The proofs of the twin prime conjecture and weaker Polignac's conjecture

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Abstract: This paper advanced a new method of appointedly covering prime circles with the levels of mathematical screen, and showed the twin prime conjecture and weaker Polignac's conjecture to be true with the proof by contradiction.

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1. Introduction. After C. Goldbach and L. Euler presented their famous conjecture [**1**, **2** p421] by a letter in 1742, which states that is every number > 2 the sum of two distinct primes?[1], A. de Polignac presented yet his famous conjecture in 1849 [**1**, **2** p424, **4** p11], One named it the twin primes conjecture, which states

Conjecture 1: *Even number 2 is the difference of two consecutive primes in an infinitude of ways.*

Then he advanced extendedly another famous conjecture also in that paper, it states

Conjecture 2: *Every even number is the difference of two consecutive primes in an infinitude of ways.*

One named it Polignac's conjecture. Conjecture 2 is a very stronger conjecture, one couldn't now not only show it but decide whether it is true as well. And one have advanced a slightly weaker conjecture which is analogous as that one, we name it a weaker Polignac's conjecture, it states

Conjecture 3: *Every even number is the difference of two primes in an infinitude of ways.*

Owing to Conjecture 3 doesn't need that two primes are in order, that is, it is allowed that there are some other primes between these two primes, its condition is weaker than Conjecture 2 and it is possible to show it. Although Conjecture 3 is weaker, yet it still includes Conjecture 1 as a special one.

In this paper we will advance a method of appointedly covering prime circles with the levels of mathematical screen, and show Conjectures 3 and 1 to be true with the proof by contradiction. Some symbols using in the paper is analogous as those in ref.[**5, 6, 7, 8**].

2. Preparative works. Prior to showing this important problem in number theory or in mathematics, we should do some things.

[1]Note: The author consider that owing to that the number 1 was judged as a prime number in early years, so Goldbach wrote $4 = 1 + 3$ and $6 = 1 + 5$ [**3**, p7]; after that time, one wrong consider that he wrote $4 = 2 + 2$ and $6 = 3 + 3$ [**2** p421].

(1) **A sequence of odd numbers.** Firstly, we should write a sequence of odd numbers except 1, and denote ordinal number of odd numbers by i and its increment by i'. According to Euclid's prime theorem, there are infinitely many odd prime numbers in that sequence, we denote the ordinal number of an odd prime number in the sequence of odd prime numbers by s and its increment by σ and σ'. That is

$$\begin{array}{rccccccccccc} odd\ \ number: & 3, & 5, & 7, & 9, & 11, & 13, & 15, & 17, & 19, & 21, & \cdots, \\ ordinal\ \ number\ \ i: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \cdots, \\ ordinal\ \ number\ \ s: & 1 & 2 & 3 & - & 4 & 5 & - & 6 & 7 & - & \cdots. \end{array}$$

(2) **An additive graph of the difference formulae of odd numbers.** Secondly, we can draw a graph with two-color circles indicating odd numbers. This graph is analogous as Fig.3 in ref.[6], n heading numbers was cut off from the sequence of odd numbers and rearranged vertically in the left line, then, an ordinal sequence of odd numbers was arranged paralleling with the former in the right, both lines construct a column for an even number, and there were imaginatively a minus sign between correspondingly two odd numbers in the same row in this column indicating this even number. we denote ordinal number of odd number vertically laying in the left line of the first column by k which is starting at 2, and its increment by k'. There are two kinds of backed circles colored with black and snow, the black is to indicate a prime number, the snow to odd composite number. For all n, here $n = 1, 2, \cdots$, we can draw out some analogous columns consisted of odd numbers and get an addition of these number circles (Fig.1).

In Fig.1 we draw a horizontal dotted line between the row $k = n$ and the row $k = n + 1$ for every column, which is parting all the difference formulae of odd numbers into two parts: one part, laying up over the dotted line, contains n difference formulae of odd numbers for the nth column, in which there include some difference formulae consisted of two primes, its number is Polignac-Xu's number for that even number; another one, laying down under the dotted line, contains infinitely many of rest difference formulae.

