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Precision Balance Method for Cupped Wave Gyro Based on Cup-bottom Triming

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Abstract: The mechanical balance process is the key process to eliminate the quadrature error and improve the performance of the cupped wave gyro. The conventional mechanical balance method for cupped wave gyro based on cup-wall trimming requires high control accuracy of trimming quantity, which increases the production cost and decreases the fabrication efficiency in large extent. However, it is hard to reach the high balance accuracy with the natural frequency split of mHz grade by using the conventional method. In this paper, the lumped mass dynamic model of the cupped wave gyro is built by discretization method, and the effects of different position trimming on the natural frequency are analyzed. It is pointed out that trimming off a tiny quantity of material from cup-wall causes large variation of the natural frequency is the main reason for the low accuracy of the conventional mechanical balance method. Then, a precision balance method for cupped wave gyro based on cup-bottom trimming is presented and the entire procedures of this method are given. The static balance process and dynamic balance process of the precision balance method are simulated by the finite element software. The simulation result shows that the precision balance method based on cup-bottom trimming brings less additional natural frequency split in the static balance process, minimizes the natural frequency split to mHz grade and rectify the angle of mode offset to 0.1° grad in the dynamic balance process, furthermore, the method decreases the requirement for control accuracy of trimming quantity evidently. The research work provides references for structure optimization design and balance process plan of the cupped wave gyro.

Key words: cupped wave gyro, precision balance, trimming, modal analysis, natural frequency split

1 Introduction

The cupped wave gyro is a novel solid-state wave gyro, which senses the angular rate based on the inertia effect of the elastic wave in the cupped resonator. The cupped wave gyros have high operation accuracy, low cost, and good shock and vibration resistance as other solid-state wave gyros do[1], moreover, the cupped structure simplifies the fabrication of the resonator[2]. The innalabs holding's Coriolis vibration gyroscope(CVG) and the Watson industries' vibrating structure gyroscope(VSG) are typical cupped wave gyros, and they are ideal for applications in a wide range of projects including stabilization, control and instrumentation for military and civilian projects worldwide[3–4].

The fabrication error of the resonator always destroys the symmetry of the resonator structure and leads to the splitting of natural frequencies and the offset of mode, which is the main quadrature error source of the gyro[5]. The mechanical balance on the resonator could minimize the splitting of natural frequencies and rectify the offset of mode, which is a key process to improve the performance of the gyro. The conventional mechanical balancing procedure generally involves the removal or addition of mass, for the cupped wave gyros, it is based on the cup-wall trimming, to be exact, trimming the grooves on the top of the cupped resonator[6–7]. In recent research, we find that trimming off a tiny quantity of material from the cup-wall causes large variation of the natural frequency. Therefore, it is necessary to control the trimming quantity accurately in the balance process. However, in the precision dynamic balance process, minimizing the natural frequency split to mHz grade relies on extremely high trimming precision, which needs special equipments and technology. All the above conditions will increase the production cost and decrease the production efficiency.

In this paper, at first, the work principle of the cupped wave gyro is introduced and the lumped mass dynamic model of the resonator is built. By analyzing the effects of different position trimming on the natural frequency, a

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precision balance method based on cup-bottom trimming for cupped wave gyro is presented. Then, with the help of the finite element analysis(FEA) software ANSYS, we simulate the static balance and dynamic balance process of a cupped resonator with fabrication errors by the precision balance method. The simulation result shows that the precision balance method based on cup-bottom trimming could get higher balance accuracy with lower trimming accuracy, and the splitting of natural frequencies could reach mHz grade. The conclusions of this paper could provide reference for structure optimization design and balance process plan of the cupped resonator.

2 Principle of Precision Balance Method

2.1 Work principle of the cupped wave gyro

The basic structure of the cupped wave gyro is presented in Fig. 1, which comprises a circular plate (cup-bottom), a cylindrical shell (cup-wall) of stepped thickness, and a rigid substrate. The active and sense piezoelectric electrodes are attached to the cup bottom, the overall structure is axisymmetrical about z axis.

