

## Gait Definition and Successive Gait-transition Method based on Energy Consumption for a Quadruped

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**Abstract:** In nature, to realize the smooth motion for different speeds, the continuous gait transition is usually required for the quadrupeds. Thus, the gait simulation of quadrupeds is a requisite step to obtain the stable and energy-efficient gait for the walking machines. In this paper, the definitions of the two gait parameters, phasic difference and duty factor are presented, which can determine the gait of the quadrupeds. Then, several typical gaits of the quadrupeds are analyzed such that the seven standard gaits and corresponding parameters are summarized. Additionally, the variance law of the two parameters, which determine the relationship of gait transition, is analyzed. Furthermore, the quadruped gait derivative spectrum (QGDS) is proposed and the gait definition of the quadrupeds is presented. To minimize the power consumption, the choice criterion of gait, the optimal gait in terms of the motion speed, duty factory, and power consumption for the walking machines, is developed. Last, the continuous variance of the gait is implemented by the simulation of the gait transition from walk to trot, which evaluate the choice criterion and transition of gait.

**Key words:** gait definition, standard gaits, quadruped, gait-transition

### 1 Introduction\*

The realization of a stable, rapid and smooth gait planning for the legged robot is one of important research areas, especially for the coordinated planning for multi-legged robot. Locomotion, one of the basic functions of an animal, is realized by the legs, which allow the animal to move on the rough terrain. Therefore, a variety of studies have been done on the motion characteristics of legged biology to achieve the motion control of the walking machine.

To accomplish the gait plane, the definition and selection of the gait are the two prerequisites. Therefore, the first step is to quantitatively represent the gait. Quadrupeds are one of the most representative terrestrial animals, and they move with several gaits, such as walking, trotting and galloping. These locomotion patterns have characteristics, such as the motion sequence of four legs, the support and swing time of each leg and the motion period<sup>[1], [2]</sup>.

In nature, the locomotion of quadrupeds on the ground is very complex. Some researchers have paid much attention to the classification and definition of the gaits of quadrupeds at different motion speeds<sup>[2]-[6]</sup>. However, the definitions mentioned above are qualitative descriptions such that no uniform definition is proposed for all the gaits. LEWIS<sup>[3]</sup> gave photos of quadruped locomotion. Many researchers have defined the gaits, but it is difficult to

classify and define the gaits because, even with the same animal under the same conditions, the relationship between the legs is not uniform. In this paper, some subordinate factors are ignored, and the standard gaits for different speeds and conditions are defined and analyzed. All the gaits of quadrupeds can evolve from these standard gaits.

The animal gait is variable. In nature, quadruped walking animals such as horses or cats change their gait to suit their walking speed. This fact is very important for realizing smooth walking and smooth running. However, most of the current walking machines are periodic with invariable gait parameters, and they can only be used on a relatively flat terrain. Some methods of gait transition have been proposed<sup>[7]-[10]</sup>. ABERNETHY, et al<sup>[8]</sup>, determined the attention demands of natural and imposed gait, as well as the attention costs of transitions between the walking and running co-ordination patterns with experiments. YANG<sup>[9]</sup> proposed two-phase discontinuous gaits as a new fault-tolerant gait for quadruped robots suffering from a locked joint failure, which prevents a joint of a leg from moving and makes it locked in a known place. MASAKADO, et al<sup>[10]</sup>, defined 6 standard gaits and developed the gait-transitions from a standard gait to a standard gait. However, the requirement of the gait switch is the discrete definition of the gait. These methods are not natural because the transitions are made in a feed-forward manner. Thus, the results are restricted by several facts: the special control rule, the amount of the restricted legs.

Moreover, the gaits of quadrupeds are selected according to the situation and locomotion requirements, where energy

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expenditure is one of the most important factors<sup>[11]</sup>. ITO, et al<sup>[12]</sup>, proposed a locomotion control scheme of quadrupeds that enables the locomotion patterns to change according to the energy evaluation.

In order to detect the relationship between gait selection and situation, different kinds of quadruped motion under different surroundings and situations are considered. We can find that the qualifications for gait selection are ①motion speed, ②the stability of the body, and ③traction of the quadruped.

In this paper, some standard gaits of quadrupeds are analyzed first, the relationship of two parameters which denotes a gait is studied, and the parameters are phasic differences between legs and duty factor of each leg. With the energy evaluation of the quadruped locomotion, the function of the duty factor with the speed is figured out. Thus, the optimized gait can be selected under any motion speed. In addition, when the speed is changed, the gaits are transferred with parameters that are continuously optimized.

