e_1 < O_2$. By the definition of $\mathbf{I}_1$, there are $O_1 \in \mathbf{O}_1$ and $h_{2'} \in H_{\langle \mathbf{O}_2 \rangle}$ such that $e_1 < O_1$ and $h_1 \perp_{e_1} h_{2'}$. Since $h_{2'} \in H_{\langle \mathbf{O}_2 \rangle}$ and $O_2 \in \mathbf{O}_2$, $h_{2'} \cap O_2 \neq \varnothing$. Since $h_2 \in H_{\langle \mathbf{O}_2 \rangle}$ by Definition 4-2(1), we have $h_2 \cap O_2 \neq \varnothing$ as well, so that since we are supposing that $e_1 < O_2$, $h_{2'} \equiv_{e_1} h_2$. Therefore, by the transitivity of undividedness, $h_1 \perp_{e_1} h_2$. This contradicts Definition 4-2(3). □

This completes the circle and the proof of Theorem 5-1. The theorem provides, in my judgment, additional support for the stability of the BST idea of EPR-like "funny business," and for the view that the very austerity of BST theory can be helpful in articulating what is "funny" about EPR-like quantum-mechanical phenomena.

6 Appendix

This appendix considers some loose ends.

6.1 Simplest and more general no-prior-common-cause-like-locus funny business

The existence of the simplest kind of no-prior-common-cause-like-locus funny business (Definition 4-1) certainly implies existence of no-prior-common-cause-like-locus funny business (Definition 4-2) of the more general kind. The question is, Under what conditions can we have the more general kind of no-prior-common-cause-like-locus funny business without also having the simplest kind? It appears that it requires some kind of infinity to distinguish the two. Roughly described example: Let OW_0 be a BST structure such that each history is a two-dimensional Minkowski space-time. Stipulate for OW_0 an enumerated set of binary choice points e_i (so each e_i has two immediate outcomes, say + and $-$). The choice points e_i are stipulated as evenly spaced along a hyperplane, so that no single point event covers (has in its causal past) more than a finite number of e_i. Let these e_i be all the choice points in OW_0. They all belong to all of the histories of OW_0, and furthermore, a history of OW_0 is uniquely determined by specifying one of + or $-$ for each i. Now define OW_1 by "omitting" the history that is all e_i+. OW_1 is itself a BST structure. There is funny business in OW_1 all right, which could be witnessed by taking a single chain down to e_1+ as defining $\mathbf{O}_1$, and taking an infinite set of chains, one down to each remaining e+, as $\mathbf{O}_2$. Let h_1 be some history of OW_1 in which e_1 goes +, and let h_2 be the history in which e_1 goes minus while all the other e_i go +. This combination satisfies the definition of no-prior-common-cause-like-locus funny business (the generalized form of Definition 4-2).

There is in OW_1, however, no no-prior-common-cause-like-locus funny business of the *simplest* kind (Definition 4-1). This can be seen as follows. Take any pair of outcome chains O_1 and O_2 in OW_1. The outcome chain O_1 determines for each e_i in its past exactly one of + and $-$, and ditto for O_2. If these determinations disagree on any e_i that they both cover, then e_i serves as a cause-like locus in the common past of O_1 and O_2, so that in this case there is no no-prior-common-cause-like-locus funny business of the simplest kind. Suppose, however, that the determinations made by O_1 and O_2 agree on every e_i in their common past (including the case where there are no e_i in their common past). Then consider the history defined by agreeing with each e_i below O_1, and agreeing with each e_j below O_2 (this is so far a consistent stipulation because of the supposal), and being all $-$ (i.e., minus) on the remaining e_k. Both outcome events occur in this history, and so they are after all consistent, so that Definition 4-2(2) in the definition of "no-prior-common-cause-like-locus funny business" is not satisfied. So you cannot find a case of no-prior-common-cause-like-locus funny business of the simplest kind in this case. The point of the infinity is that you never "need" the all-e_i+ missing history as a witness to the consistency of any single pair of outcome chains. In contrast, given a version of *OW* with only finitely many e_i, a single missing history (say, all e_i+) leads to a case of no-prior-common-cause-like-locus funny business of the simplest kind: Take one outcome chain O_1 covering exactly the first e_1+, and another outcome chain O_2 covering exactly the remaining e_j+. They are inconsistent, but without a prior common cause-like locus.