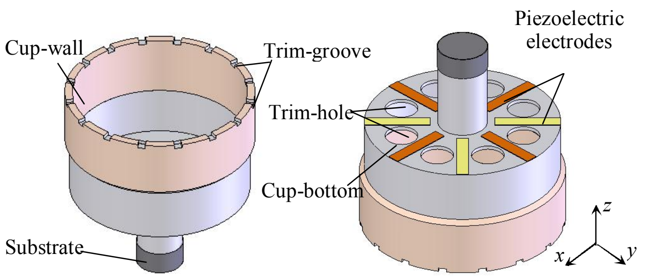
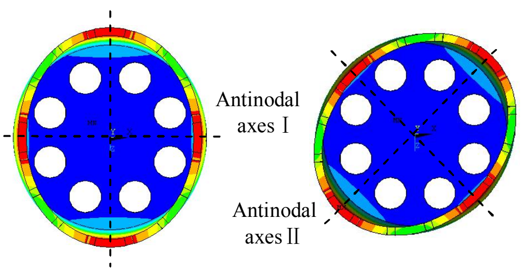


Fig. 1. Basic structure of the cupped wave gyro

According to the work principle of cupped wave gyro mentioned in Ref. [8] and other angular rate gyros[6], we know that the resonator vibration is activated by the piezoelectric electrodes. The active mode is shown in Fig. 2(a). When the gyro is rotating about z-axis, the sense mode is induced by the Coriolis force, as shown in Fig. 2(b), and piezoelectric electrodes sense the vibration and output signal proportional to angular rate.



(a) Active mode (b) Sense mode

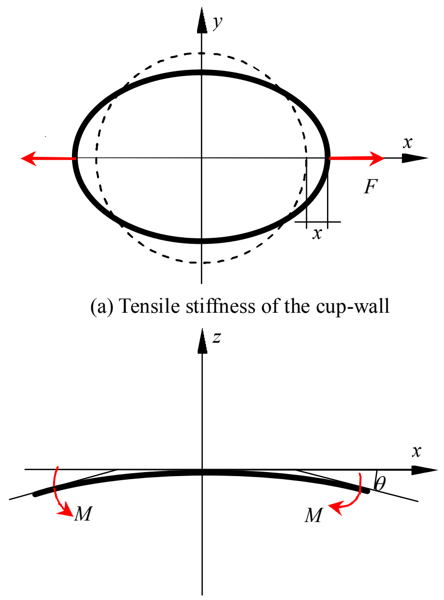
Fig. 2. Cupped wave gyro's work modes

The active mode and the sense mode are two work modes of the gyro. The work modes of a perfect resonator have the same natural frequencies, and the mode shapes are standing waves with four antinodes. Furthermore, the antinodal axes are coincident to the piezoelectric electrodes.

2.2 Natural frequency sensitivity to trimming position

The fabrication error of the resonator causes the natural frequency split (the split of the two work mode frequencies) and the mode offset (the offset of antinodal axes from electrodes), which is the main quadrature error source of the gyro. The aim of mechanical balance is to minimize the natural frequency split and the mode offset to the adjustable range of the electrical balance[9–10], which is the key process to improve the gyro's performance.

In order to plan the mechanical balance procedure effectively, firstly, we must analyze the effect of different position trimming on the natural frequency of the resonator. According to the characteristics of the cupped resonator structure and the mode shape, we can build the lumped mass dynamic model of the cupped wave gyro by discretization modeling method[11]. First, the bending angle θ of the cup-bottom is defined as the generalized displacement, the drive moment M of the piezoelectric electrodes is defined as the generalized force, besides, k_t is defined as the tensile stiffness of the cup-wall, k_b is defined as the bending stiffness of the cup-bottom. The principle of stiffness definition is shown in Fig. 3.



(a) Tensile stiffness of the cup-wall

(b) Bending stiffness of the cup-bottom

Fig. 3. Principle of stiffness definition

Then we can simplify the resonator to a one-DOF system of second-order, as shown in Fig. 4. By neglecting the damp and the generatrix bending of the cup-wall, the simplified dynamic model of the gyro can be expressed as follows:

$$\ddot{\theta}\left[m_2^*\frac{R^2}{4}+m_1^*\left(\frac{R^2}{4}+H^2\right)\right]+\theta(k_b+k_tH^2)=M, \quad (1)$$

where m_1^* is the lumped mass of thick portion of the cup-wall (neglecting the mass of the thin portion), m_2^* is the lumped mass of the cup-bottom. H is the distance from the center of thick portion of the cup-wall to the cup-bottom, and R is the radius of the resonator.

