Any Classical Description of Nature Requires Classical Electromagnetic Zero-Point Radiation

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Abstract

Any attempt to describe nature within classical physics requires the presence of Lorentz-invariant classical electromagnetic zero-point radiation so as to account for the Casimir forces between parallel conducting plates at low temperatures. However, this zero-point radiation also leads to classical explanations for a number of phenomena which are usually regarded as requiring quantum physics. Here we provide a cursory overview of the classical electromagnetic theory which includes classical zero-point radiation, and we note the areas of agreement and disagreement between the classical and quantum theories, both of which contain Planck's constant $\hbar$.

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I.   INTRODUCTION

Although classical physics provides satisfactory explanations for many phenomena in mechanics and electromagnetism, there seems to be little interest in classical explanations for phenomena which involve Planck's constant $\hbar$. Thus although there are natural classical explanations for the stable ground state of hydrogen, for the blackbody radiation spectrum, for Casimir forces, for specific heats of solids, and for diamagnetism, these explanations are not mentioned in the physics textbooks. The root cause for this neglect is the failure of modern physicists to allow the possibility of classical electromagnetic zero-point radiation. In this article we start out by discussing the experimentally observed Casimir forces where measurements have become increasingly accurate in recent years.   We note that these experiments demand the presence of classical electromagnetic zero-point radiation if we attempt to explain nature within classical electromagnetic theory.   We then point out that the presence of classical zero-point radiation has significant implications for thermal behavior and atomic structure.

Some physicists will object that we already have perfectly good quantum explanations for these phenomena so that classical explanations are superfluous.   To these physicists who like the quantum theory explanations, we would simply repeat the words of Sherlock Holmes: "I don't mean to deny that the evidence is in some ways very strong in favour of your theory; I only wish to point out that there are other theories possible."[1]

The classical theory which includes classical electromagnetic zero-point radiation has in the past been termed "random electrodynamics"[2] or "stochastic electrodynamics"[3] or "classical electron theory with classical electromagnetic zero-point radiation."[4][5]   The theory corresponds to the classical electron theory of H. A. Lorentz but with a change in the boundary conditions to include classical electromagnetic zero-point radiation.   In recent years, the theory has had notable successes in the simulation work for the hydrogen atom by Cole and Zou[6] and also in the relativistic work which provides an entirely new perspective on blackbody radiation.[7]

II. CASIMIR FORCES AND CLASSICAL ELECTROMAGNETIC ZERO-POINT RADIATION

A. Casimir Forces

The need for classical electromagnetic zero-point radiation within a classical theory seems most transparent when we try to explain the experimentally observed Casimir forces between conducting parallel plates. Casimir forces are forces associated with the discrete normal mode structure of waves in a finite volume.[8] Thus if we consider the thermal motion of a one-dimensional string of length L which has fixed end points at $x = 0$ and $x = L$, the random thermal motion can be expressed in terms of the oscillations of the normal modes of the string with random phases between the modes. Thermal wave motion will have a characteristic energy $U(\omega_n)$ associated with each mode of (angular) frequency $\omega_n = 2\pi n v/(2L)$, $n = 1, 2, ...$ where v is the speed of the waves on the string. If we imagine the string passing through a small hole in a partition located at some point x between the fixed end points, $0 < x < L$, then the small hole will enforce a node in the string's oscillations, and therefore the partition will experience forces due to the oscillations of the string on the two different sides of the partition. In general the partition will experience a net force because the normal modes on opposite sides of the partition are associated with different lengths x and $L - x$. This net force is a Casimir force on the partition arising from the differences in energies $U(\omega_n)$ for different frequencies associated with the different lengths x and $L - x$ of the string. An analogous situation arises for any wave system where the boundary conditions enforce a nodal structure.

