

**Neutrinos and Their Charged Cousins:
Are They Secret Sharers?**

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Masses and mixings of quarks and leptons differ wildly from one another. Thus it is all the more challenging to search for some hidden attribute that they may share

“Neutrinos they are very small. They have no charge and have no mass and do not interact at all.” So wrote the renowned author John Updike, erring twice in one couplet. Indeed, neutrinos are the most storied of particles. Think of Pauli’s famous letter proposing them as a “desperate remedy” for the apparent failure of energy conservation in beta decay. He waited 25 years for his neutrinos to be observed. Think of Ray Davis’ good news and bad: that he had succeeded in detecting solar neutrinos, but that they were too few. And only once did I witness a standing ovation at a physics conference — at Neutrino-98 in Takayama, Japan, where scientists at Super-Kamiokande announced the observation of atmospheric neutrino oscillations, an effect I had anticipated almost two decades earlier.¹ A few years later scientists at SNO, the Sudbury Neutrino Observatory in Canada, announced their triumphant resolution of Ray’s decades-old solar neutrino problem. More front-page neutrino stories are sure to follow!

Neutrinos, once ghostlike and elusive, are now mundane. They are seen and studied from many sources: reactors, accelerators, radioactive decays, Earth’s interior, cosmic-ray interactions in the stratosphere and from two stars: the sun and supernova 1987a. What we’ve learned about them is succinctly described by three left-handed neutrino states and a relatively small number of adjustable parameters, but many vexing questions remain:

¹ S.L. Glashow *in* Quarks & Leptons, Cargèse 1979 (Plenum, 1980, New York) pp. 687-713, see p. 707.

Is the neutrino mass spectrum normal or inverted? Are the masses lepton-number violating, *i.e.*, Majorana; or lepton-number conserving, *i.e.*, Dirac; or are they a bit of both, thereby putting light singlet states, so-called sterile neutrinos, into play? Will astrophysical observations or the Katrin experiment or searches for neutrinoless double beta decay, determine the neutrino mass scale? Is there observable CP violation in the neutrino sector? Will future data belie our simple model? Let me put aside these questions and begin my tale with a brief historical digression. Recall the five eras of lepton hadron symmetry:

- $(1 \times 1)^2$ During the 1920s atoms and their nuclei seemed to be made up from just two elementary constituents: protons and electrons — one hadron and one lepton, although neither word had yet been coined.
- $(1 \times 2)^2$ The discovery of neutrons and the invention of neutrinos led to the second era: one doublet of nucleons and one of leptons.
- $(1 \times 3)^2$ The detection of strange particles and muons in cosmic rays led Bob Marshak to propose the Kiev symmetry in 1959. The fundamental Sakata triplet of hadrons (p, n, Λ) was likened to the lepton triplet (ν, e, μ) . This era was short lived. Sakata's triplet was replaced by three quarks, but the discovery of a second neutrino in 1963 undid Marshak's symmetry.
- $(2 \times 2)^2$ Soon after Gell-Mann and Zweig devised quarks, James Bjorken and I proposed the existence of yet one more. Our reasoning was purely aesthetic: with charm, two quark doublets would accompany the two lepton doublets, thus restoring lepton-hadron symmetry. Six years passed before John Iliopoulos, Luciano Maiani and I offered substantive and convincing arguments for the existence of charm, another four before the experimental discovery of charmonium.

$(3 \times 2)^2$ Mere months after the discovery of the J/Ψ , groups led by Marty Perl and Leon Lederman spotted half the members of a third family of fundamental fermions. Top quarks and tau neutrinos would show up decades later. In the current and longest-lived era of lepton-hadron symmetry, there are three doublets of quarks and and three of leptons.

Fermion masses and mixings were much simpler in the two-doublet era than now. Back then, with only two families and no evidence for neutrino masses, the ‘flavor problem’ involved only seven parameters: four quark masses, two charged lepton masses and Cabibbo’s angle. But when 2×2 matrices became 3×3 , things got complicated. Quark masses and mixings involve six masses and four Cabibbo-Kobayashi-Maskawa (CKM) parameters. Meanwhile, observations of solar and atmospheric oscillations showed that neutrinos have small but consequential masses. Thus the lepton sector involves ten analogous quantities: six lepton masses and four Pontecorvo-Maki-Nakagawa-Sakata (PMNS) parameters, as well as two infuriatingly inaccessible Majorana phases.* All twenty of the flavor parameters are either measured or constrained. And yet, frustratingly, no significant relationship among them strikes the eye nor has any been deduced from a plausible theoretical framework.

The CKM matrix has little in common with its leptonic analog. All three quark mixing angles are small: Cabibbo’s is about 13° , the others much smaller. Contrariwise, atmospheric neutrino oscillations seem nearly maximal and the solar oscillation angle is large as well. But could there be some common attribute hiding amongst the masses of quarks and leptons, if not amongst their mixings?

The three charged leptons are widely disparate in mass. So are the three up-type quarks and the three down-type quarks. Denote the masses in each category by B (for biggest), M and S (for smallest). We consider several shared measures of their disparity which might also characterize neutrino masses. In order of descending strength, they are:

* Here we assume there to be three relevant neutrino states with lepton number violating (Majorana) masses.