(3) **An important conclusion.** In order to serve T. M. Apostol's problem: "Is there an even number > 2 which is not the difference of two primes?" [**4**, p11], in the ref.[5-8], we have calculated Goldbach-Xu's numbers for some even numbers with a generalized hierarchial recursion algorithm. There are some difference formulae over the dotted line for every column, that is, Polignac-Xu's number is ≥ 1, *i.e.*, $\#\{D^{(n)}\}_{PX} \geq 1$, where the subscript PX is to distinguish it from Goldbach-Xu's number $\#\{D^{(n)}\}_{GX}$, and by our calculation it is oscillatingly increased as the even number increases. So we can conclude

Conclusion 4: *For every even number its Polignac-Xu's number is* ≥ 1 *and is positively, oscillatingly increased as the even number increases.*

(4) **Some convention of using symbols.** In order to conveniently indicate its significant for some mathematical items, we use some symbols to denote them. For an odd number laying at a sit in the nth column, we use a front superscript n to denote it, a front subscript l to denote it is in the left line, a front subscript r to denote it in the right, we use a letter o to denote an odd number, specially, use a letter p to denote an odd prime, and the ordinal number i for indicating an odd number which is in the ith row in right line in the nth column sits at a back superscript, the ordinal number k for indicating an odd number which is in the kth row in left line in the nth column sits at a back superscript also, the ordinal