For a conducting partition in a conducting-walled box, electromagnetic waves will lead to Casimir forces. The possibility of Casimir forces between conductors was first proposed by H. B. G. Casimir[9] in 1948 in connection with the normal modes for electromagnetic radiation between conducting parallel plates. Any spectrum of random radiation will lead to forces on a conducting partition in a conducting-walled box. One of the familiar spectra for random electromagnetic radiation is the Rayleigh-Jeans spectrum where the energy $U_{RJ}(\omega, T)$ per normal mode at (angular) frequency ω and temperature T is independent of the frequency ω and is given by $U_{RJ}(\omega, T) = k_B T$. This spectrum leads to an attractive Casimir force F_{RJ} between conducting parallel plates of area $A = L \times L$ and separation d, where $d << L$; the

force is proportional to the temperature T and to the plate area $A = L \times L$, and inversely proportional to the third power of the separation d between the plates[10]

$$F_{RJ} = -\frac{\zeta(3)k_B TA}{4\pi d^3} \tag{1}$$

where $\zeta(3)$ is a numerical constant. According to this formula, the Casimir force should vanish as the temperature T goes to zero. However, experimental measurements[11] show clearly that the Casimir forces do not vanish at low temperature, but rather become independent of temperature. Within classical physics, the only natural explanation for the experimentally measured Casimir forces between uncharged conducting plates at low temperature is the existence of temperature-independent random radiation. This radiation has been termed classical electromagnetic zero-point radiation.

B. Spectrum of Classical Electromagnetic Zero-Point Radiation

What is the natural spectrum for classical electromagnetic zero-point radiation? This radiation should correspond to the state of lowest possible energy, the vacuum state. And our qualitative notion is that the vacuum should be as featureless as possible; in an inertial frame, it should be homogeneous, isotropic, scale invariant, and indeed Lorentz invariant. It turns out that there is a unique spectrum (unique up to one multiplicative constant) of random classical radiation which satisfies these requirements.[3][12] The spectrum has an energy $U_0(\omega)$ per normal mode given by

$$U_0(\omega) = \mathsf{const} \times \omega/c \tag{2}$$

where const is an unknown constant. We mentioned that *any* spectrum of random classical radiation will lead to Casimir forces between conducting parallel plates. The force between parallel conducting plates of area $A = L \times L$ separated by a small distance d, $d << L$, in the presence of the classical electromagnetic zero-point spectrum of Eq. (2) is given by

$$F_0 = \mathsf{const} \times \frac{\pi^2 A}{120d^4} \tag{3}$$

Indeed, it is found that this formula describes the experimental measurements provided that

$$\mathsf{const} = 1.58 \times 10^{-26} J \cdot m \tag{4}$$

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Thus the experimentally observed Casimir forces at low temperature are accounted for by the Lorentz-invariant spectrum of classical electromagnetic zero-point radiation given in Eq. (2) provided that the constant takes the value in Eq. (4).

C. Planck's Constant $\hbar$ and Classical Electromagnetic Zero-Point Radiation

The constant appearing in Eq. (4) was obtained from a purely classical analysis of the Casimir forces between conducting parallel plates. No aspects of energy or action quanta are involved. However, the numerical value of the constant as well as the spectrum of Eq. (2) are familiar from a very different theory, namely from quantum theory. Thus we can either continue to work with $\mathsf{const} = 1.58 \times 10^{-26} J \cdot m$ or we can instead everywhere in the classical analysis replace const by the familiar expression $\hbar c/2$

$$\mathsf{const} = \hbar c/2 \tag{5}$$

since both have the same numerical value.

Of course, there is a danger in introducing Planck's constant. Planck's constant $\hbar$ has been associated with "quantum phenomena" for so long that it is often referred to as "a quantum constant," and some physicists believe that the mere presence of Planck's constant in a theory indicates that the theory is a "quantum" theory. However, Planck's constant is simply a numerical value which in itself does not indicate the type of theory where it appears. As a numerical value, Planck's constant may appear in any theory. Indeed, Planck's constant $h = 2\pi\hbar$ was first introduced in 1899 before there was any mention of quantum theory.

In the present discussion based upon classical electromagnetic theory, Planck's constant $\hbar$ is introduced simply as a numerical value setting the scale of classical electromagnetic zero-point radiation. As we have emphasized above, we can avoid Planck's constant altogether simply by always writing the expressions in terms of the const appearing in Eq. (4) or by writing out its numerical value. The zero-point radiation of Eq. (2) is regarded as random classical radiation with a Lorentz-invariant spectrum which appears as a homogeneous solution of Maxwell's equations. Thus the solutions of Maxwell's equations in terms of sources can be expressed as integrals over the sources using the retarded Green function of the wave equation (thus providing the particular solution) plus zero-point radiation as

the homogeneous solution of Maxwell's equations.