F1: For each category (charged leptons, up-type quarks and down-type quarks) the ratios B/M and M/S are both large, ranging from 17 to 140. Here we suppose the corresponding neutrino mass ratios to be greater than ten.

F2: Here we impose a weaker constraint, requiring only one neutrino mass ratio, B/S , to exceed ten.

F3: An even weaker indicator of mass disparity is for the three masses not to form a triangle or equivalently, for B to exceed $M + S$. Charged lepton masses easily satisfy this inequality, as do those of quarks of either charge. Here we assume the same for neutrino masses.

F1 implies F2. For the known values of Δ_s and Δ_a , F2 implies F3. We exhibit the implications of each feature on three observables: Σ , the sum of the absolute values of the neutrino masses (which may be determined astrophysically); m_β , the effective mass of the electron neutrino (which may be determined from studies of the beta-decay endpoint); and $m_{\beta\beta}$, the e - e element of the neutrino mass matrix (which may be determined from the rate of neutrinoless double beta decay). In a conventional notation, the three neutrino observables are given by:

$$\begin{aligned}\Sigma &\equiv |m_1| + |m_2| + |m_3|, \\ m_\beta &\approx (c_3^2|m_1|^2 + s_3^2|m_2|^2)^{1/2}, \\ m_{\beta\beta} &\approx |c_3^2m_1 + s_3^2m_2|,\end{aligned}$$

where $\Sigma > m_\beta \geq m_{\beta\beta}$. We use the following experimental data for our analyses: $s_3^2 \equiv \sin^2 \theta_{12} \approx 0.3$, $s_2^2 \equiv \sin^2 \theta_{13} \leq 0.03$, $|m_3^2 - m_1^2| \approx 2400 \text{ meV}^2$ and $m_2^2 - m_1^2 \approx 77 \text{ meV}^2$.

Should neutrino masses share feature F1 with their charged counterparts, one neutrino must be nearly massless and the hierarchy must be normal. We obtain (with masses in meV):

$$\begin{aligned}\text{F1 (normal)} &: \quad \Sigma \approx 58, \quad m_\beta \approx 5, \quad m_{\beta\beta} \approx 3, \\ \text{F1 (inverted)} &: \quad \text{Not possible.}\end{aligned}$$

Should they share feature F2, one neutrino must be nearly massless, but the hierarchy may be either normal or inverted. For the normal case, the consequences are the same as for F1. We obtain:

$$\begin{aligned} \text{F2 (normal)} : \quad & \Sigma \approx 58, \quad m_\beta \approx 5, \quad m_{\beta\beta} \approx 3, \\ \text{F2 (inverted)} : \quad & \Sigma \approx 98, \quad m_{\beta\beta} \leq m_\beta \approx 49. \end{aligned}$$

Should they share feature F3, the hierarchy may be either normal or inverted and we obtain:

$$\begin{aligned} \text{F3 (normal)} : \quad & \Sigma \approx 113, \quad m_{\beta\beta} < m_\beta \approx 28, \\ \text{F3 (inverted)} : \quad & \Sigma \approx 98, \quad m_{\beta\beta} \leq m_\beta \approx 49. \end{aligned}$$

Even this weakest indication of neutrino mass disparity severely constrains the three neutrino observables. If it is satisfied, neither endpoint effects in tritium decay nor neutrinoless double beta decay are likely to be observed in the foreseeable future.

Conversely, either a measurement of the electron neutrino mass, or a detection of neutrinoless double beta decay, or a convincing astrophysical argument that Σ much exceeds 100 meV would imply that none of the above features of charged fermion masses characterize neutrino masses. They would have to differ from those of quarks and charged leptons in virtually every imaginable fashion: in their magnitudes and mixing parameters, their degree of disparity, as well as the manner of their origin.

Suppose the *Katrin* experiment finds the electron neutrino mass to exceed 200 meV (the lower limit of its sensitivity)². Then the three neutrino masses would not only form a triangle, but one which is equilateral to a precision of 3% or better! Thus are we led a fourth and quite different feature which could be shared by quark and lepton mass matrices:

² See: www-ik.fzk.de/tritium/.

F4: All three quark mixing angles are small and the off-diagonal entries of M_d are small in the up-quark basis. Here we assume the latter property to characterize M_ν as well. Although two neutrino mixing angles are **not** small, the off-diagonal entries of M_ν will be small in the charged lepton basis if and only if the three neutrino masses are nearly degenerate.

Experimenters should be pleased were neutrinos to share this feature. All three neutrino observables would be relatively large, satisfying:

$$m_{\beta\beta} \approx m_\beta \approx \Sigma/3 \geq 100 \text{ meV}.$$

The inequality enforces neutrino mass degeneracy to within 10%, thus ensuring the diagonal entries of M_ν to be nearly equal and its off-diagonal entries to be at least an order of magnitude smaller. Thus F4 requires M_ν to have a simple “One plus Zee³” form — a multiple of the unit matrix augmented by three far smaller (but non-zero) off-diagonal entries.⁴ But can anyone suggest a sensible theoretical scheme yielding this most pleasing of textures?

³ A. Zee, Phys. Lett. 93B (1980) 389.

⁴ S.L. Glashow, arXiv: 0912.4976 [hep-ph].