6.2 Anomalies

Belnap 2002, note 30, observed that space-like relatedness between two initials $\mathbf{I}_1$ and $\mathbf{I}_2$ is defined pointwise, so that $\mathbf{I}_1$ SLR $\mathbf{I}_2$ can hold even when $\mathbf{I}_1$ and $\mathbf{I}_2$ are inconsistent (no history contains them both). I observed that this is also a case of primary SLR modal-correlation funny business in the sense of Definition 1-2. Making contact with language of the Bell literature, such a case would have a causal structure analogous to a case in which you could not simultaneously initialize to make a certain measurement on the left and a certain measurement on the right, so that it would be causal-structurally like a failure of "parameter independence."

What, in the other two versions of funny business, answers to the inconsistency of $\mathbf{I}_1$ and $\mathbf{I}_2$ in the case of primary SLR modal-correlation funny business? Consider first "some cause-like locus not in the past" funny business in the sense of Definition 1-3: $\mathbf{I} \subseteq h$ and $h \underline{\perp}_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$ (which means $h \perp_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$ and $\forall e[e \in \mathbf{I} \rightarrow \exists h_1[h_1 \in H_{\langle \mathbf{O} \rangle}$ and $h \perp_e h_1]]$) even though no member of $\mathbf{I}$ lies in the causal past of any member of $\mathbf{O}$. It is tempting to say that in this case "before" $\mathbf{I}$ both h and $\mathbf{O}$ are possible, but "after" $\mathbf{I}$ at least one of h and $\mathbf{O}$ becomes "henceforth" impossible. But this language presumes that $H_{[\mathbf{I}]} \cap H_{\langle \mathbf{O} \rangle} \neq \varnothing$, which does not follow. The example that comes to mind is infinite, namely, that described in Figure 4 of Belnap 2002. The only history of which $\mathbf{I}_1$ is a subset is h_ω, which makes $H_{[\mathbf{I}]}$ inconsistent with O_2. Probably any counterexample *has* to be infinite. At the same time it seems as if one finds an example of two initials that, while inconsistent with each other, are nevertheless part of a primary SLR modal correlation, then the example has to be *doubly* infinite, so that, say, each initial determines its own unique history.

When, however, $H_{[\mathbf{I}]} \cap H_{\langle \mathbf{O} \rangle} \neq \varnothing$, the tempting language seems acceptable, where we unpack the "before" possibility merely as the compossibility of each of h and $\mathbf{O}$ with $H_{[\mathbf{I}]}$. The "after" impossibility means simply that no immediate outcome of $\mathbf{I}$ (no member of $\Pi_{\mathbf{I}}$) is consistent with both h and the occurrence of $\mathbf{O}$.

Passing now to the third and fourth of the BST versions of funny business, what about no-prior-screener-off funny business (Definition 3-3) and no-prior-common-cause-like-locus funny business (Definition 4-2)? As far as I can see, these formulations do not permit the isolation of any special cases. It seems that special cases are generated only by those formulations of funny business in which initials explicitly figure.

6.3 Reduction of relevant splitting

In the presence of no funny business, the need for and the complications of the definition of $h \perp\!\!\!\!\perp_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$ disappear. Let NFB be an acronym for "no funny business." Then we have the following.

6-1 FACT. *(Reduction of "relevant splitting")* Under the hypothesis of no funny business, the entire "action" of an initial $\mathbf{I}$ with respect to a history h_1 and an outcome event $\mathbf{O}$ can be concentrated in some single point event in $\mathbf{I}$. That is, NFB and $h_1 \perp\!\!\!\!\perp_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$ together imply $\exists e[e \in \mathbf{I}$ and $h_1 \perp_e H_{\langle \mathbf{O} \rangle}]$.