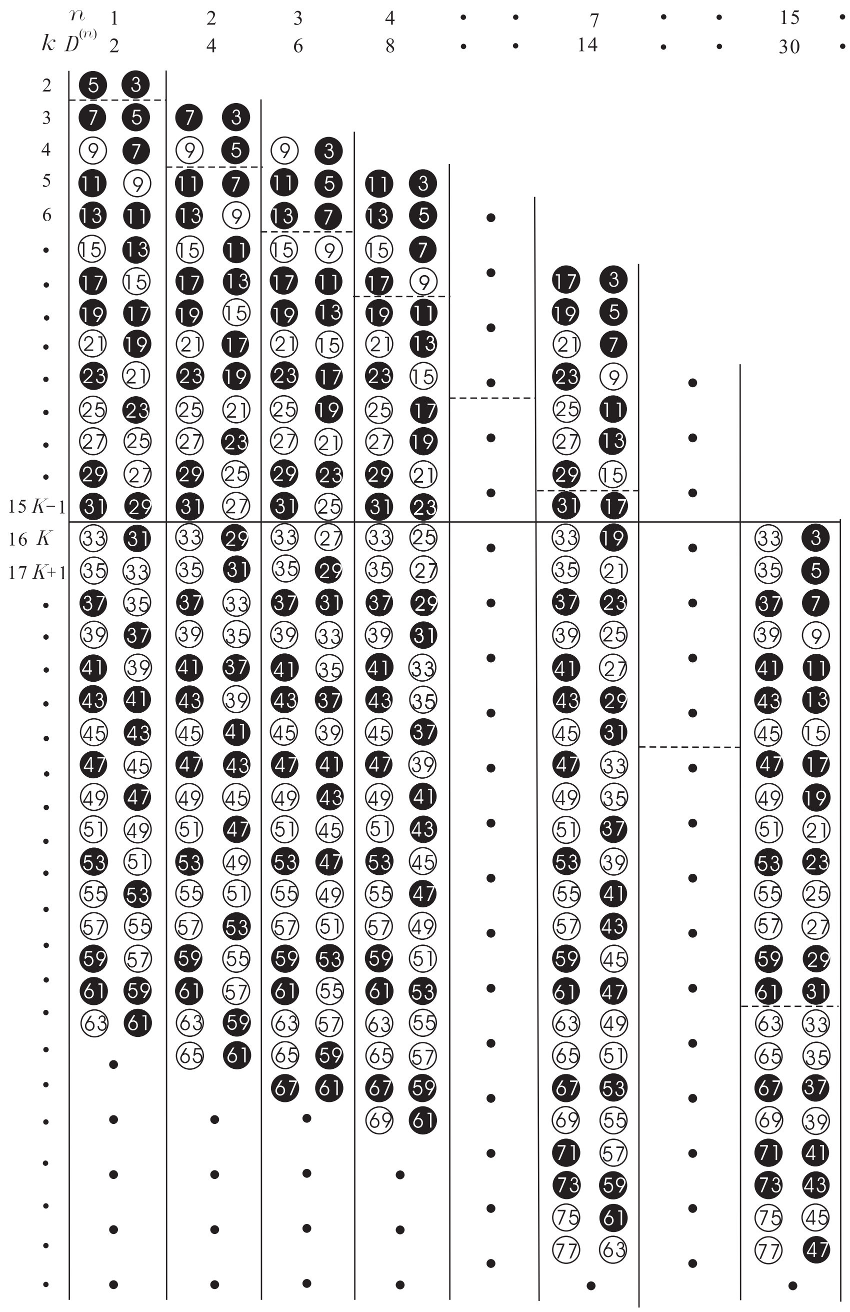

FIGURE 1. There will appear some "appointed" couples of two primes after rearranged vertically the sequence of odd numbers.

number s for displaying an odd prime number sits at a back subscript, that is, ${}^{n}_{l}o^{k}$, ${}^{n}_{l}p^{k}_{s}$, ${}^{n}_{r}o^{i}$, or ${}^{n}_{r}p^{i}_{s}$.

Furthermore, we use symbol "$\vdash\!\dashv$" to indicate the prime meet prime in the same row in the nth column, that is, there is ${}^{n}_{l}p^{k}_{s+\sigma} \vdash\!\dashv {}^{n}_{r}p^{i}_{s}$ probably in the kth row of the nth column, and use symbol "$\rightharpoonup$" to indicate a prime circle is being covered by a level of mathematical screen, that is, ${}^{n}_{l}p^{k}_{s+\sigma} \rightharpoonup {}^{n}_{l}\widetilde{p}^{k}_{s+\sigma}$.

(5) **About the levels of mathematical screen.** And here the term "screen" possesses mathematical significant. We define "snow" is zero level of the screen and "black" is infinite level, there are integer levels between them, and we use symbols to indicate them as follow

$\mathfrak{S}(0)$: snow,

$\mathfrak{S}(1)$: the first level of mathematical screen (note: with 30% of the screen indicates it in fig.2),

$\mathfrak{S}(2)$: the second level (with 40% indicates it),

$\mathfrak{S}(n)$: the nth level (with $(10n + 20)\%$ indicates it).

Such that the potence of the set of the levels of screens $\#\mathbb{S}$ is equal to that of the set of nonnegative integers $\#\mathbb{N}$, *i.e.*, $\#\mathbb{S} = \#\mathbb{N}$; therefore, owing to $n \in \mathbb{N}$, the number of the levels of mathematical screen (expect the zero level) is sufficiently for monogamously covering prime circles in all n columns (including the case of $n \to \infty$).

(6) **Three relationship formulae.** From Fig.1 we can see that there are some important relationship formulae between n, i and k. There are

(1) $$k = i + n.$$

(2) $${}^{n}_{l}p^{k+k'}_{s+\sigma} = 2n + {}^{n}_{r}p^{k+k'-n}_{s}.$$

(3) $${}^{n}_{l}p^{k}_{s+\sigma} = {}^{n}_{r}p^{k}_{s+\sigma}.$$

3. The proof of conjecture 3.

Proof. We use the proof by contradiction. Assume that Conjecture 3 is not true, thus, it could be deduced that there are four possible cases.

(a) The first cases may be assumed that every even number is the difference of two primes but not in an infinitude of ways. It is directly in contradiction with Conclusion 4. The first case indicates that for all even number its Polignac-Xu's number is finitely, but Conclusion 4 indicates that when an even number tens infinitely its Polignac-Xu's number also tens infinitely. Both are in contradiction each other, and the assumption is wrong.

(b) The second cases may be assumed that all sufficiently larger even numbers are the difference of two primes in an infinitude of ways but not sufficiently larger even numbers aren't in an infinitude of ways.

