

# COSMETIC CROSSINGS OF GENUS ONE KNOTS

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**ABSTRACT.** We show that for genus one knots the Alexander polynomial and the homology of the double cover branching over the knot provide obstructions to cosmetic crossings. As an application we prove the nugatory crossing conjecture for the negatively twisted, positive Whitehead doubles of all knots. We also verify the conjecture for several families of pretzel knots and all genus one knots with up to 10 crossings.

*Keywords:* cosmetic crossing, nugatory crossing, Thurston norm, pretzel knot, Whitehead double.

*Mathematics Subject Classification:* 57M25, 57M27, 57M10.

## 1. INTRODUCTION

A fundamental open question in knot theory is the question of when a crossing change on an oriented knot changes the isotopy class of the knot. A crossing disc for an oriented knot  $K \subset S^3$  is an embedded disc  $D \subset S^3$  such that  $K$  intersects  $\text{int}(D)$  twice with zero algebraic intersection number. A crossing change on  $K$  can be achieved by twisting  $D$  or equivalently by performing appropriate Dehn surgery of  $S^3$  along the crossing circle  $\partial D$ . The crossing is called nugatory if and only if  $\partial D$  bounds an embedded disc in the complement of  $K$ . A non-nugatory crossing on a knot  $K$  is called cosmetic if the oriented knot  $K'$  obtained from  $K$  by changing the crossing is isotopic to  $K$ . Clearly, changing a nugatory crossing doesn't change the isotopy class of a knot. The nugatory crossing conjecture (Problem 1.58 of Kirby's list [1]) asserts that the converse is true: if a crossing change on a knot  $K$  yields a knot isotopic to  $K$  then the crossing is nugatory. In other words, there are not any knots in  $S^3$  that admit cosmetic crossings.

In the case that  $K$  is the trivial knot an affirmative answer follows from a result of Gabai [4] and work of Scharlemann and Thompson [10]. The conjecture is also known to hold for 2-bridge knots by work of Torisu [11], and for fibered knots by work of Kalfagianni [5]. For knots of braid index three a weaker form of the conjecture, requiring that the crossing change happens on a closed 3-braid diagram, is discussed by Wiley in [12].

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In this paper we study cosmetic crossings on genus one knots and we show that the Alexander polynomial and the homology of the double cover branching over the knot provide obstructions to cosmetic crossings. Our main results are the following.

**Theorem 3.1.** *Let  $K$  be an oriented genus one knot. If  $K$  admits a cosmetic crossing then it is algebraically slice. In particular,  $\Delta_K(t) \doteq f(t)f(t^{-1})$ , where  $f(t) \in \mathbb{Z}[t]$  is a linear polynomial.*

**Theorem 4.2.** *Let  $K$  be an oriented genus one knot and let  $Y_K$  denote the double cover of  $S^3$  branching over  $K$ . If  $K$  admits a cosmetic crossing then the homology group  $H_1(Y_K) := H_1(Y_K, \mathbb{Z})$  is a finite cyclic group.*

Given a knot  $K$  let  $D_+(K, n)$  denote the  $n$ -twisted, positive clasped Whitehead double of  $K$ . As an application of Theorems 3.1 and 4.2 we prove that, for every  $K$  and  $n < 0$ , the knot  $D_+(K, n)$  satisfies the nugatory crossing conjecture (Corollary 5.1). We also prove the conjecture for several families of pretzel knots (Corollary 5.2) and 22 out of the 23 genus one knots with up to twelve crossings (see Section 5).

Throughout the paper we will discuss oriented knots in an oriented  $S^3$  and we work in the smooth category.

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## 2. CROSSING CHANGES AND ARCS ON SURFACES

In this section we use a result of Gabai [4] to prove that a cosmetic crossing change on a knot  $K$  can be realized by twisting along an essential arc on a minimum genus Seifert surface of  $K$  (Proposition 2.3). For genus one knots such an arc will be non-separating on the surface. In the next sections this will be our starting point for establishing connections between cosmetic crossings and knot invariants determined by Seifert matrices.

Let  $K$  be an oriented knot in  $S^3$  and  $C$  be a crossing of sign  $\epsilon$ , where  $\epsilon = 1$  or  $-1$  according to whether  $C$  is a positive or negative crossing (see Figure 1). A *crossing disc* of  $K$  corresponding to  $C$  is an embedded disc  $D \subset S^3$  such that  $K$  intersects  $\text{int}(D)$  twice, once for each branch of  $C$ , with zero algebraic intersection number. The boundary  $L = \partial D$  is called a *crossing circle*. Performing  $(-\epsilon)$ -surgery on  $L$ , changes  $K$  to another knot  $K' \subset S^3$  that is obtained from  $K$  by changing the crossing  $C$ .