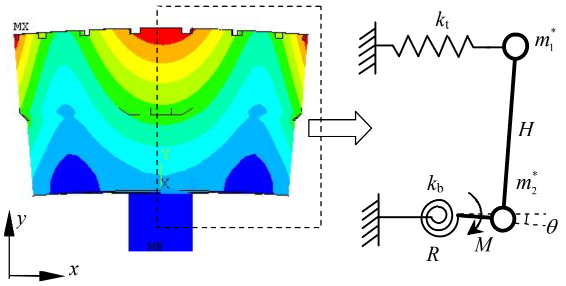


Fig. 4. Lumped mass dynamic model of the cupped wave gyro

The natural frequency of the above model can be expressed as

$$\omega=\sqrt{\frac{K^*}{m^*}},\tag{2}$$

where K^* is the equivalent stiffness of the system, m^* is the equivalent mass of the system, and

$$K^*=k_tH^2+k_b,\quad m^*=m_1^*\left(\frac{R^2}{4}+H^2\right)+m_2^*\frac{R^2}{4}.$$

On the basis of the static analysis of the cylindrical shell in the theory of plate and shell[12], when a pair of symmetrical force F is effected on a cylindrical shell, as shown in Fig. 3(a), the largest tensile displacement is about $x=0.149[FR^3(1-\mu^2)]/(4EI_1)$. Then the tensile stiffness can be expressed as follows:

$$k_t=\frac{F}{x}=\frac{2.24Eb_1z_1^3}{R^3(1-\mu^2)},\tag{3}$$

where E is the Young's modulus, μ is the Poisson's ratio, b_1 is the width, and z_1 is the thickness of the thick portion of the cup-wall.

In order to analyze the stiffness of the cup-bottom conveniently, we could consider the structure between the circular holes on the cup-bottom as cantilever beams. On the basis of the static analysis of cantilever beam in the mechanics theory of materials[13], when a moment M is effected on a cantilever beam, as shown in Fig. 3(b), the bending angle is about $\theta=MR/(EI_2)$. Then the bending stiffness can be expressed as follows:

$$k_b=\frac{M}{\theta}=\frac{Eb_2z_2^3}{12R},\tag{4}$$

where b_2 is the width and z_2 is the thickness of the equivalent beam of the cup-bottom.

For the mechanical balance, the trimming process is to remove a tiny quantity of material from the designated position of the resonator. This tiny quantity of material affects the whole mass little, but leads to local stress concentration and affects significantly on the stiffness of the resonator. Therefore, while analyzing the effect of trimming quantity on the variation of the natural frequency, we can neglect the variation of the whole mass but pay more attention on the variation of stiffness of the local structure.

While cup-wall trimming, the natural frequency sensitivity to the mass of trimmed material can be deduced as follows:

$$\begin{aligned}S_1&=\frac{\partial\omega}{\partial m_1}=\frac{\partial\omega}{\partial k_t}\frac{\partial k_t}{\partial m_1}=\left(\frac{K^*}{m^*}\right)^{-\frac{1}{2}}\frac{\Delta k_t}{\Delta b_1}\frac{\Delta b_1}{\Delta m_1}H^2=\\&\left(\frac{K^*}{m^*}\right)^{-\frac{1}{2}}\frac{\partial k_t}{\partial b_1}\frac{\Delta b_1}{\Delta b_1z_1a\rho}H^2=\left(\frac{K^*}{m^*}\right)^{-\frac{1}{2}}\frac{2.24Ez_1^2}{R^3(1-\mu^2)}\frac{H^2}{a\rho}.\end{aligned}\tag{5}$$

While cup-bottom trimming, the natural frequency sensitivity to the mass of trimmed material can be deduced as follows:

$$\begin{aligned}S_2&=\frac{\partial\omega}{\partial m_2}=\frac{\partial\omega}{\partial k_b}\frac{\partial k_b}{\partial m_2}=\left(\frac{K^*}{m^*}\right)^{-\frac{1}{2}}\frac{\Delta k_t}{\Delta b_2}\frac{\Delta b_2}{\Delta m_2}=\\&\left(\frac{K^*}{m^*}\right)^{-\frac{1}{2}}\frac{\partial k_t}{\partial b_2}\frac{\Delta b_2}{\Delta b_2z_2a\rho}=\left(\frac{K^*}{m^*}\right)^{-\frac{1}{2}}\frac{Ez_2^2}{12R}\frac{1}{a\rho}.\end{aligned}\tag{6}$$

In Eqs. (5) and (6), the trimming mass from the cup-wall is $\Delta m_1=\Delta b_1z_1a\rho$, the trimming mass from the cup-bottom is $\Delta m_2=\Delta b_2z_2a\rho$, where a is the width of the trimming tool.