For this possible case, all even number $< D$, here D is a sufficiently larger number, only is finite many of the difference formulae of two primes not in an infinitude of ways. That is, we may assume that when $k \geq K$, every even number $< D$ doesn't be expressed as the difference of two primes (Note: When an even number approaches to D, n is to approach to N, here N also is a sufficiently larger number). This will introduce some error conclusion.

Let us return to Fig.1, for $n = 1$, suppose that there is a last pair of twin primes in the $(K - 1)$th row in the 1st column, according to the assumption of the proof by contradiction, there weren't any "appointed" pair, the prime meet prime in same row, of two primes in all rows while $k \geq K$. Furthermore, we would extend the extent of columns as larger as all ones for $n \leq N$ and suppose that there isn't any "appointed" pair when $k \geq K$ there.

Now let us consider a sufficiently larger number K, and draw a horizontal real line between the $(K-1)$th row and the $k=K$th row. And there must not be the prime meet prime in Kth row of the 1st column because there isn't any three odd numbers are consecutive primes except 3, 5, 7. And if there, then, appears the prime meet prime in consequence rows under the horizontal line, that is, there is ${}^{1}_{l}p^{K+k'}_{s+\sigma+1} \vdash\!\dashv {}^{1}_{r}p^{K-1+k'}_{s+\sigma}$ in the $(K+k')$th row, we cover ${}^{1}_{r}p^{K-1+k'}_{s+\sigma}$ and ${}^{1}_{l}p^{K-1+k'}_{s+\sigma}$ for all pairs of k' and σ with $\mathfrak{S}(1)$, at this time, there are ${}^{1}_{r}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{1}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma}$ and ${}^{1}_{l}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{1}_{l}\widetilde{p}^{K-1+k'}_{s+\sigma}$.

Meanwhile, we continuously cover ${}^{n}_{r}p^{K-1+k'}_{s+\sigma}$ and ${}^{n}_{l}p^{K-1+k'}_{s+\sigma}$ for all $n>1$. And there are ${}^{n}_{r}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{n}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma}$ and ${}^{n}_{l}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{n}_{l}\widetilde{p}^{K-1+k'}_{s+\sigma}$.

Next, for $n=2$ and $k \geq K+1$, if there appears the prime meet prime in the $(K+1)$th row of the 2nd column, that is, there is ${}^{2}_{l}p^{K+1}_{s+1} \vdash\!\dashv {}^{2}_{r}p^{K-1}_{s}$, because ${}^{2}_{r}p^{K-1}_{s}$ must be a prime, we would only cover ${}^{2}_{l}p^{K+1}_{s+1}$ and ${}^{2}_{r}p^{K+1}_{s+1}$ with $\mathfrak{S}(2)$. And there are ${}^{2}_{l}p^{K+1}_{s+1} \rightharpoonup {}^{2}_{l}\widetilde{p}^{K+1}_{s+1}$ and ${}^{2}_{r}p^{K+1}_{s+1} \rightharpoonup {}^{2}_{r}\widetilde{p}^{K+1}_{s+1}$ (Fig.2).

Then we continuously cover ${}^{n}_{l}p^{K+1}_{s+1}$ and ${}^{n}_{r}p^{K+1}_{s+1}$ for all $n>2$. And there are ${}^{n}_{l}p^{K+1}_{s+1} \rightharpoonup {}^{n}_{l}\widetilde{p}^{K+1}_{s+1}$ and ${}^{n}_{r}p^{K+1}_{s+1} \rightharpoonup {}^{n}_{r}\widetilde{p}^{K+1}_{s+1}$