**Definition 2.1.** A crossing supported on a crossing circle  $L$  of an oriented knot  $K$  is called *nugatory* if  $L = \partial D$  also bounds an embedded disc in the complement of  $K$ . This disc and  $D$  form an embedded 2-sphere that decomposes  $K$  into a connected sum where some of the summands may be trivial. A non-nugatory crossing on a knot  $K$  is called *cosmetic* if the oriented knot  $K'$  obtained from  $K$  by changing  $C$  is isotopic to  $K$ ; that is,

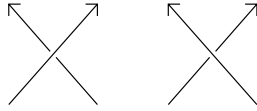


FIGURE 1. Left: a positive crossing. Right: a negative crossing.

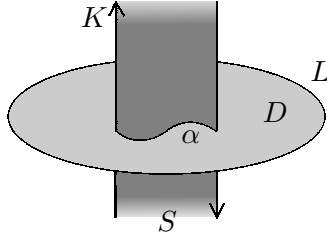


FIGURE 2. The crossing arc  $\alpha = S \cap D$ .

there exists an orientation-preserving diffeomorphism  $f : S^3 \rightarrow S^3$  with  $f(K) = K'$ .

For a link  $J$  in  $S^3$  we will use  $\eta(J)$  to denote a regular neighborhood of  $J$  in  $S^3$  and we will use  $M_J := \overline{S^3 \setminus \eta(J)}$  to denote the closure of the complement of  $\eta(J)$  in  $S^3$ .

**Lemma 2.2.** *Let  $K$  be an oriented knot and  $L$  a crossing circle supporting a crossing  $C$  of  $K$ . Suppose that  $M_{K \cup L}$  is reducible. Then  $C$  is nugatory.*

*Proof.* An essential 2-sphere in  $M_{K \cup L}$  must separate  $\eta(K)$  and  $\eta(L)$ . Thus in  $S^3$ ,  $L$  lies in a 3-ball disjoint from  $K$ . Since  $L$  is unknotted, it bounds a disc in the complement of  $K$ .  $\square$

Let  $K$  be an oriented knot and  $L = \partial D$  a crossing circle supporting a crossing  $C$ . Let  $K'$  denote the knot obtained from  $K$  by changing  $C$ . Since the linking number of  $L$  and  $K$  is zero,  $K$  bounds a Seifert surface in the complement of  $L$ . Let  $S$  be a Seifert surface that is of minimum genus among all such Seifert surfaces in the complement of  $L$ . Since  $S$  is incompressible, after an isotopy we can arrange so that the closed components of  $S \cap D$  are homotopically essential in  $D \setminus K$ . But then each such component is parallel to  $\partial D$  on  $D$  and by further modification we can arrange so that  $S \cap D$  is a single arc  $\alpha$  that is properly embedded on  $S$  as illustrated in Figure 2. The surface  $S$  gives rise to Seifert surfaces  $S$  and  $S'$  of  $K$  and  $K'$ , respectively.

**Proposition 2.3.** *Suppose that  $K$  is isotopic to  $K'$ . Then  $S$  and  $S'$  are Seifert surfaces of minimal genus for  $K$  and  $K'$ , respectively.*

*Proof.* If the crossing is nugatory then  $L$  bounds a disc in the complement of  $S$  and the conclusion is clear. Suppose the crossing is cosmetic; by Lemma 2.2,  $M_{K \cup L}$  is irreducible. We can consider the surface  $S$  properly embedded

in  $M_{K \cup L}$  so that it is disjoint from  $\partial\eta(L) \subset \partial M$ . The assumptions on irreducibility of  $M_{K \cup L}$  and on the genus of  $S$  imply that the foliation machinery of Gabai [4] applies. In particular,  $S$  is taut in the Thurston norm. The manifolds  $M_K$  and  $M_{K'}$  are obtained by Dehn filling of  $M_{K \cup L}$  along  $\partial\eta(L)$ . By [4, Corollary 2.4],  $S$  can fail to remain taut in the Thurston norm (i.e. genus minimizing) in at most one of  $M_K$  and  $M_{K'}$ . Since we have assumed that  $C$  is a cosmetic crossing,  $M_K$  and  $M_{K'}$  are homeomorphic (by an orientation preserving homeomorphism). Thus  $S$  remains taut in both of  $M_K$  and  $M_{K'}$ . This implies that  $S$  and  $S'$  are Seifert surfaces of minimal genus for  $K$  and  $K'$ , respectively.  $\square$

By Proposition 2.3, a crossing change of a knot  $K$  that produces an isotopic knot corresponds to a properly embedded arc  $\alpha$  on a minimum genus Seifert surface  $S$  of  $K$ . We observe the following.