Perhaps the reader can obtain a sense of what is involved in classical zero-point radiation by envisioning the more familiar situation involving thermal radiation at nonzero temperature. Suppose that an experimenter sets up his electromagnetic sources in a laboratory full of classical thermal radiation at temperature $T > 0$. Then in order to describe the electromagnetic fields in the lab, the experimenter would include both the fields due to the sources which he has manipulated plus the fields due to the thermal radiation which were already present and which the experimenter did not introduce intentionally. And as every experimenter knows, finite-temperature behavior will alter his sources. Thus the sources which are introduced by the experimenter are influenced by the radiation which is already present when the experimenter arrives in his lab. This radiation, which is already present when the experimenter sets up his equipment, corresponds to the homogeneous boundary condition on Maxwell's equations used by the experimenter. Zero-point radiation is analogous to thermal radiation as radiation which is not introduced by the experimenter but which is always present and which can influence the sources which are arranged by an experimenter.

III. IMPLICATIONS OF CLASSICAL ELECTROMAGNETIC ZERO-POINT RADIATION

A. Linear Systems

1. Linear Oscillator

Because classical electromagnetic zero-point radiation must be present in any classical electromagnetic theory which accounts for the experimentally observed Casimir forces between conductors, we also expect zero-point radiation to influence every classical electromagnetic system. For example, if we picture a particle of charge e and mass m at the end of a spring oscillating along the x-axis so that the system has a natural mechanical oscillation frequency ω_0, then we expect that the system will both be damped as the oscillating charge emits radiation and be pushed into motion by the random zero-point radiation. Thus in the nonrelativistic point-dipole approximation, we expect the system to satisfy the equation

of motion

$$m\frac{d^2x}{dt^2} = -m\omega_0^2 x + \frac{2}{3}\frac{e^2}{c^3}\frac{d^3x}{dt^3} + e\mathbf{E}_x(0,t) \tag{6}$$

where the mass times the acceleration equals the spring restoring force plus the radiation damping force plus the zero-point radiation driving force. This is a linear stochastic equation which can easily be solved. For a small electric charge e, the charge actually cancels out of the expressions for the average values. The average position and momentum of the system are both zero, but the mean square of the displacement $< x^2 >$ and mean square of the momentum $< p^2 >=< (mv)^2 >$ are given by[4][13]

$$< x^2 >= \frac{\mathsf{const}/c}{m\omega_0} = \frac{1}{2}\frac{\hbar}{m\omega_0} \tag{7}$$

and

$$< p^2 >= \frac{\mathsf{const}}{c} \times m\omega_0 = \frac{1}{2}\hbar m\omega_0 \tag{8}$$

while the average energy is given by

$$U = \frac{\mathsf{const}}{c} \times \omega_0 = (1/2)\hbar\omega_0 \tag{9}$$

We have included the expressions involving const so as to remind the reader that these expression arise from the balance between the driving force from classical zero-point radiation and the damping from the radiation reaction force. However, it is clear that these expressions are identical with those which appear in the quantum mechanics of the harmonic oscillator. It turns out that the average values of all of the products of oscillator position and momentum given by the classical calculations are identical with the expectation values of the *symmetrized* operator products of the corresponding quantum oscillator.[13]

2. Physical Systems Described by Linear Oscillators

There are a number of physical systems which are traditionally described in terms of molecules modeled as harmonic oscillators. These include the van der Waals forces between molecules and also the van der Waals forces between molecules and conducting or dielectric walls.[14] The specific heats of solids involve molecules which are often described by harmonic oscillators.[15] Finally, the diamagnetism of molecules can be described in terms of the behavior of linear oscillator systems.[16] Because of the general connection[13] between

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the average values of products of classical oscillator position and momentum with the symmetrized operator products of the corresponding quantum variables, all of these phenomena have natural classical descriptions within classical electromagnetic theory which includes classical electromagnetic zero-point radiation.

3. *Disagreement for Nonrelativistic Nonlinear Non-Coulomb Systems*

One needs to be circumspect about the areas of agreement and disagreement between classical and quantum theories. Despite the very close agreement for linear systems between quantum theory and classical theory with classical zero-point radiation, the theories part company for nonrelativistic nonlinear non-Coulomb systems. Thus rotator specific heats are quite different within quantum theory and classical theory with classical zero-point radiation.[17] Furthermore, classical nonlinear oscillator systems scatter random radiation toward the Rayleigh-Jeans spectrum[18] whereas quantum systems do not.