While $\Delta m_1=\Delta m_2$, we can get the ratio of the two sensitivities by Eqs. (5) and (6):

$$\frac{S_1}{S_2}=\frac{12(1-\mu^2)}{2.24}\left(\frac{H}{R}\right)^2\left(\frac{z_1}{z_2}\right)^2.\tag{7}$$

Referring to the structure parameters of the usual cupped wave gyros[8–9], the ratio of H to R is about 1–1.5, the ratio of z_1 to z_2 is about 2–4, and the Poisson's ratio μ is about 0.1–0.3. According to Eq. (7), the ratio of S_1 to S_2 is about 20–100, which proves that the natural frequency sensitivity to cup-wall trimming is much greater than the sensitivity to cup-bottom trimming.

Let a constant definition of trimming mass, trimming on grooves of cup-wall will cause larger quantity of natural frequency variation, and trimming on holes of cup-bottom will obtain higher definition of natural frequency variation.

2.3 Precision balance method based on cup-bottom trimming

The same as all the solid-state gyros, the most common mechanical balance process of the cupped wave gyro is trimming on specific positions of the resonator, and the process includes static balance and dynamic balance. The aim of static balance process is to eliminate the centroid eccentricity and the aim of the dynamic balance is to minimize the natural frequency split and to rectify the mode offset. There are always additional natural frequency split in the static balance process, therefore, the static balance process should be done before the dynamic balance process[6], and the trimming quantity in the dynamic balance process should be distributed along the central axis symmetrically.

According to the deduction in section 2.2, we know that trimming off a tiny quantity of material from the cup-wall causes large variation of the natural frequency of the resonator. For the conventional mechanical balance method based on cup-wall trimming, in the static balance process, trimming on cup-wall causes large additional natural frequency split, in the dynamic process, trimming on cup-wall can minimize the natural frequency split to a great extent, but high balance precision always requires extremely high trimming precision based on special equipments and technology.

For the sake of reducing the fabrication cost and improving the mechanical balance precision, a precision balance method for cupped wave gyro based on cup-bottom trimming is proposed in this paper. This method could get higher mechanical balance precision and efficiency with lower trimming definition, including three key solving procedures as follows.

(1) Static balance. Trimming on the holes of the cup-bottom to eliminate the centroid eccentricity, and bring less additional natural frequency split.

(2) Primary dynamic balance. Trimming on the grooves of the cup-wall to minimize the majority of natural frequency split.

(3) Precision dynamic balance. Trimming on the holes of the cup-bottom to eliminate the residual natural frequency split and to rectify the mode offset.

3 Simulation of Precision Balance Method

For the complexity of the resonator structure and the balance process, we could only analyze the effect of different position trimming on the natural frequency of the resonator qualitatively by the lumped mass dynamic model, but could not calculate the variation of the natural frequency and the mode offset in balance process by analytical method.

Therefore, in this section, with the help of the finite element software ANSYS, we simulate the precision balance method based on cup-bottom trimming, calculate the variation of the natural frequency and the mode offset quantitatively, evaluate the validity of this balance method and plan a feasible mechanical balance process for the cupped resonator.

The flow chart of the simulation is shown in Fig. 5.