PROOF. Suppose NFB and $h_1 \perp\!\!\!\!\perp_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$. By NFB (in the form that says that for every cause-like locus for $\mathbf{O}$ with respect to h_1, some part of $\mathbf{I}$ lies in the past of some part of $\mathbf{O}$), choose $e \in \mathbf{I}$ and $O \in \mathbf{O}$ such that $e < O$. We show that $h_1 \perp_e H_{\langle \mathbf{O} \rangle}$. To this end, let $h_2 \in H_{\langle \mathbf{O} \rangle}$; it suffices to show that $h_1 \perp_e h_2$. By relevance, choose $h_{2'} \in H_{\langle \mathbf{O} \rangle}$ such that $h_1 \perp_e h_{2'}$. $h_2 \in H_{\langle O \rangle}$ and $h_{2'} \in H_{\langle O \rangle}$ by the definition of $H_{\langle \mathbf{O} \rangle}$, so $h_{2'} \equiv_e h_2$ by these two plus the fact that $e < O$. Hence, $h_1 \perp_e h_2$ by the transitivity of undividedness. Hence $h_1 \perp_e H_{\langle \mathbf{O} \rangle}$ as required. $\square$

6.4 Generalization to three or more outcome events?

It is noteworthy that the no-prior-splitter version of funny business does not, whereas the no-prior-screener-off version does seem to suggest a natural generalization to cases of three or more outcome events. But this is more appearance than reality. One should take into account the Uffink 1999 consideration of the problem of saying something common-cause-like in the case of three events each pair of which are independent. The translation into modal terms in BST seems to be as follows.

> There are three outcome events such that each pair is consistent, but not all three taken together. Say $h_1 \in (H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle})$, $h_2 \in (H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_3 \rangle})$, and $h_3 \in (H_{\langle \mathbf{O}_2 \rangle} \cap H_{\langle \mathbf{O}_3 \rangle})$, whereas $(H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle} \cap H_{\langle \mathbf{O}_3 \rangle}) = \varnothing$.

In this case, what would count as a case of no-prior-common-cause-like-locus funny business? First reformulate in binary terms, since $\mathbf{O}_1 \cup \mathbf{O}_2$ is itself an outcome event and since $(H_{\langle \mathbf{O}_1 \rangle} \cap H_{\langle \mathbf{O}_2 \rangle}) = H_{\langle \mathbf{O}_1 \cup \mathbf{O}_2 \rangle}$. So there are three threats of binary no-prior-common-cause-like-locus funny business. To avoid the threat, there must in each case be an appropriate common cause. Thus, to escape three-termed funny business, each of the following must hold.

1. $\exists e \exists O_1 \exists O_2 [h_1 \perp_e h_2$ and $O_1 \in (\mathbf{O}_1 \cup \mathbf{O}_2)$ and $O_2 \in \mathbf{O}_3$ and $e < O_1$ and $e < O_2]$.
2. $\exists e \exists O_1 \exists O_2 [h_1 \perp_e h_3$ and $O_1 \in (\mathbf{O}_1 \cup \mathbf{O}_3)$ and $O_2 \in \mathbf{O}_2$ and $e < O_1$ and $e < O_2]$.
3. $\exists e \exists O_1 \exists O_2 [h_2 \perp_e h_3$ and $O_1 \in (\mathbf{O}_2 \cup \mathbf{O}_3)$ and $O_2 \in \mathbf{O}_1$ and $e < O_1$ and $e < O_2]$.

There seems to be no reason for supposing that the witnesses to (1)–(3) must overlap. If they did, that would presumably count as a "common common cause." So this is a reflection that coheres with the results of Szabo, Redei, *et al*. that the existence of common causes does not by any means imply the existence of common common causes, neither in detail nor in spirit.

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