B. The Classical Hydrogen Atom

One hundred years ago, Rutherford[19] published his work proposing the nuclear model of the atom. Instead of the plum-pudding model for the atom, consisting of a continuous "jelly" of positive charge with embedded negative point electrons, the atom rather followed a "planetary" model, consisting of a small, heavy, positive nucleus with electrons outside. However, it was realized at the time that electrons in Coulomb orbit around the heavy nucleus would radiate energy as electromagnetic radiation, and so it was thought that they would spiral into the nucleus as they lost energy. At the time of Rutherford's experiments, physicists were not aware of the idea of classical electromagnetic zero-point radiation. The presence of this random zero-point radiation, which we now know is required to exist in a classical theory so as to account for Casimir forces, changes the perspective on the old problem of atomic collapse. The presence of classical electromagnetic zero-point radiation raises the possibility that atomic structure is due to a balance between the loss of energy as electrons radiate and the pick-up of energy as electrons experience the random forces of the zero-point radiation. This basic model is the same as that used above in Eq. (6) when discussing linear oscillator systems. The nonrelativistic model for hydrogen corresponds to

the equation

$$m\frac{d^2\mathbf{r}}{dt^2} = -\frac{e^2\widehat{\mathbf{r}}}{r^2} + \frac{2}{3}\frac{e^2}{c^3}\frac{d^3\mathbf{r}}{dt^3} + e\mathbf{E}(0,t) \qquad (10)$$

where the mass times acceleration equals the Coulomb force attracting the electron to the nucleus plus the radiation damping force plus the random force due to the zero-point radiation.

In contrast to Eq. (6) for a linear system which was easy to solve analytically, the stochastic differential equation (10) for the hydrogen atom has never been solved analytically. However, the ground state has been solved by numerical simulation. In 2003, Cole and Zou[6] followed the motion of an electron described by equation (10) and found that the electron did not plunge into the nucleus or go far from the nucleus; indeed, the probability distribution for the electron's distance from the nucleus agreed closely with the familiar result given by the Schroedinger ground state. There are no free parameters in Cole and Zou's calculation; the values for the electron mass, charge, and scale of zero-point radiation are all fixed by other experiments. The work is a striking suggestion of the power of a classical theory in describing some parts of atomic physics.

There is also a revealing controversy associated with the calculation of the hydrogen atom ground state. Cole and Zou's calculation provides a numerical probability distribution for the hydrogen ground state and suggests that the classical hydrogen atom with zero-point radiation is stable over the time. On the other hand, Marshall and Claverie[20] in 1980 set up the same nonrelativistic calculation in terms of action-angle variables. They never computed any ground state distribution, but rather it was concluded that there could be no stable ground state for the classical hydrogen atom in classical electromagnetic zero-point radiation. The zero-point radiation was viewed as "too strong," so that the electrons in Coulomb orbit around the nucleus were ionized through the plunging orbits of small angular momentum. Thus the work beginning with Marshall and Claverie's analysis suggested the opposite situation from that of the old problem of atomic collapse in classical theory.

However, there is a failure in Marshall and Claverie's calculation as applied to nature. Plunging elliptical orbits of small angular momentum do not exist in nature! This situation often comes as a shock to physicists who are familiar with the nonrelativistic classical mechanics of Coulomb and Kepler orbits. Relativity changes the orbits of mechanical motion most severely for orbits of small angular momentum.[21] Within the relativistic mechanics of a point mass held in a Coulomb or Kepler orbit by a force $\mathbf{F} = -\,e^2\widehat{\mathbf{r}}/r^2$, any orbit which

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has small angular momentum, $L < e^2/c$, must plunge into the nucleus while conserving energy and angular momentum! It should be emphasized that this last sentence involves pure relativistic mechanics, not electromagnetism, and there is no energy loss or gain due to radiation emission or absorption.[21]

Thus we find that the calculations of Marshall and Claverie are modified by relativity at precisely the point where they suggest that the electron is ejected from the atom. We conclude that Cole and Zou's numerical simulations have indeed found the nonrelativistic approximation to the ground state of the classical hydrogen atom in classical electromagnetic zero-point radiation.[22]

C. Blackbody Radiation and Relativity

The beginning of the twentieth century saw the introduction of quantum ideas in connection with the problem of blackbody radiation. Indeed today, those textbooks which still introduce quanta from a historical[23] rather than an axiomatic[24] perspective still discuss the classical physics of radiation normal modes and energy equipartition within nonrelativistic statistical mechanics.[23] The blackbody radiation problem troubled physicists all though the first quarter of the century. In addition to the now-famous quantum calculations, there were attempts to derive the equilibrium spectrum of thermal radiation from classical scattering calculations[18] and from equilibrium classical particle motion.[25] However, the physicist in the first quarter of the twentieth century were unaware of two important aspects of classical physics: classical electromagnetic zero-point radiation and the importance of relativity.