## 2 Standard Gaits of Quadrupeds

A gait is a combination of actions in a progressive motion that requires each one of the supporting members of the body to be either alone or in association with another supporting member. It usually can be divided into three steps: leg lift from the ground in its regular sequence, leg swing in the direction of the movement, leg placement on the ground, resulting in continue motion<sup>[2], [3]</sup>.

The gaits are varied relative to the structure of quadruped's leg and the terrain. Here some standard gaits for different motion speed are concluded, and named walk, amble, trot, pace, transverse-canter, rotator-canter and gallop. The terminologies are used to define the presented gaits in this paper, which are different from those in everyday speech. In these gaits, the most extraordinary standard gait is the systems of locomotion that permits the division of a stride into two co-ordinate parts, each of which, with a reciprocation of limb action, is essentially a repetition of the other<sup>[13]</sup>.

To facilitate a study of the regular gaits, numbers with some combination are adopted to designate the support legs and the order. We use 1, 2, 3 and 4 or symbols  $\Delta$ ,  $\blacktriangle$ ,  $\circ$  and  $\bullet$  to denote left-froe leg, right-fore leg, left-hind leg and right-hind leg respectively.

### 2.1 Walk

Of the various methods of animal motion, walk claims the first consideration. Walk is characterized by an immutable sequence of limb movements, common to man and beast alike; also, walk is employed by all terrestrial vertebrates<sup>[3]</sup>.

During the walk, the constant habit of a quadruped is to travel on the surface of the ground to support and propel itself with all the four feet. The successive foot-impacts are

as follows, assuming the notation to commence with the landing of the left-fore foot. The diagonal hind-foot will follow, and it is lifted synchronously with the left-fore foot landing on the ground. Then the lateral fore-foot will follow next, and under the normal conditions of regular progress this fore-foot will be lifted simultaneously with the suspended hind-foot being placed on the ground.

This analysis determines the successive methods of support afforded by the feet of a quadruped during a normal stride of the walk to be 123-134-124-234-123 as in Fig. 1.

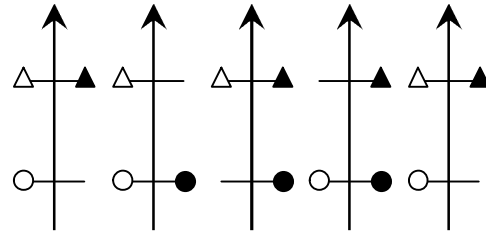


Fig. 1 Successive methods of support feet under the walk

According to a mathematical analysis developed by McGhee and Frank<sup>[4]</sup>, only three of the gaits perhaps exist for the foot placement of an animal or machine. Therefore, the animal or machine will be statically stable at all time. Furthermore, among these three, there exists a unique optimum gait that maximizes static stability, which is walk.

### 2.2 Amble

The amble is similar to the walk. Practically, amble has the same regularity of intervals but without the same sequence of foot impacts as walk. The back-foot is lifted from the ground simultaneously with its lateral foot being placed down, and the fore-foot is lifted from the ground simultaneously with its diagonal foot being placed. The successive methods of support feet under the amble are 124-134-123-234-124 as in Fig. 2.

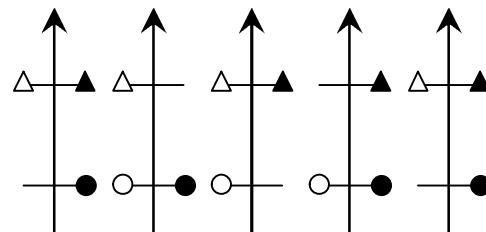


Fig. 2 Successive methods of support feet under the amble

Amble is also similar to the walk; during the amble, all four of the animal's feet are employed for the purposes of support and propulsion.

The walk and the amble are probably the only two gaits used by the elephant in its natural state<sup>[3]</sup>. The use of amble gaits is nearly ubiquitous among primates<sup>[13]-[15]</sup>.

### 2.3 Trot

The trot is a system of progress in which each pair of diagonal feet is alternately lifted with synchronicity, thrust

forward, and again placed on the ground. In this gait, the animal moves with two support feet, and each pair of diagonal feet with a half support time. The successive method of support afforded by the feet of a horse during a normal stride of the trot is 14-23-14 as in Fig. 3.

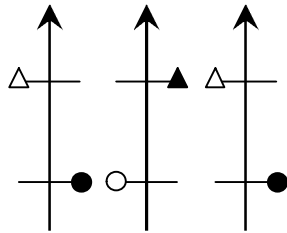


Fig. 3 Successive methods of support feet under the trot

The trot is demonstrated by the ox, wapiti, eland, fallow-deer, dog and cat, in their respective seriates<sup>[3]</sup>. Human walking, with a successive swing of the limbs, is similar to the trot.