And if there is ${}^{2}_{l}p^{K+1+k'}_{s+1+\sigma'} \vdash\!\dashv {}^{2}_{r}p^{K-1+k'}_{s+\sigma'}$, we cover ${}^{2}_{r}p^{K-1+k'}_{s+\sigma'}$ and ${}^{2}_{l}p^{K-1+k'}_{s+\sigma'}$ for all pairs of k' and σ', and there are ${}^{2}_{r}p^{K-1+k'}_{s+\sigma'} \rightharpoonup {}^{2}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma'}$ and ${}^{2}_{l}p^{K-1+k'}_{s+\sigma'} \rightharpoonup {}^{2}_{l}\widetilde{p}^{K-1+k'}_{s+\sigma'}$. And we consecutively cover ${}^{n}_{r}p^{K-1+k'}_{s+\sigma'}$ and ${}^{n}_{l}p^{K-1+k'}_{s+\sigma'}$ for all $n>2$ with that level of the screen, and there are ${}^{n}_{r}p^{K-1+k'}_{s+\sigma'} \rightharpoonup {}^{n}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma'}$ and ${}^{n}_{l}p^{K-1+k'}_{s+\sigma'} \rightharpoonup {}^{n}_{l}\widetilde{p}^{K-1+k'}_{s+\sigma'}$.

Following, for $n=3$ and $k \geq K+2$, if there appears the prime meet prime in the $(K+2)$th row of the 3nd column, that is, there is ${}^{3}_{l}p^{K+2}_{s+\sigma} \vdash\!\dashv {}^{3}_{r}p^{K-1}_{s}$, because ${}^{3}_{r}p^{K-1}_{s}$ must be a prime, we would only cover ${}^{3}_{l}p^{K+2}_{s+\sigma}$ and ${}^{3}_{r}p^{K+2}_{s+\sigma}$ with $\mathfrak{S}(3)$. And there are ${}^{3}_{l}p^{K+2}_{s+\sigma} \rightharpoonup {}^{3}_{l}\widetilde{p}^{K+2}_{s+\sigma}$ and ${}^{3}_{r}p^{K+2}_{s+\sigma} \rightharpoonup {}^{3}_{r}\widetilde{p}^{K+2}_{s+\sigma}$. Then we continually cover ${}^{n}_{l}p^{K+2}_{s+\sigma}$ and ${}^{n}_{r}p^{K+2}_{s+\sigma}$ for all $n>3$ with that level. And there are ${}^{n}_{l}p^{K+2}_{s+\sigma} \rightharpoonup {}^{n}_{l}\widetilde{p}^{K+2}_{s+\sigma}$ and ${}^{n}_{r}p^{K+2}_{s+\sigma} \rightharpoonup {}^{n}_{r}\widetilde{p}^{K+2}_{s+\sigma}$.

And if there appear the prime meet prime in the $(K+2+k')$th row of that column, that is, there are ${}^{3}_{l}p^{K+2+k'}_{s+\sigma+\sigma'} \vdash\!\dashv {}^{3}_{r}p^{K-1+k'}_{s+\sigma}$, we will cover ${}^{3}_{r}p^{K-1+k'}_{s+\sigma}$ and ${}^{3}_{l}p^{K-1+k'}_{s+\sigma}$, and cover ${}^{n}_{r}p^{K-1+k'}_{s+\sigma}$ and ${}^{n}_{r}p^{K-1+k'}_{s+\sigma}$ for all $n>3$ and all pairs of k' and σ with with $\mathfrak{S}(3)$. Such that there are ${}^{3}_{r}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{3}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma}$, ${}^{3}_{l}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{3}_{l}\widetilde{p}^{K-1+k'}_{s+\sigma}$, ${}^{n}_{r}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{n}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma}$ and ${}^{n}_{r}p^{K-1+k'}_{s+\sigma} \rightharpoonup {}^{n}_{r}\widetilde{p}^{K-1+k'}_{s+\sigma}$.

Then, we do this for ever and ever for all n, here $n=4, 5, \cdots, n$.