The mere presence of classical electromagnetic zero-point radiation alters our ideas of classical statistical mechanics. Indeed a number of derivations of the Planck spectrum for blackbody radiation have been given within classical physics based upon the presence of classical electromagnetic zero-point radiation, some using as their starting point precisely the earlier calculations which (in the absence of classical zero-point radiation) led to the Rayleigh-Jeans spectrum.[26] However, all of those calculations left a nagging doubt because of the scattering calculations using nonrelativistic charged mechanical systems; all of these calculations show that nonrelativistic nonlinear scattering systems push classical radiation toward the Rayleigh-Jeans spectrum.[18]

The importance of relativity appeared above in validating the work of Cole and Zhou against the conclusion of Claverie and Marshall. The importance of relativity also appears in understanding the spectrum of blackbody radiation. Only in 2010 was it pointed out that a relativistic scattering system will not scatter classical electromagnetic zero-point radiation toward the Rayleigh-Jeans spectrum.[27] This result is absolutely crucial. All of the calculations leading to the Rayleigh-Jeans spectrum for classical thermal radiation involve mixtures of nonrelativistic physics and relativistic electromagnetic radiation.[18] None of the calculations leading to the Rayleigh-Jeans law holds up as a fully relativistic calculation.[28]

Indeed most recently, it has been shown that the Planck spectrum for thermal radiation follows from the presence of classical zero-point radiation and the structure of relativistic spacetime within classical physics.[7] Zero-point radiation is the unique spectrum of random classical radiation which is Lorentz invariant and scale invariant. Zero-point radiation is required in the classical theory so as to account for the experimentally observed Casimir forces. Now classical electromagnetism is invariant under not only relativistic transformations but also conformal transformations which include dilatations and proper conformal transformations.[29] In an inertial frame, classical thermal radiation is carried into thermal radiation at a different temperature by a time-dilating conformal transformation, while the spectrum of classical zero-point radiation is invariant under time-dilating conformal transformations and indeed under all conformal transformations. However, if we consider thermal radiation not just in an inertial frame but in a general non-inertial, static coordinate frame, then time-dilating conformal transformations carry zero-point radiation into thermal radiation at finite non-zero-temperature. We can use this connection between zero-point radiation and thermal radiation in a coordinate frame undergoing uniform relativistic acceleration through flat spacetime (a Rindler frame[30]) to give a derivation of the Planck spectrum in an inertial frame by taking the zero-acceleration limit.[7] The Planck spectrum is connected directly with zero-point radiation and relativity in classical physics.

D. Speculations Regarding Wave-Like Aspects of Particles and Line Spectra

Although classical physics can give satisfying classical explanations for some phenomena involving Planck's constant $\hbar$, there are at present no calculations which give a definite explanation for the experimentally observed diffraction effects for particles passing through

small slits. Clearly classical electromagnetic zero-point radiation interacts with any conducting or dielectric surface, and this interaction changes the correlation function for the electromagnetic fields compared to free space. Also, qualitatively, the motion of charged particles near slits will be influenced by the correlation function for the classical electromagnetic zero-point radiation near the slits. Perhaps one day we will be able to calculate this influence within a classical theory. In any case, it is obvious that if the number of slits is changed, then the correlation function for zero-point radiation will change, and hence the pattern of particles passing through the slits will change. Indeed, the influence of surfaces which change the correlation functions for zero-point radiation is well-understood for charged harmonic oscillator systems at rest outside plane surfaces where the changes in the correlation function lead to van der Waals forces.[14]

Furthermore, there is at present no definitive explanation for the experimentally observed line spectra. Within classical physics, we expect line spectra to correspond to some sort of resonance behavior. In the hydrogen atom, a highly excited electron radiates away more energy than it picks up from the zero-point radiation, and indeed the spectra of hydrogen do indeed approach the traditional spectral frequencies calculated in the absence of zero-point radiation. This is the idea which is involved in the correspondence principle. However, when the electron is excited but near the ground state in energy, it seems hard to calculated the radiation emitted by the electron as it loses energy by radiation emission and absorbs energy from the classical electromagnetic zero-point radiation. Cole and Zou have pointed out that the Coulomb potential holds fascinating nonlinear resonances for electrons in a circularly polarized electromagnetic driving wave.[31] However, there is at present no explanation within classical phyisics for the line spectra of atoms.