**2.4 Pace**

In the pace the legs of the animal are used in lateral pairs, instead of diagonal pairs, as in the trot. The successive method of support afforded by the feet in one stride of the pace is 13-24-13 as in Fig. 4.

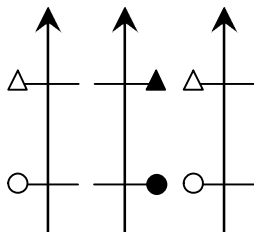


Fig. 4 Successive methods of support feet under the rack

This is an awkward, and to the rider exceedingly disagreeable, method of locomotion, so horses are rarely trained to use; when they are used, it is for traction, some slight advantage at a point of time over the trot. But this gait is natural to the camel, the giraffe and a few other animals<sup>[3]</sup>.

**2.5 Transverse-canter**

The transverse-canter has quite a different sequence of foot-fallings compared to the gaits introduced above, and during a portion of the stride the support is always only one leg. Assuming that the left-fore leg lands and raises first, the right-fore leg follows as soon as the left rises from the ground. Under the usual conditions on level ground, the remaining half with a substitution of the front feet for the back will be executed in practically the same manner. Besides the right leg following the left leg (Left-Transverse-Canter, LTC), some animals like to move their limbs with the left leg following the right one (Right-Transverse-Canter, RTC). The support feet

successive methods are 1-2-3-4-1 and 1-4-3-2-1 as in Fig. 5.

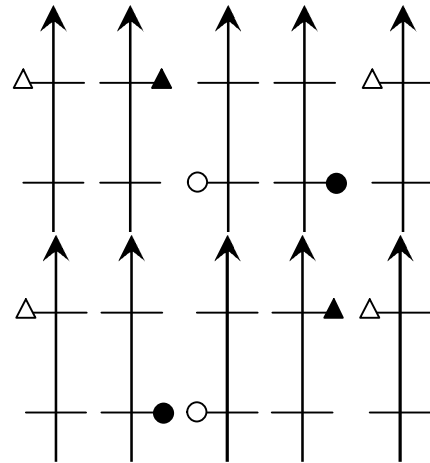


Fig. 5 Successive methods of support feet under the gait of LTC and RTC

The transverse-canter is employed by the horse and by the greatest number of other animals, both horny and soft-footed<sup>[3]</sup>.

**2.6 Rotator –canter**

There is another canter, in which the limb movements and consequent foot-fallings succeed each other in a rotative manner, which may be roughly represented by a circle, and it is called the “rotator-canter”. The rotator-canter has the same regularity of intervals as the transverse-canter, but without the same sequence of foot-fallings. Similar to the transverse-canter, there are two kinds of rotator-canter that are decided based on the animal’s habit, in which the sequence of the foot-fallings are implemented clockwise (clockwise-rotator-canter, CRC) or anticlockwise (anticlockwise-rotator-canter, ARC). The support feet successive methods are 1-2-4-3-1 or 1-3-4-2-1 as in Fig. 6.

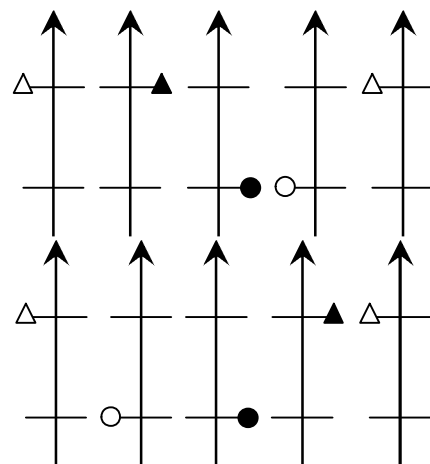


Fig. 6 Successive methods of support feet under the gait of CRC and ARC

The rotator-canter is adopted by the dog, the deer and

some other animals.

### 2.7 Gallop

The word “gallop”, in various forms of spelling, is now almost universally employed to designate the most rapid of all quadrupedal movements. The action is adopted by nearly all animals in one or the other of methods, when caprice, persuasion, or necessity, they exercise their utmost power for the attainment of their greatest speed.

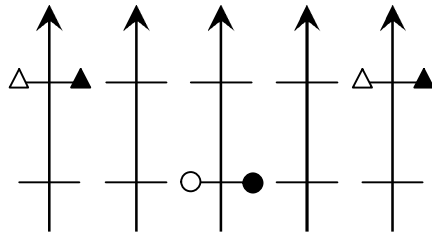


Fig. 7 Successive methods of support feet under the gallop

In the gallop the legs of the animal are used in fore-and-aft pairs, instead of diagonal pairs, as in the trot, lateral pairs, as in the pace. The feet are alternately lifted in synchrony and thrust forward and placed again on the ground. In this gait, the animal moves with two support feet with a long period of no support motion. The successive methods of support feet under the gallop are shown in Fig. 7.