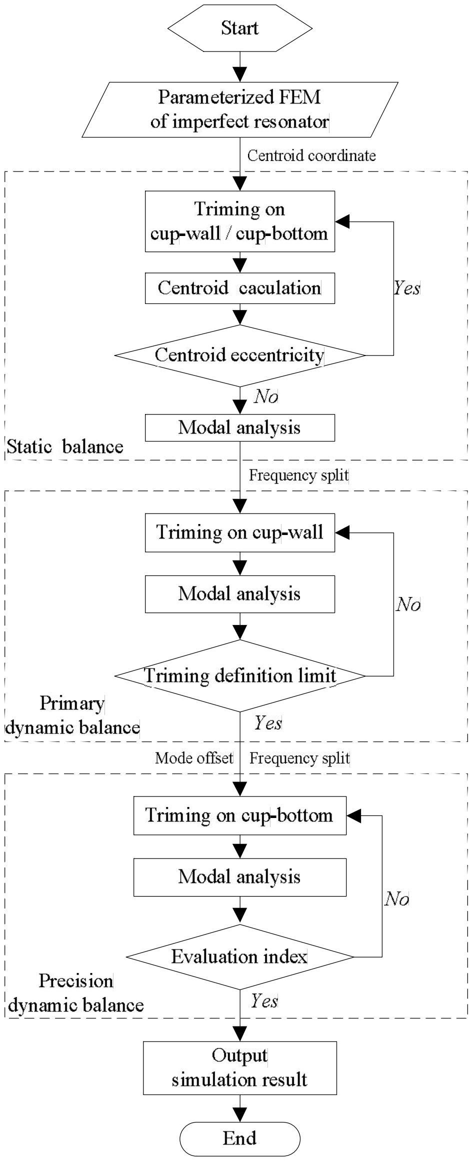


Fig. 5. Flow chart of the simulation

3.1 Finite element model and modal analysis of the imperfect cupped resonator

The parameterized 3D model of the cupped resonator with form and position errors is built in the software ANSYS, including sixteen grooves on cup-wall and eight holes on cup-bottom. Table 1 describes the geometry parameters. The roundness error of inner surface is 0.05 mm, the roundness error of outer surface is 0.005 mm, and the coaxial error is 0.05 mm.

Table 1. Geometry parameters of imperfect resonator

Parameter	Value
Height & radius of cup-wall H, R_1/mm	18, 12.5
Height & thickness of thicker portion of cup-wall b_1, h_1/mm	10, 1

Radius & thickness of cup-bottom R_2, h_2/mm	12.8, 0.3
Width & depth of grooves a, b/mm	1, 0.5
Radius of holes r/mm	2.25

The finite element model(FEM) is built by using sweep method, as shown in Fig. 6. The properties of FEM are shown in Table 2.

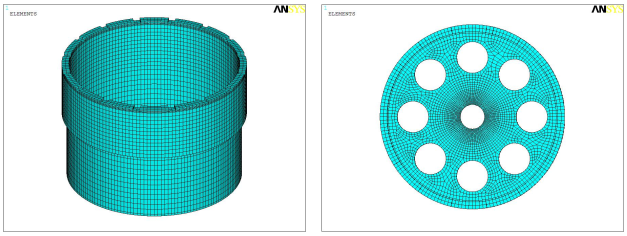


Fig. 6. FEM of the cupped resonator

Table 2. FEM properties of imperfect resonator

Parameter	Value
Young's modulus E/GPa	210
Poisson' s ratio μ	0.28
Density ρ/(kg · m^{-3})	7 780
Mass m/g	7.75
Centroid coordinate (x_0, y_0)/(mm, mm)	(0.057, 0)
Type of element	Solid45

We define the all-DOF constraints on substrate of the cupped resonator and make modal analysis on the FEM of the resonator by using Block Lanczo method. According to the solution result, the natural frequency of active mode and sense mode are 4 185.722 Hz and 4 186.679 Hz, the natural frequency split is 0.975 Hz, the angle of mode offset is about 4.9°. The modal contour is shown in Fig. 7. On the basis of the effect of natural frequency split and mode offset on control system, and refer to the evaluation index of a Russian solid-state wave gyro[6,14], we set the target of the mechanical balance as follows.

(1) Natural frequency split less than 0.01 Hz;

(2) Mode offset angle less than 1°.