**Lemma 2.4.** *If  $\alpha$  is inessential on  $S$ , then the crossing is nugatory.*

*Proof.* Recall that  $\alpha$  is the intersection of a crossing disc  $D$  with  $S$ . Since  $\alpha$  is inessential, it separates  $S$  into two pieces, one of which is a disc  $E$ . Consider  $D$  as properly embedded in a regular neighborhood  $\eta(S)$  of the surface  $S$ . The boundary of a regular neighborhood of  $E$  in  $\eta(S)$  is a 2-sphere that contains the crossing disc  $D$ . The complement of the interior of  $D$  in that 2-sphere gives a disc bounded by the crossing circle  $L = \partial D$  with its interior disjoint from the knot  $K = \partial S$ .  $\square$

### 3. OBSTRUCTING COSMETIC CROSSINGS IN GENUS ONE KNOTS

A knot  $K$  is called *algebraically slice* if it admits a Seifert surface  $S$  such that the Seifert form  $\theta : H_1(S) \times H_1(S) \rightarrow \mathbb{Z}$  vanishes on a half-dimensional summand of  $H_1(S)$ ; such a summand is called a *metabolizer* of  $H_1(S)$ . If  $S$  has genus one, then the existence of a metabolizer for  $H_1(S)$  is equivalent to the existence of an essential oriented simple closed curve on  $S$  that has zero self-linking number. If  $K$  is algebraically slice, then the Alexander polynomial  $\Delta_K(t)$  is of the form  $\Delta_K(t) \doteq f(t)f(t^{-1})$ , where  $f(t) \in \mathbb{Z}[t]$  is a linear polynomial with integer coefficients and  $\doteq$  denotes equality up to multiplication by a unit in the ring of Laurent polynomials  $\mathbb{Z}[t, t^{-1}]$ . For more details on these and other classical knot theory concepts we will use in this and the next section, the reader is referred to [2] or [7].

**Theorem 3.1.** *Let  $K$  be an oriented genus one knot. If  $K$  admits a cosmetic crossing, then it is algebraically slice. In particular, there is a linear polynomial  $f(t) \in \mathbb{Z}[t]$  such that  $\Delta_K(t) \doteq f(t)f(t^{-1})$ .*

*Proof.* Let  $K'$  be a knot that is obtained from  $K$  by a cosmetic crossing change  $C$ . By Proposition 2.3, there is a genus one Seifert surface  $S$  such that a crossing disc supporting  $C$  intersects  $S$  in a properly embedded arc  $\alpha \subset S$ . Let  $S'$  denote the result of  $S$  after the crossing change. Since  $C$  is a cosmetic crossing, by Lemma 2.4,  $\alpha$  is essential. Further, since the genus

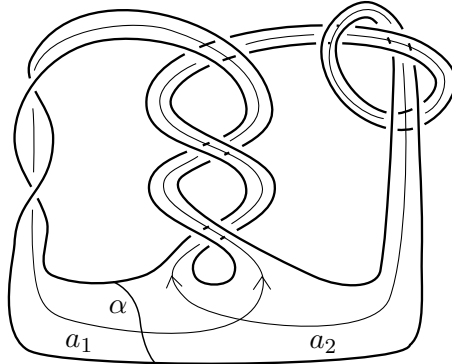


FIGURE 3. A genus one surface  $S$  with generators  $a_1$  and  $a_2$  of  $H_1(S)$  and a non-separating arc  $\alpha$ .

of  $S$  is one,  $\alpha$  is non-separating. We can find a simple closed curve  $a_1$  on  $S$  that intersects  $\alpha$  exactly once. Let  $a_2$  be another simple closed curve so that  $a_1$  and  $a_2$  intersect exactly once and the homology classes of  $a_1$  and  $a_2$  form a symplectic basis for  $H_1(S) \cong \mathbb{Z} \oplus \mathbb{Z}$ . Note that  $\{a_1, a_2\}$  form a corresponding basis of  $H_1(S')$ . See Figure 3.