## 3 Relationship Between the Phasic Difference and Duty Factor

### 3.1 Nonstandard gaits of quadruped

In natural, when the animals move, the motion gait is not one of standard gaits. Also, not all the quadrupeds walk with the same frequency of the swinging leg and support time landing leg.

Such as walk, sometimes animals walk quickly, and the supports are not just three feet in the standard walk, but the alternations of two and three feet in the way that each foot is lifted in the regular succession from the ground in advance of its posterior foot placed down. Similarly, when an animal is walking very slowly, such as crawling, the supports are not furnished alternately by two and three feet or just three feet, but by alternations of three and four feet. In contrast with the quick walk, each foot is placed in regular succession on the ground in advance of preceding foot being lifted.

Besides the trot, animals move quickly or slowly than the trot. With a quick trot under a shorter support time than half stride periods, the body of the animal makes a transit, without support, twice during each stride. The slow trot with long support time is seldom found in nature. When it is selected, the procedure results in throwing the duty of support alternately on two feet and on four feet.

The same phenomenon also appears in amble, pace and

other standard gaits. So the relationship and variation trend between the standard gaits should be found.

### 3.2 Two parameters of the gait

Usually, multipeds have manifold motion types, such as walk, trot, pace and gallop. The motion type of animals is termed gait, which is denoted by a phasic difference  $\varphi$  and a duty factor  $\beta$ <sup>[5]</sup>.

The phasic difference is the ratio of the phase of the different legs trailing the first leg to the motion period. We assume that the first motion leg is the left-front leg. Thus, the phasic difference of the right-front leg is  $\varphi_{12}$ , and that of the left-back and right-back are  $\varphi_{13}$  and  $\varphi_{14}$ , respectively. The quadruped has seven standard gaits as introduced above that appear when the animal moves in an invariable velocity<sup>[16]</sup>.

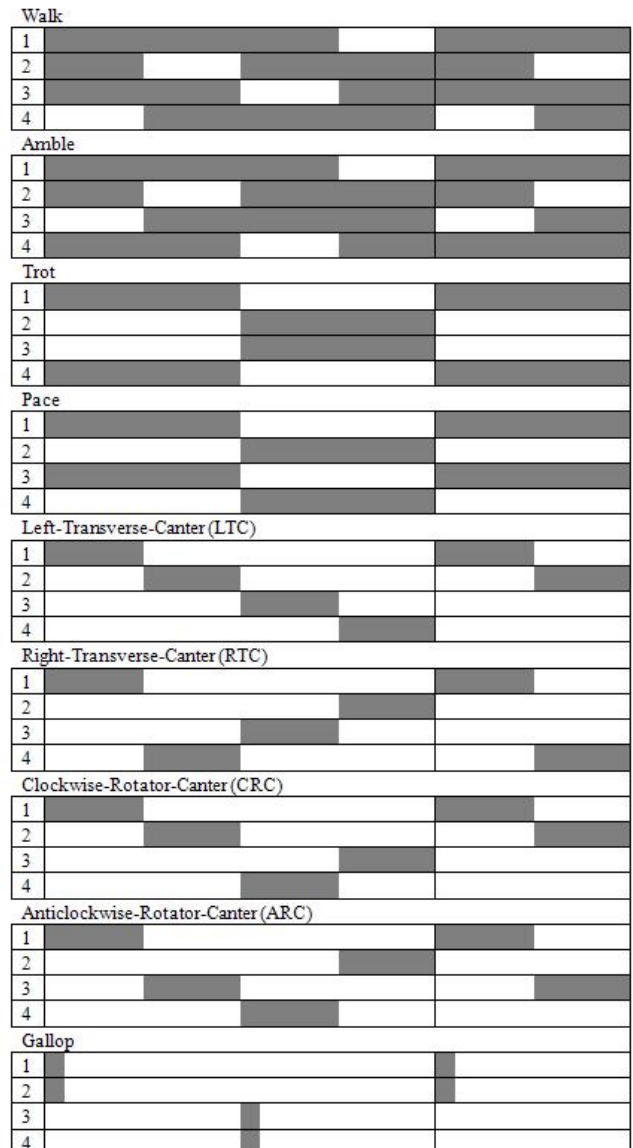


Fig. 8 Motion patterns of standard gaits.

In this section, we discuss the relationship between the phasic difference and the duty factor. The time variations of the standard gaits (three basic gaits, trot, pace, gallop, and

four evolutive gaits, walk, amble, transverse-canter, and rotator-canter) are shown in Fig. 8 with motion patterns. The black line means tile standing phase, and the white line indicates the swing phase.