For example, let us see in Fig.2, There are some clearly examples. There is ${}^{1}_{l}43^{21}_{13} \vdash\!\dashv {}^{1}_{r}41^{20}_{12}$ in the row $k=21$ of the 1st column, then, we cover ${}^{1}_{r}41^{20}_{12}$ and ${}^{1}_{l}41^{20}_{12}$ with $\mathfrak{S}(1)$, and cover ${}^{n}_{l}41^{20}_{12}$ and ${}^{n}_{r}41^{20}_{12}$ for all $n>1$; and there is ${}^{1}_{l}61^{30}_{17} \vdash\!\dashv {}^{1}_{r}59^{29}_{16}$ in the row $k=30$ of that column, and we cover ${}^{1}_{r}59^{29}_{16}$ and ${}^{1}_{l}59^{29}_{16}$ with that level of screen, and cover ${}^{n}_{l}59^{29}_{16}$ and ${}^{n}_{r}59^{29}_{16}$ for all $n>1$. And there are ${}^{n}_{r}\widetilde{41}^{20}_{12}$, ${}^{n}_{l}\widetilde{41}^{20}_{12}$, ${}^{n}_{r}\widetilde{59}^{29}_{16}$, and ${}^{n}_{l}\widetilde{59}^{29}_{16}$ for all $n=1, 2, \cdots$.

Next, there is ${}^{2}_{l}47^{23}_{14} \vdash\!\dashv {}^{2}_{r}43^{21}_{13}$ in the row $k=23$ of the 2nd column, then, we cover ${}^{2}_{r}43^{21}_{13}$ and ${}^{2}_{l}43^{21}_{13}$ with $\mathfrak{S}(2)$, such that there are ${}^{2}_{r}\widetilde{43}^{21}_{13}$ and ${}^{2}_{l}\widetilde{43}^{21}_{13}$. And we consequently cover ${}^{n}_{r}43^{21}_{13}$ and ${}^{n}_{l}43^{21}_{13}$ for all $n>2$, thus, there are ${}^{n}_{r}\widetilde{43}^{21}_{13}$ and ${}^{n}_{l}\widetilde{43}^{21}_{13}$ for all $n=2, 3, \cdots$.

And there is ${}^{3}_{l}37^{18}_{11} \vdash\!\dashv {}^{3}_{r}31^{15}_{10}$ in the row $k=18$ of the 3rd column, owing to the odd number ${}^{3}_{r}o^{15} = {}^{3}_{r}31^{15}_{10}$ must be a prime, we cover ${}^{3}_{l}37^{18}_{11}$ and ${}^{3}_{r}37^{18}_{11}$ and ${}^{n}_{l}37^{18}_{11}$ and ${}^{n}_{r}37^{18}_{11}$ with $\mathfrak{S}(3)$. Continually, there are ${}^{3}_{l}53^{26}_{15} \vdash\!\dashv {}^{3}_{r}47^{23}_{14}$ in the row $k=26$ and