The Seifert matrices of  $S$  and  $S'$  with respect to these bases are

$$V = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{and} \quad V' = \begin{pmatrix} a - \epsilon & b \\ c & d \end{pmatrix}$$

respectively, where  $a, b, c, d \in \mathbb{Z}$  and  $\epsilon = 1$  or  $-1$  according to whether  $C$  is a positive or a negative crossing. The Alexander polynomials of  $K, K'$  are given by

$$\Delta_K(t) \doteq \det(V - tV^T) = ad(1 - t)^2 - (b - ct)(c - tb),$$

$$\Delta_{K'}(t) \doteq (a - \epsilon)d(1 - t)^2 - (b - ct)(c - tb).$$

Since  $K$  and  $K'$  are isotopic we must have  $\Delta_K(t) \doteq \Delta_{K'}(t)$  which easily implies that  $d = \text{lk}(a_2, a_2) = 0$ . Hence  $K$  is algebraically slice and

$$\Delta_K(t) \doteq (b - ct)(c - tb) = (-t)(b - ct)(b - ct^{-1}) \doteq (b - ct)(b - ct^{-1}).$$

Setting  $f(t) = b - ct$  we obtain  $\Delta_K(t) \doteq f(t)f(t^{-1})$  as desired. Note that since  $|b - c|$  is the intersection number between  $a_1$  and  $a_2$ , by suitable orientation choices, we may assume that  $c = b + 1$ .  $\square$

Recall that the determinant of a knot  $K$  is defined by  $\det(K) = |\Delta_K(-1)|$ . As a corollary of Theorem 3.1 we have the following.

**Corollary 3.2.** *Let  $K$  be a genus one knot. If  $\det(K)$  is not a perfect square then  $K$  admits no cosmetic crossings.*

*Proof.* Suppose that  $K$  admits a cosmetic crossing. By Theorem 3.1  $\Delta_K(t) \doteq f(t)f(t^{-1})$ , where  $f(t) \in \mathbb{Z}[t]$  is a linear polynomial. Thus, if  $K$  admits cosmetic crossings we have  $\det(K) = |\Delta_K(-1)| = [f(-1)]^2$ .  $\square$

## 4. FURTHER OBSTRUCTIONS: HOMOLOGY OF DOUBLE COVERS

Here we will derive further obstructions to cosmetic crossings in terms of the homology of the double branched cover of the knot. Given  $m \in \mathbb{Z}$  let  $\mathbb{Z}_m = \mathbb{Z}/m\mathbb{Z}$  denote the cyclic abelian group of order  $|m|$ . The following lemma will be useful to us.

**Lemma 4.1.** *If  $H$  denotes the abelian group given by the presentation*

$$H \cong \left\langle c_1, c_2 \mid \begin{array}{l} 2xc_1 + (2y+1)c_2 = 0 \\ (2y+1)c_1 = 0 \end{array} \right\rangle,$$

then we have

- (1)  $H \cong 0$ , if  $y = 0$  or  $y = -1$ .
- (2)  $H \cong \mathbb{Z}_d \oplus \mathbb{Z}_{\frac{(2y+1)^2}{d}}$ , if  $y \neq 0, -1$  and  $\gcd(2x, 2y+1) = d$  where  $1 \leq d \leq 2y+1$ .

*Proof.* The proof is an easy linear algebra exercise left to the reader.  $\square$

**Theorem 4.2.** *Let  $K$  be an oriented genus one knot and let  $Y_K$  denote the double cover of  $S^3$  branching over  $K$ . If  $K$  admits a cosmetic crossing, then the homology group  $H_1(Y_K)$  is a finite cyclic group.*

*Proof.* Suppose that a genus one knot  $K$  admits a cosmetic crossing yielding an isotopic knot  $K'$ . The proof of Theorem 3.1 shows that  $K$  and  $K'$  admit Seifert matrices of the form

$$V = \begin{pmatrix} a & b \\ b+1 & 0 \end{pmatrix} \quad \text{and} \quad V' = \begin{pmatrix} a-\epsilon & b \\ b+1 & 0 \end{pmatrix} \quad (1)$$

respectively, where  $a, b \in \mathbb{Z}$  and  $\epsilon = 1$  or  $-1$  according to whether  $C$  is a positive or a negative crossing. In particular we have

$$\Delta_K(t) \doteq \Delta_{K'}(t) \doteq b(b+1)(t^2+1) - (b^2 + (b+1)^2)t. \quad (2)$$