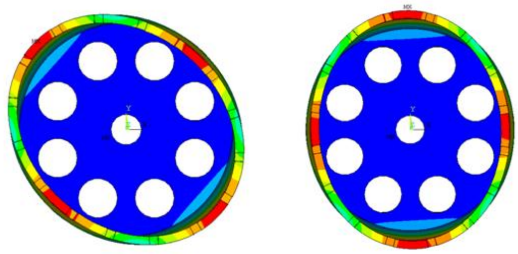
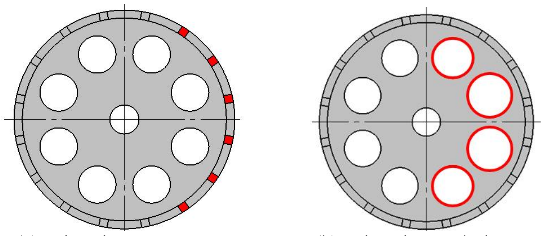


Fig. 7. Modal contour of the imperfect resonator before mechanical balance

3.2 Static balance and modal analysis of the imperfect cupped resonator

The trimming position of static balance should be chosen in the same side with the centroid (0.057 3, 0), and the trimming mass should be distributed separately to avoid large variation of local stiffness.



(a) Trimming on grooves (b) Trimming on holes

Fig. 8. Schematic view of two static balance methods

In this section, we simulate the static balance process by trimming grooves on cup-wall and trimming holes on cup-bottom separately, the simulation method in ANSYS is to vary the values of the grooves' depth and the holes' radii. The trimming positions are marked in Fig. 8. The modal analysis is applied on the FEM of the resonator, after static balance. The solution result is laid out in Table 3.

Table 3. Comparison of two static balance methods

Course	Natural frequency f/Hz		Frequency variation Δf/Hz		Frequency split δf/Hz	Trimming mass Δm/mg	Centroid coordinate (x_0, y_0)/(mm, mm)
	Antinodal axes Ⅰ	Antinodal axes Ⅱ	Antinodal axes Ⅰ	Antinodal axes Ⅱ			
Before trimming	4 186.697	4 185.722	–	–	0.975	–	(0.057 3, 0)
Trimming on grooves	4 129.538	4 121.809	57.159	63.913	7.729	36.12	(0.000 6, 0)
Trimming on holes	4 184.917	4 183.797	1.780	1.925	1.120	53.13	(0.000 7, 0)

By analyzing the solution result, we know that, both of these two methods can eliminate the centroid eccentricity of the imperfect resonator. Trimming on grooves causes much larger natural frequency variation and larger natural frequency split than trimming on holes. Therefore, trimming on cup-bottom is an ideal method of static balance. The modal contour of the imperfect resonator after static balance is shown in Fig. 9.

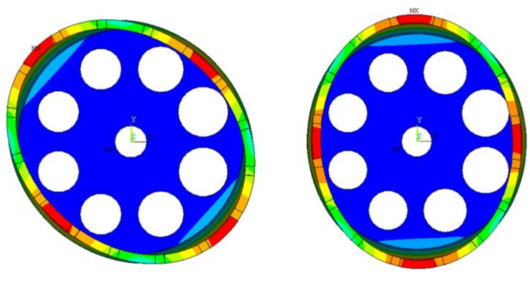


Fig. 9. Modal contour of the imperfect resonator

after static balance

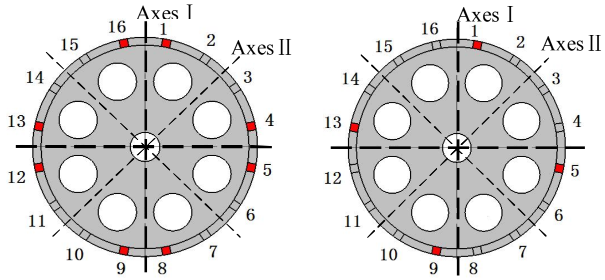
3.3 Primary dynamic balance and modal analysis of the imperfect cupped resonator

As mentioned in section 2.3, we select trimming on the grooves of the cup-wall as the primary dynamic balance method to minimize the major natural frequency split after static balance based on cup-bottom trimming. The grooves to be trimmed should be located near the antinodal axes of higher natural frequency. According to Table 3, the natural frequency of antinodal axes Ⅰ is 1.12 Hz higher than axes Ⅱ, therefore, the stiffness of antinodal axes Ⅰ should be decreased.

The particular procedures of the primary dynamic balance simulation are as follows.

(1) Trim on eight-groove for three times. Vary the depths of grooves 1, 4, 5, 8, 9, 12, 13 and 16 equally as one trimming procedure, as described in Fig. 10(a), and make modal analysis after every trimming procedure.