IV.   DISCUSSION

In this article, we have pointed out that any attempt at a classical explanation of nature must included classical electromagnetic zero-point radiation to account for the experimentally observed Casimir forces between conducting parallel plates. The spectrum of classical zero-point radiation can be determined up to one multiplicative constant by symmetry requirements, such as Lorentz invariance or scale invariance or conformal invariance. The scale of the classical zero-point radiation is determined by fitting Casimir-force experiments. The

numerical value obtained for the scale of zero-point radiation is familiarly given in terms of the number appearing in Planck's constant $\hbar$. Thus within classical theory, Planck's constant $\hbar$ enters the theory as a number setting the scale of the homogeneous solution of Maxwell's equations corresponding to classical electromagnetic zero-point radiation.

Once classical electromagnetic zero-point radiation is introduced into the classical theory, there are implications for van der Waals forces, specific heats, diamagnetism, atomic structure, and blackbody radiation. In some cases, the classical calculations are in agreement with quantum theoretical calculations, and in some cases they are not. For mechanical systems described in terms of free fields or linear oscillator systems, there is agreement between the classical and quantum average values. For nonrelativistic nonlinear non-Coulomb systems, there is disagreement between the classical and the quantum calculations. For the ground state of hydrogen, there is fascinating agreement between classical numerical simulation calculations and the Schroedinger ground state. Also, classical physics gives a simple and powerful explanation for the blackbody radiation spectrum in terms of zero-point radiation and relativistic theory.

Planck's constant $\hbar$ is a number which can appear in classical or quantum theories. The constant $\hbar$ appears in all theories which include zero-point radiation or zero-point energy. Within quantum theory, Planck's constant is related to commutators of operators which then lead to zero-point energy for quantized mechanical systems and zero-point radiation for quantized fields. Within classical electron theory with classical electromagnetic zero-point radiation (stochastic electrodynamics), Planck's constant appears as a scale factor for classical zero-point radiation, and Planck's constant then reappears in all systems with electromagnetic interactions. Thus Planck's constant $\hbar$ is introduced at very different points in quantum as compared to classical theory. What should we say about the limit $\hbar \to 0$ which is often called "the classical limit"? The limit $\hbar \to 0$ turns the quantum theory of noncommuting operators into a classical theory with commuting variables, but any idea of zero-point energy or zero-point radiation has disappeared along with quantum operator behavior. On the other hand, the limit $\hbar \to 0$ removes the zero-point energy from any classical theory, and therefore this limit turns the classical electron theory with classical electromagnetic zero-point radiation into the classical electron theory of H. A. Lorentz[32] which was used in the years around 1900. Clearly the more recent classical electron theory which includes classical zero-point radiation with a scale set by $\hbar$ can explain far more of

nature than is possible with the older classical electron theory where zero-point radiation is omitted and $\hbar$ is regarded as zero.

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(Springer-Verlag, New York 1977), pp. 49-51. See also, See for example, B. F. Schutz, *A First Course in General Relativity* (Cambridge U. Press 1986), p. 150.

[31] D. C. Cole and Y. Zou, "Analysis of Orbital Decay Time for the Classical Hydrogen Atom Interacting with Circularly Polarized Electromagnetic Radiation," Physical Review E **69**, 016601(12) (2004); "Subharmonic resonance behavior for the classical hydrogen atomic system," Journal of Scientific Computing, Vol. **39**, 1-27 (2009).

[32] Traditional classical electron theory is described by H. A. Lorentz, *The Theory of Electrons* (Dover, New York 1952). This volume is a republication of the second edition of 1915 based on Lorentz's Columbia University lectures of 1909. Note 6, p. 240, gives Lorentz's explicit assumption on the boundary conditions.