Presentation matrices for  $H_1(Y_K)$  and  $H_1(Y_{K'})$  are given by

$$V + V^T = \begin{pmatrix} 2a & 2b+1 \\ 2b+1 & 0 \end{pmatrix}, \quad \text{and} \quad V' + (V')^T = \begin{pmatrix} 2a-2\epsilon & 2b+1 \\ 2b+1 & 0 \end{pmatrix} \quad (3)$$

respectively. It follows that Lemma 4.1 applies to both  $H_1(Y_K)$  and  $H_1(Y_{K'})$ . By that lemma,  $H_1(Y_K)$  is either cyclic or  $H_1(Y_K) \cong \mathbb{Z}_d \oplus \mathbb{Z}_{\frac{(2b+1)^2}{d}}$ , with  $b \neq 0, -1$  and  $\gcd(2a, 2b+1) = d$  where  $1 < d \leq 2b+1$ . Similarly,  $H_1(Y_{K'})$  is either cyclic or  $H_1(Y_{K'}) \cong \mathbb{Z}_{d'} \oplus \mathbb{Z}_{\frac{(2b+1)^2}{d'}}$ , with  $\gcd(2a-2\epsilon, 2b+1) = d'$  where  $1 < d' \leq 2b+1$ . Since  $K$  and  $K'$  are isotopic, we have  $H_1(Y_K) \cong H_1(Y_{K'})$ . One can easily verify this can only happen in the case that  $\gcd(2a, 2b+1) = \gcd(2a-2\epsilon, 2b+1) = 1$  in which case  $H_1(Y_K)$  is cyclic.  $\square$

It is known that for an algebraically slice knot of genus one every minimum genus surface  $S$  contains a metabolizer (compare [8, Theorem 4.2]) After completing the metabolizer to a basis of  $H_1(S)$  we have a Seifert matrix  $V$  as in (1) above.

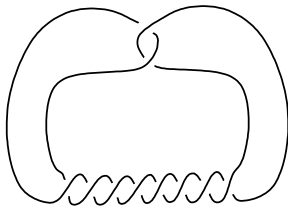


FIGURE 4. The  $(-4)$ -twisted negative clasped double of the unknot  $D_-(O, -4)$ .

**Corollary 4.3.** *Let  $K$  be an oriented, algebraically slice knot of genus one. Suppose that a genus one Seifert surface of  $K$  contains a metabolizer leading to a Seifert matrix  $V$  as in (1) so that  $b \neq 0, -1$  and  $\gcd(2a, 2b + 1) \neq 1$ . Then  $K$  cannot admit a cosmetic crossing.*

*Proof.* Let  $d = \gcd(2a, 2b + 1)$ . As in the proof of Theorem 4.2, we use Lemma 4.1 to conclude that  $H_1(Y_K) \cong \mathbb{Z}_d \oplus \mathbb{Z}_{\frac{(2b+1)^2}{d}}$  and hence is non-cyclic unless  $d = 1$ . Now the conclusion follows by Theorem 4.2.  $\square$

5. APPLICATIONS

**5.1. Twisted Whitehead doubles.** Given a knot  $K$  let  $D_+(K, n)$  denote the  $n$ -twisted Whitehead double of  $K$  with a positive clasp and let  $D_-(K, n)$  denote the  $n$ -twisted Whitehead double of  $K$  with a negative clasp.

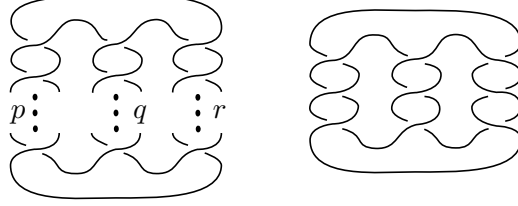
**Corollary 5.1.** *Given a knot  $K$ , the Whitehead double  $D_+(K, n)$  admits no cosmetic crossing if either  $n < 0$  or  $|n|$  is odd. Similarly  $D_-(K, n)$  admits no cosmetic crossing if either  $n > 0$  or  $|n|$  is odd.*