(2) Trim on four-groove for three times. Vary the depths of grooves 1, 5, 9 and 13 equally as one trimming procedure, as described in Fig. 10(b), and make modal analysis after every trimming procedure.



(a) Eight-groove trimming (b) Four-groove trimming

Fig. 10. Schematic view of primary dynamic balance

Table 4 shows the solution result of the primary dynamic balance simulation.

Table 4. Solution result of the primary dynamic balance simulation

Course	Natural frequency f/Hz		Frequency split	Trimming mass of each groove
	Antinodal axes Ⅰ	Antinodal axes Ⅱ	δf/Hz	Δm/μg
Before dynamic balance	4 184.917	4 183.797	1.120	–
Trim on eight-groove for the 1st time	4 182.197	4 181.931	0.266	185.55
Trim on eight-groove for the 2nd time	4 181.322	4 181.101	0.221	74.22
Trim on eight-groove for the 3rd time	4 180.805	4 180.743	0.062	26.69
Trim on four-groove for the 1st time	4 180.804	4 180.758	0.046	5.94
Trim on four-groove for the 2nd time	4 180.683	4 180.655	0.028	0.74
Trim on four-groove for the 3rd time	4 180.547	4 180.582	−0.035	0.74

By analyzing the solution result, we can see that, after trimming on eight-groove for three times, the natural frequency of antinodal axes Ⅰ has decreased more than antinodal axes Ⅱ, in other word, the natural frequency split has decreased. After trimming on eight-groove for three times, the natural frequency split has decreased further.

However, in the last two steps of the simulation, the natural frequency split drops from 0.028 Hz to −0.035 Hz, but the trimming mass is only about 0.74 μg. Given that the natural frequency split should be decreased to less than 0.01 Hz, the trimming mass must be controlled to 0.1 μg grade. With practical equipment and technology, trimming mass of 0.1 μg grade is almost impossible. The last two steps of the simulation are invalid in reality.

Therefore, trimming on grooves of the cup-wall could not meet the high precision requirements of the mechanical balance, but it can minimize the natural frequency split to a great extent, it is an ideal method of primary dynamic balance method.

After primary dynamic balance, the natural frequency split is about 0.046 Hz and the angle of mode offset is about 0.66°. The modal contour is shown in Fig. 11. The residual natural frequency split will be eliminated in precision dynamic balance process.

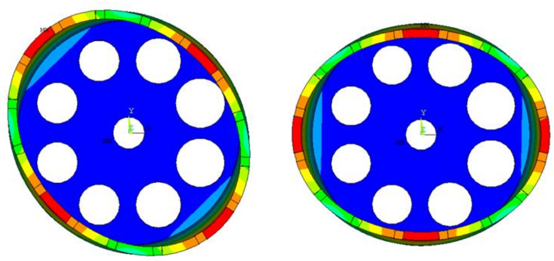


Fig. 11. Modal contour of the imperfect resonator after primary dynamic balance

3.3 Precision dynamic balance and modal analysis of the imperfect cupped resonator

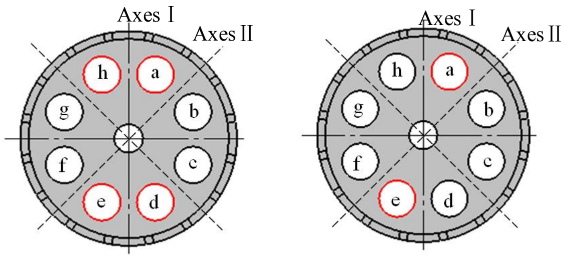
As mentioned in section 2.3, we select trimming on the holes of the cup-bottom as the precision dynamic balance method to eliminate the residual natural frequency split after primary dynamic balance based on cup-wall trimming. The holes to be trimmed should be located near the antinodal axes of higher natural frequency. According to Table 4, the natural frequency of antinodal axes Ⅰ is 0.046 Hz higher than axes Ⅱ, therefore, the stiffness of antinodal axes Ⅰ should be decreased.

The particular procedures of the precision dynamic balance simulation are as follows.

(1) Trim on four-hole for two times. Vary the radii of holes a, d, e, and h equally as one trimming procedure, as

described in Fig. 12(a), and make modal analysis after every trimming procedure.

(2) Trim on two-hole for two times. Vary the radii of hole a and e equally as one trimming procedure, as described in Fig. 12(b), and make modal analysis after every trimming procedure.