The phasic differences of nine standard gaits are shown in Table 1. With the table, we can find a relationship between the phasic difference  $\varphi_{12}$ ,  $\varphi_{13}$  and  $\varphi_{14}$  as follows.

The legs of animals have two motion phases when moving: standing phase and swing phase. In the standing phase, the role of the leg is to keep contact with the ground and to propel the body. During the swing phase, the leg is swung up and returned to the starting point for the next

standing phase. Thus, the duty factor  $\beta$  represents the rate of the standing period to the total period of one stride motion<sup>[17]</sup>. Changing the duty factor implies changing the gait; the gait is changed to suit the walking speed<sup>[18]</sup>. For example, when the quadruped move with a trot gait, it is observed that the duty factor is near to 0.5. The basic manner of this gait is that two pairs of diagonal legs make two phases alternatively with a half phase shift.

By studying the different gaits of dogs, cats and horses, we can give out the familial duty factors opposite of the phasic differences to finish Table 1.

**Table 1. Phasic differences and duty factors of standard gaits.**

Phasic difference and duty factor	Walk	Pace	Amble	Trot	RTC	LTC	ARC	CRC	Gallop
$\varphi_{12}$	1/2	1/2	1/2	1/2	3/4	1/4	3/4	1/4	1
$\varphi_{13}$	3/4	1	1/4	1/2	1/2	1/2	1/4	3/4	1/2
$\varphi_{14}$	1/4	1/2	3/4	1	1/4	3/4	1/2	1/2	1/2
$\beta$	3/4	1/2	3/4	1/2	1/4	1/4	1/4	1/4	$\approx 0$

### 3.2 Quadruped gait derivative spectrum

As the standard gaits and their consecutive phases introduced above are not invariably followed even by the same animal, perhaps as a consequence of inequalities on the surface of the track, it will be a safer plan to give a broader significance to the gaits. For example, in the trot or pace, one of the inphase legs has precedence over another in the gait.

Thus, there should be some relationship between the phasic difference and the duty factor when varying another. The locomotion of the animals has symmetry, and here the gaits of quadrupeds are divided into two types: the motion is eudipleural when  $\beta > 0.5$  and fore-and aft symmetric when  $\beta < 0.5$ .

When  $0.5 < \beta < 0.75$ , keeping the phasic difference  $\varphi_{12} = \varphi_{34} = 1/2$ , the eudipleural gaits, walk, pace, amble and trot can be transited with the phasic difference between the front-left-leg and back-right-leg successively increasing. The variety of the phasic difference  $\varphi_{12}$ ,  $\varphi_{13}$  and  $\varphi_{14}$  with the duty factor are shown in Fig. 9 with a black line. When  $\beta > 0.75$ , the phasic difference, the black dash line in the Fig. 9, is invariable.

When the duty factor is less than 0.5, the gaits of the quadrupeds almost have fore-and-aft symmetry. The analysis of animal gaits with a duty factor less than 0.5, such as the transverse-canter, rotator-canter and gallop, demonstrates two systems of gait: one in which the foot-impacts as transverse-canter individually succeed each other in a way that may be conveniently represented by the points of a cross and the other in which the limb movements and consequent foot-fallings as rotator-canter, succeed each other in a rotative manner, which may be roughly represented by a circle.

Keeping the phasic difference  $\varphi_{13} = \varphi_{24} = 1/2$ , the transverse-gaits, trot, left-transverse-canter, gallop and right-transverse-canter can be transited with the phasic difference  $\varphi_{12}$  successively increasing. The variation of  $\varphi_{12}$ ,  $\varphi_{13}$  and  $\varphi_{14}$  with the duty factor are shown in Fig. 9 with a dark gray line.

In rotator-gaits, pace, clockwise-rotator-canter, gallop and anticlockwise-rotator-canter, when the phasic difference is kept at  $\varphi_{14} = \varphi_{23} = 1/2$ , they can be transited with the phasic difference  $\varphi_{12}$  successively increasing. The variation of  $\varphi_{12}$ ,  $\varphi_{13}$  and  $\varphi_{14}$  with the duty factor are shown in Fig. 9 with an gray line.

In this paper, the patterns in Fig. 9 are named quadruped gait derivative spectrum (QGDS) to describe the phase differences of three legs relative to the first leg. With these patterns in Fig. 9, the phasic differences of the legs corresponding to the duty factor can be accepted; thereby the gait for such a situation can be quickly confirmed. When the quadruped wants to change the motion speed, with the current and goal duty factor, the gait can be transformed in series.