*Proof.* A Seifert surface of  $D_+(K, n)$  obtained by plumbing an  $n$ -twisted annulus with core  $K$  and a Hopf band gives rise to a Seifert matrix  $V_n = \begin{pmatrix} -1 & 0 \\ -1 & n \end{pmatrix}$  [7, Example 6.8]. Thus the Alexander polynomial is of the form

$$\Delta_n \doteq -n(t^2 + 1) + (1 + 2n)t \tag{4}$$

Suppose now that  $D_+(K, n)$  admits a cosmetic crossing. Then  $\Delta_n$  should be of the form shown in equation (2). Comparing the leading coefficients in the expressions (2) and (4) we obtain  $|n| = |b(b + 1)|$  which implies that  $|n|$  should be even. We have shown that if  $|n|$  is odd then  $D_+(K, n)$  admits no cosmetic crossing changes. Suppose now that  $n < 0$ . Since the Seifert matrix  $V_n$  only depends only on  $n$  and not on  $K$ ,  $D_+(K, n)$  is  $S$ -equivalent to the positively clasped,  $n$ -twisted double of the unknot. This is a positive knot and it has non-zero signature [9]. Hence  $D_+(K, n)$  is not algebraically slice and by Theorem 3.1 it cannot admit cosmetic crossings.

A similar argument holds for  $D_-(K, n)$ .  $\square$

FIGURE 5.  $P(p, q, r)$  with  $p, q$  and  $r$  positive and  $P(3, 3, -3)$ .

**5.2. Pretzel knots.** Let  $K$  be a three string pretzel knot  $P(p, q, r)$  with  $p, q$  and  $r$  odd (see Figure 5). The knot determinant is given by  $\det(K) = |pq + qr + pr|$  and if  $K$  is non-trivial then it has genus one. It is known that  $K$  is algebraically slice if and only if  $pq + qr + pr = -m^2$ , for some odd  $m \in \mathbb{Z}$  [6].

**Corollary 5.2.** *The knot  $P(p, q, r)$  with  $p, q$  and  $r$  odd does not admit cosmetic crossings if one of the following is true:*

- (1)  $pq + qr + pr \neq -m^2$ , for every odd  $m \in \mathbb{Z}$ .
- (2)  $q + r = 0$  and  $\gcd(p, q) \neq 1$ .
- (3)  $p + q = 0$  and  $\gcd(p, r) \neq 1$ .

*Proof.* In case (1) the result follows from Theorem 3.1 and the discussion above. For case (2) recall that there is a genus one surface for  $P(p, q, r)$  for which a Seifert matrix is  $V_{(p,q,r)} = \frac{1}{2} \begin{pmatrix} p+q & q+1 \\ q-1 & q+r \end{pmatrix}$  [7, Example 6.9]. Suppose that  $q + r = 0$ . If  $\gcd(p, q) \neq 1$ , then  $\gcd(p+q, q) \neq 1$  and the conclusion in case (2) follows by Corollary 4.3. Case (3) is similar.  $\square$

$K$	$\det(K)$	$K$	$\det(K)$	$K$	$\det(K)$
$3_1$	3	$9_2$	15	$11a_{362}$	39
$4_1$	5	$9_5$	23	$11a_{363}$	35
$5_2$	7	$9_{35}$	27	<b><math>11n_{139}</math></b>	<b>9</b>
<b><math>6_1</math></b>	<b>9</b>	<b><math>9_{46}</math></b>	<b>9</b>	$11n_{141}$	21
$7_2$	11	$10_1$	17	$12a_{803}$	21
$7_4$	15	<b><math>10_3</math></b>	<b>25</b>	$12a_{1287}$	37
$8_1$	13	$11a_{247}$	19	$12a_{1166}$	33
$8_3$	17	$11a_{343}$	31	-	-

TABLE 1. Genus one knots with at most 12 crossings.



**5.3. Low crossing knots.** Table 1, obtained from KnotInfo [3], gives the 23 knots of genus one with at most 12 crossings with the values of their determinants. There are four knots with square determinant. Thus Corollary 3.2 excludes cosmetic crossings for all but the knots  $6_1$ ,  $9_{46}$ ,  $10_3$  and  $11n_{139}$  on Table 1. Now  $6_1$  and  $10_3$  are 2-bridge knots and by [11] they don't admit cosmetic crossings. The knot  $9_{46}$  is isotopic to  $P(3, 3, -3)$  of Figure 5 which by Corollary 5.2 cannot have cosmetic crossings. The knot  $K = 11n_{139}$  is isotopic to  $P(-5, 3, -3)$ . As in the proof of Corollary 5.2 we calculate  $H_1(Y_K) \cong \mathbb{Z}_9$ . Thus Theorems 3.1 and 4.2 fail to settle the nugatory crossing conjecture for  $11n_{139}$ . Since  $K$  is neither fibered nor a 2-bridge knot the results of [5, 11] also fail to settle the conjecture.

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