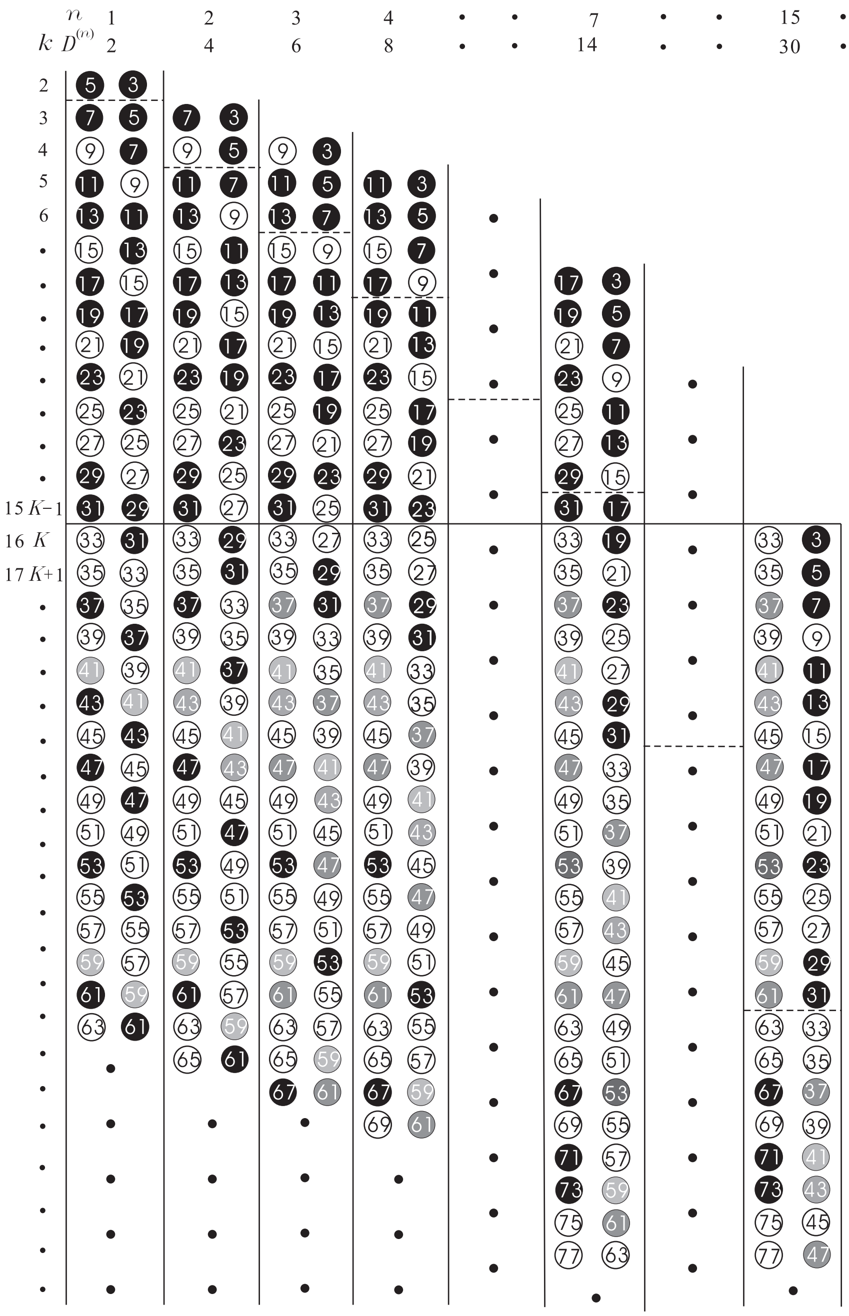

FIGURE 2. There were no Polignac-Xu's number over the dotted line and were not any uncovered prime circle in the interval (32, 64) under this line while $n = 15$.

${}^{3}_{l}67^{33}_{18} \vdash\!\dashv {}^{3}_{r}61^{30}_{17}$ in the row $k = 33$, we should also cover ${}^{3}_{r}47^{23}_{14}$ and ${}^{3}_{l}47^{23}_{14}$, and ${}^{3}_{r}61^{30}_{17}$ and ${}^{3}_{l}61^{30}_{17}$, such that there are ${}^{3}_{l}\widetilde{37}^{18}_{11}$ and ${}^{3}_{r}\widetilde{37}^{18}_{11}$, ${}^{3}_{r}\widetilde{47}^{23}_{14}$ and ${}^{3}_{l}\widetilde{47}^{23}_{14}$, and ${}^{3}_{r}\widetilde{61}^{30}_{17}$ and ${}^{3}_{l}\widetilde{61}^{30}_{17}$.

And we would lastly cover ${}^{n}_{l}37^{18}_{11}$ and ${}^{n}_{r}37^{18}_{11}$, ${}^{n}_{r}47^{23}_{14}$ and ${}^{n}_{l}47^{23}_{14}$, and ${}^{n}_{r}61^{30}_{17}$ and ${}^{n}_{l}61^{30}_{17}$ for all $n > 3$, thus, there are ${}^{n}_{l}\widetilde{37}^{18}_{11}$ and ${}^{n}_{r}\widetilde{37}^{18}_{11}$, ${}^{n}_{r}\widetilde{47}^{23}_{14}$ and ${}^{n}_{l}\widetilde{47}^{23}_{14}$, and ${}^{n}_{r}\widetilde{61}^{30}_{17}$ and ${}^{n}_{l}\widetilde{61}^{30}_{17}$ for all n=3, 4, $\cdots$.