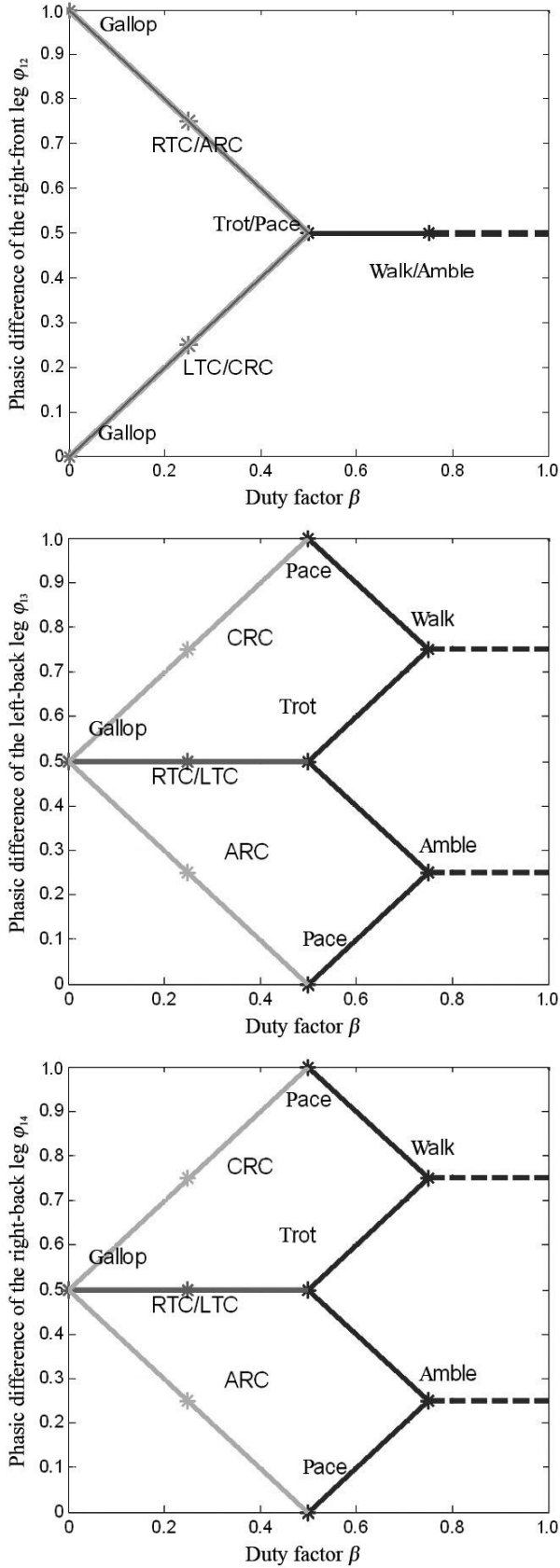


Fig. 9 Quadruped gait derivative spectrum.

#### 4 Relationship Between the Speed and Duty Factor

Quadrupeds move with the gaits introduced above. These

locomotion patterns are selected depending on the situation and locomotion requirement, where energy expenditure is one of the most important factors<sup>[11], [19]</sup>. In order to detect the relationship between running speed and oxygen consumption, HOYT trained horses to walk, trot and gallop on a motorized treadmill and to extend their gaits on command. From their result, it is evident that there exists an optimal speed for each gait, at which it costs the least oxygen consumption per meter, and the optimal speed increases for gaits with a lower duty factor. Thus, changing gaits and selecting speed can minimize energy consumption.

In Ref. [12], ITO gave the energy consumption per unit distance as

$$E = K_1 \frac{\beta}{1-\beta} v + K_2 \frac{1-\beta}{\beta} \frac{1}{v}, \quad (1)$$

where the first term denotes energy-to-move, and the second term denotes energy-to-support. As the speed  $v$  increases, the first term increases, while the second term decreases. It can be explained that there exists an optimal speed  $v_0$  that minimizes the Eq. (1). The optimal speed  $v_0$  is given by

$$v_0 = \frac{1-\beta}{\beta} \sqrt{\frac{K_2}{K_1}}. \quad (2)$$

The energy-to-move during one locomotion period is denoted here by  $E_T$ . We evaluate the energy-to-move by the energy consumption in the viscosity of muscles, thus,  $E_T$  becomes<sup>[12]</sup>

$$E_T = 4 \int_{T_{sp}} \eta \omega_{sp}^2 dt + 4 \int_{T_{sw}} \eta \omega_{sw}^2 dt, \quad (3)$$

where  $T_{sp}$  and  $T_{sw}$  denote the duration of the support and swing phase, respectively,  $\omega_{sp}$  and  $\omega_{sw}$  are the angular velocity in support and swing phase, respectively, and  $\eta$  denotes the viscosity coefficient of the muscles. Then, the limb swing amplitude  $A$  can be calculated from the distance moving in the support phase:

$$A = \arctan\left(\frac{v T_{sp}}{Y/2}\right), \quad (4)$$

where  $Y$  is the height of the body. Approximating around the upright standing posture, Eq. (4) becomes

$$A \approx \left(\frac{v T_{sp}}{Y/2}\right). \quad (5)$$

The relation

$$T_{sp} = \beta T \quad (6)$$

and

$$T_{sw} = (1 - \beta)T \quad (7)$$

gives

$$\omega_{sp} = \frac{2A}{T_{sp}} = \frac{v}{Y} \quad (8)$$

and

$$\omega_{sw} = \frac{2A}{T_{sw}} = \frac{\beta}{1 - \beta} \frac{v}{Y}. \quad (9)$$