(a) Four-hole trimming (b) Two-hole trimming

Fig. 12. Schematic view of precision dynamic balance

Table 5 shows the solution result of the primary dynamic balance simulation.

Table 5. Solution result of the precision dynamic balance simulation

Course	Natural frequency f /Hz		Frequency split	Trimming mass of each hole
	Antinodal axes I	Antinodal axes II	δf/Hz	Δm/μg
Before precision dynamic balance	4 180.804	4 180.758	0.046	–
Trim on four-hole for the 1st time	4 180.496	4 180.485	0.011	1 718
Trim on four-hole for the 2nd time	4 180.449	4 180.454	−0.005	134
Trim on two-hole for the 1st time	4 180.434	4 180.438	−0.004	56
Trim on two-hole for the 2nd time	4 180.421	4 180.426	−0.005	56

By analyzing the solution result, we can see that, after trimming on four-hole for two times, the natural frequency split is 0.05 Hz, which can fulfill the evaluation index of the mechanical balance, but the angle of mode offset is about 2.1°, which should be minimized further by trimming on two holes. The modal contour of the resonator after trimming on four-hole for two times is shown in Fig. 13.

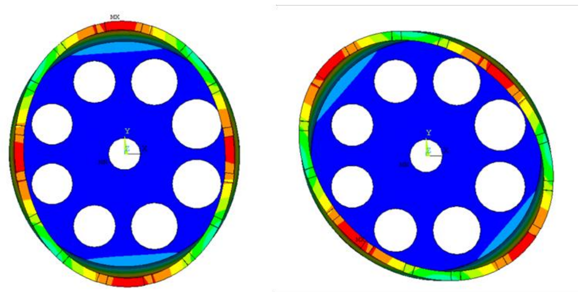


Fig. 13. Modal contour of the imperfect resonator after the second time trimming on four holes

When trimming on holes a and e, controlling the definition of trimming mass at 50 μg grade, the natural frequencies of the two antinodal axes vary without obvious difference. The natural frequency split is decreased to 0.004 Hz, the angle of mode offset is rectified to about 0.23°, as shown in Fig. 14.

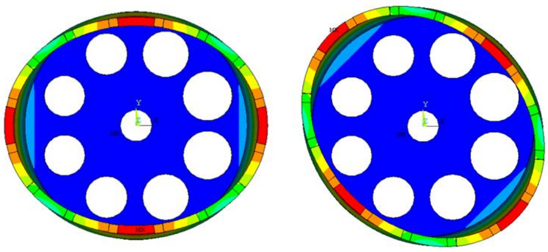


Fig. 14. Modal contour of the imperfect resonator after the precision balance

In the real mechanical balance process, the trimming mass can be controlled to 50 μg grade easily with universal equipment. Therefore, trimming on holes of the cup-bottom can fulfill the high precision evaluation index of the mechanical balance, which is an ideal method for precision dynamic balance.

4 Conclusions

(1) The effect of different position trimming on the natural frequency of the resonator can be analyzed qualitatively by the lumped mass dynamic model of the cupped resonator. The natural frequency sensitivity of cup-wall trimming is much greater than the sensitivity to cup-bottom trimming.

(2) For the static balance process, trimming on cup-wall causes much larger natural frequency variation and natural frequency split than trimming on cup-bottom. Therefore, trimming on cup-bottom is an ideal method of static balance.

(3) For the dynamic balance process, trimming on cup-wall (composed of eight-groove trimming and four-groove trimming) can minimize the natural frequency

split to 10^{-2} grade rapidly, which is an ideal method of primary dynamic balance. Trimming on cup-bottom (composed of four-hole trimming and two-groove trimming) can minimize the natural frequency split to 10^{-3} grade and rectify the mode offset to 0.1° grade without high trimming mass definition, which is an ideal method of precision dynamic balance.

(4) The simulation result proves that, the precision balance method for the cupped wave gyro based on cup-bottom trimming can remedy the low balance precision of the conventional mechanical balance method. The optimization procedures of mechanical balance for the cupped wave gyro are, firstly, static balance based on cup-bottom trimming, then, primary dynamic balance based on cup-wall trimming, and precision dynamic balance based on cup-bottom trimming at last.

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