At best latter, when n turns from 1 to $K-1$, the black prime circles laying at the $(K-1)$th sit in the right line will be through all rest black circles between the Kth odd number and the $2(K-1)$th in the left, and all these prime circles in the interval [2K+1, 4K+1], no matter whether in the right line or in the left, will be covered with $K-1$ levels of screen. And there are ${}^{K-1}_{r}p^{K+k'}_{s+\sigma} \rightharpoonup {}^{K-1}_{r}\widetilde{p}^{K+k'}_{s+\sigma}$ and ${}^{K-1}_{l}p^{i+i'}_{s+\sigma} \rightharpoonup {}^{K-1}_{l}\widetilde{p}^{i+i'}_{s+\sigma}$ for all pairs of k' and σ in two lines in the $(K-1)$th column.

And we consider that there isn't the primes meet primes for ${}^{K-1}_{r}\widetilde{p}^{K+k'+k''}_{s+\sigma+\sigma'} \vdash\!\dashv {}^{K-1}_{l}p^{i+i'}_{s+\sigma}$ in the $(K-1)$th column, and its number does not be counted among Polignac-Xu's number for that even number, although there were probably infinitely many "appointed" prime pairs in infinitely far away under the dotted line.

So we can conclude three error conclusions: (i) There were not any appointed couple between two primes in all rows, *i.e.*, when an even number $D = 2n \geq 2(K-1)$, there is $\#\{D^{(n)}\}_{PX} = 0$, and there were no Polignac-Xu's number, among there, in the interval of under the horizontal real line and over the dotted line, while $n \geq K-1$. This is contradiction with Conclusion 4; (ii) There were not any prime in interval $(2K, 4K)$, for instance, if $K = 16$, there were not any prime in (32, 64) under the horizontal dotted line in fig.2 also while $n \geq K-1$. This is contradiction with a famous prime theorem, the Chebyshev-Bertrand Theorem, that there must exist at least one prime between x and $2x$, and (iii) There would be not any prime in interval $(2K, \infty)$ also under dotted line while $n \to \infty$,, this is contradiction with the Euclid Prime Theorem, that there are infinitely many primes in natural numbers. And the assumption is wrong.

(c) The third cases assumes that all of sufficiently larger even numbers and some of not sufficiently larger even numbers are the difference of two primes in an infinitude of ways but some others aren't in an infinitude of ways.

In this status, we divide all even number in to two sets, $\mathcal{A}$ and $\mathcal{B}$, the set $\mathcal{A}$ contains the former, and the set $\mathcal{B}$ contains the latter. For all even number in $\mathcal{B}$, we take analogous method as covering sufficiently larger odd primes in the columns indicating those even numbers in above subsection (b). After processed all those columns we may find that all prime number $\geq K$ in $\mathcal{B}$ have been covered. Of course, these, which lay in all columns indicating all even numbers in $\mathcal{A}$, should also be covered. And there is ${}^{n}_{l}\widetilde{p}^{K+k'}_{s+\sigma+\sigma'} \vdash\!\dashv {}^{n}_{r}p^{i+i'}_{s+\sigma}$ for all pairs of $K+k'$ and $s+\sigma$ for all columns in $\mathcal{A}$, such that there is no ${}^{n}_{l}p^{K+k'}_{s+\sigma+\sigma'} \vdash\!\dashv {}^{n}_{r}p^{i+i'}_{s+\sigma}$ in that set, and the assumption is contradiction in itself in this case and it is wrong.

(d) In the last case one assumes that every even number except 2 is the difference of two primes in an infinitude of ways. There, indeed, is no good method to show this assumption to be wrong. But if it were true, yet one can symmetrically assume that every even number except 4 is the difference of two primes in an infinitude of ways; every even number except 6 is the difference of two primes in an infinitude of ways; and go on. That will introduce a new antinomy. The assumption is wrong.

Where we should note that if there is no Fig.2, *i.e.*, if there isn't a relationship indicated as Fig.2 between relative numbers, which indicates the central point at issue, then one couldn't set his mind to a "symmetrically assume". For instance, it is impossible to set a "symmetrically assume" for Conjecture 2, and this conjecture couldn't be shown with the method used in this paper.

Up to now, we have furnished showing Conjecture 3 to be true and also Conjecture 1. □

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