Therefore, Eq. (3) becomes

$$\begin{aligned} E_T &= 4 \int_{T_{sp}} \eta \left( \frac{v}{Y} \right)^2 dt + 4 \int_{T_{sw}} \eta \left( \frac{\beta}{1 - \beta} \frac{v}{Y} \right)^2 dt \\ &= 4\eta \left[ \left( \frac{v}{Y} \right)^2 \beta T + \left( \frac{\beta}{1 - \beta} \frac{v}{Y} \right)^2 (1 - \beta)T \right] \quad (10) \\ &= 4\eta \frac{\beta T}{1 - \beta} \left( \frac{v}{Y} \right)^2 \end{aligned}$$

Then, evaluating energy-to-move per meter,

$$E_1 = \frac{E_T}{vT} = \frac{4\eta}{Y^2} \frac{\beta}{1 - \beta} v, \quad (11)$$

so the first term of Eq. (1) is

$$E_1 = K_1 \frac{\beta}{1 - \beta} v = \frac{4\eta}{Y^2} \frac{\beta}{1 - \beta} v. \quad (12)$$

Then, the parameter  $K_1 = 4\eta/Y^2$ .

In the second term, if fatigue is neglected, it can be thought that the energy to support a body in unit time, denoted by  $E_s$ , remains constant at the stationary locomotion. Then the energy-to-support in unit distance, denoted by  $E_2$ , is inversely proportional to the locomotion speed  $v$ . During the time interval  $\Delta t$ , the energy consumption is  $E_s \Delta t$ , and the traveling distance is  $v \Delta t$ , with the result that

$$E_2 = \frac{E_s \Delta t}{v \Delta t} = \frac{E_s}{v} = K_2 \frac{1 - \beta}{\beta} \frac{1}{v}. \quad (13)$$

Then, the parameter  $K_2 = \frac{\beta}{1 - \beta} E_s$ .

As a result, the optimal speed  $v_0$  can be expressed as

$$v_0 = \frac{1 - \beta}{\beta} \sqrt{\frac{\beta}{1 - \beta} \frac{E_s Y^2}{4\eta}} = \sqrt{\frac{1 - \beta}{\beta}} \sqrt{\frac{E_s Y^2}{4\eta}}. \quad (14)$$

Eq. (14) can be expressed as

$$\beta = \frac{1}{1 + \frac{4\eta}{E_s Y^2} v_0^2}. \quad (15)$$

So the relationship between the speed and duty factor can be expressed as Fig. 10.

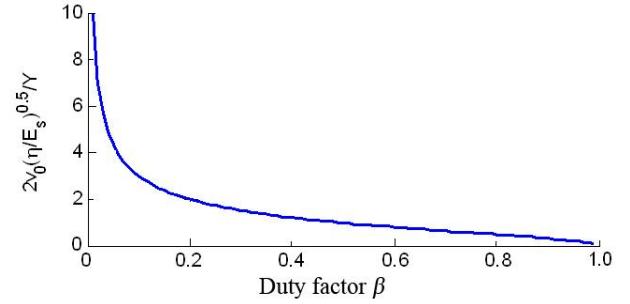


Fig. 10 Relationship between duty factor and optimal motion speed

With the analyzed patterns of motion of animals in Ref. [5], we can assume that the swing amplitude  $A$  is constant. Let leg stroke

$$R = v T_{sp}. \quad (16)$$

So,  $R$  is constant according to Eq. (4). With Eq. (6) the speed can be expressed as

$$v = \frac{R}{\beta T}, \quad (17)$$

thus, the periods of the motion with the optimal speed  $v_0$  is

$$T_0 = \frac{R}{\beta v_0} = \frac{R}{\sqrt{\beta(1 - \beta)} \sqrt{\frac{E_s Y^2}{4\eta}}}. \quad (18)$$

From Fig. 11, we can find a significant phenomenon that for the motion periods of all gaits, the value of  $T_0$  is symmetrical with  $\beta$  varying from 0 to 1. A minimal value exists when the duty factor  $\beta$  is 0.5; when the  $\beta$  is modified between 0.2 and 0.8, the change in the period is almost zero.

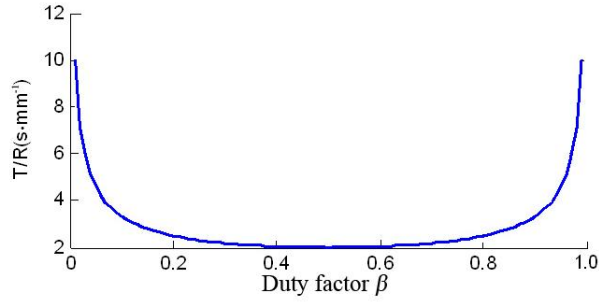


Fig. 11 Relationship between duty factor and optimal motion periods.

## 5 Gait Transition from Walk to Trot

A perfect mechanical model is considered, which involves four uniform legs. The amplitude of the leg swing is constant. The mechanical model moves with an unaltered level  $Y$  and leg stroke  $R$ .

The gait transition from the walk gait to the trot gait is considered. To realize the course of the gaits, the variation of the angle  $\theta$  of the body and line between the hip and foot should be recorded. In Fig. 12, the dotted line is the angle variation with time under the walk gait, in which  $\varphi_{12}=0.75$ ,  $\varphi_{13}=0.5$ ,  $\varphi_{14}=0.25$ , and  $\beta=0.75$ . The dashed line is the trot gait, in which  $\varphi_{12}=0.5$ ,  $\varphi_{13}=0.5$ ,  $\varphi_{14}=0$ , and  $\beta=0.5$ .

With the relationships between the phasic difference, duty factor and speed discussed above, the transition gait from the walk to trot can be given. Fig. 12 shows the locomotion pattern transitions when the desired speed increases and the gait transitions from walk to trot.

Fig. 13 shows the angular velocity in the support and swing phases  $\omega_{sp}$ ,  $\omega_{sw}$ .

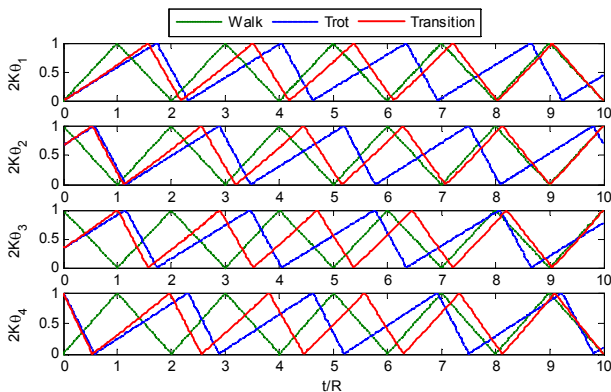


Fig. 12 Time course of four joints for a gait transition from walk to trot, here  $K = \sqrt{\eta/E_s}$

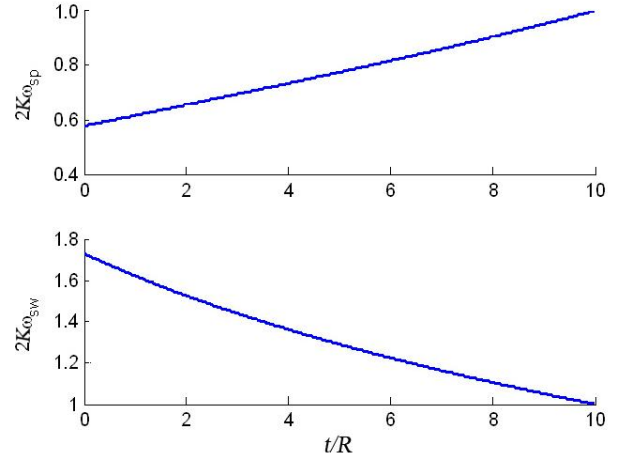


Fig. 13 Angular velocity in support and swing phase  $\omega_{sp}$ ,  $\omega_{sw}$ , here  $K = \sqrt{\eta/E_s}$

## 6 Conclusions

(1) The parameter definition of the gait planning is presented, and then the gait of the quadruped is analyzed and classified. Furthermore, the definitions for seven types of the gait is given, which is the prerequisite of the gait variance.

(2) Quadruped gait derivative spectrum for the transition of the gait is obtained through the analysis of the gait parameters of the quadruped. In addition to quantitative character of the gait, the definition of the continue gait for the quadruped is given, ensuring the continue transition of the gait.

(3) An optimal gait is usually required corresponding to the motion speed. To this end, the rule of the gait selection with respect to the speed is proposed through the study of the relationship between the speed, gait, energy.

(4) The simulation of the gait selection and transition demonstrates that the efficiency of the proposed definition.

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