Notes on dark energy interacting with dark matter and unparticle in loop quantum cosmology

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Abstract: We investigate the behavior of dark energy interacting with dark matter and unparticle in the framework of loop quantum cosmology. In four toy models, we study the interaction between the cosmic components by choosing different coupling functions representing the interaction. We found that there are only two attractor solutions namely dark energy dominated and dark matter dominated Universe. The other two models are unstable, as they predict either a dark energy filled Universe or one completely devoid of it.

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I. INTRODUCTION

Recent cosmological and astrophysical data gathered from the observations of supernovae SNe Ia [1], cosmic microwave background radiations via WMAP [2], galaxy redshift surveys via SDSS [3] and galactic X-ray [4] convincingly suggest that the observable Universe experiences an accelerated expansion phase. It is well-known that the simplest and elegant way to explain this behavior is the inclusion of Einstein's cosmological constant [5], however the two deep theoretical problems (namely the "fine-tuning" and the "coincidence" one) led to the notion of 'dark energy'. The dynamical nature (i.e. composition and origin) of dark energy, at least in an effective level, can arise from various scalar fields, such as a canonical scalar field (quintessence) [6], a phantom field [7], that is a scalar field with a negative sign of the kinetic term, or the combination of quintessence and phantom in a unified model named quintom [8]. One of the first quintom works which appeared is [9]. Recent review on dynamical DE from modified gravity perspective can be found in [10]. Also, there are other momentous works on dynamics of the scalar models for accelerating expansion phase of the universe with different methods and terminologies[11]. Recently, a new type of the scalar models have been investigated more than others [12].

One of the long-standing problems in the standard Big Bang cosmology is the initial singularity from which all matter and energy originated. Standard cosmology based on general relativity offers no resolution to this problem, however a recent quantum gravitational model of loop quantum gravity (LQG) offers a nice solution. The theory and principles of LQG when applied in the cosmological framework creates a new theoretical framework of loop quantum cosmology (LQC). The effects of loop quantum gravity can be described in two possible ways: the first one is based on the modification of the behavior of the inverse scale factor operator below a critical scale factor. This approach has been used to study quantum bounces, avoidance of singularities and to produce inflationary expansion [20]. A second approach is to add a term quadratic in density to the Friedmann equation. In LQC, the non-perturbative effects lead to correction term $-\rho^2/\rho_c$ to the standard Friedmann equation. With the inclusion of this term, the Universe bounces quantum mechanically as the energy density of matter-energy reaches the level of ρ_c (order of the Planck density). Thus the LQC is non-singular by producing a bounce before the occurrence of any potential singularity and hence transitions from a pre-Bang and after-Bang are all well-defined. The observational constraints due to the quadratic term in (1) are discussed in the literatures [21] where it is shown that the model with quadratic correction to density is consistent with the observational tests. Thus we should not worry on the solar system tests in this model. For the Universe with a large scale factor, the first type of modification (first approach) to the effective Friedmann equation can be neglected and only the second type of modification (second approach) is important [20]. Thus the dynamics of dark energy have been investigated recently in LQC using second approach [22].

In this paper, we address the problem of cosmic coincidence problem in a modern theoretical framework of loop quantum cosmology. Here we assume a non-minimal coupling between dark energy, dark matter and the unparticle component. Since the nature of dark energy and dark matter is still unknown, it is possible to have non-gravitational interactions between them. We are unsure of the form of the interaction, hence we introduce the interaction terms only phenomenologically. We construct a system of dynamical equations containing three equations for the three components. We convert them to dimensionless form and perform stability analysis. We construct four toy models and show that the dynamical equations have two possible attractor solutions i.e. dark matter dominated and dark energy dominated. Other

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dynamical systems are unstable i.e. one model predicts that everything decays and the Universe gets emptier void of energy. Another model predicts that the Universe will contain only dark energy.

The model in which dark energy interacts with two different fluids has been investigated in the literature. In [13], the two fluids were dark matter and another was unspecified. However, in another investigation [14], the third component was taken as radiation to address the cosmic-triple-coincidence problem and study the generalized second law of thermodynamics. In this article, we choose the third component as unparticle [15], following [16] thereby generalizing their study from the general relativistic cosmology to the loop quantum cosmology. An unparticle is based on the hypothesis that there could be exact scale invariant hidden sector resisted at a high energy scale. The fundamental energy scale of unparticle is far beyond the reach of today's accelerators, there is a possibility that this new unparticle sector could affect the low-energy phenomenology. An interesting feature of unparticle is that it does not have a definite mass and instead has a continuous spectral density as a result of scale invariance. Moreover, the equation of state of unparticle w_u is positive unlike dark energy and it interacts weakly with standard model particles. We consider the question how the evolution of Universe is affected when the unparticle takes part in the interaction with dark energy and dark matter.

This paper is organized as follows: in section II, we construct a cosmological scenario in which dark energy interact with dark matter and unparticle. Then we present the formalism of autonomous dynamical system which is suitable for phase space stability analysis. In section III, we study the stability of the dynamical system by choosing different coupling functions. Finally we briefly discuss our results in the last section.

II. OUR MODEL

Applying the techniques of loop quantum gravity to homologous and isotropic spacetime leads to the so-called loop quantum cosmology. Due to quantum corrections, the Friedmann equations get modified. The big bang singularity is resolved and replaced by a quantum bounce [17]. For a brief summary on loop quantum cosmology, see [18]. Considering quantum correction, the modified Friedmann equation turns out to be (in the case of $k = 0$) [18]

$$H^2 = \frac{\kappa^2}{3}\rho\Big(1 - \frac{\rho}{\rho_c}\Big), \tag{1}$$

where $\rho = \rho_m + \rho_d + \rho_u$, where ρ_m, ρ_d and ρ_u represent the energy densities of matter, dark energy and the radiation. Also $\rho_c \equiv \frac{\sqrt{3}}{16\pi^2\gamma^3}\rho_{\rm Pl}$ determines the loop quantum effects, $\rho_{\rm Pl}$ is the Planck density and γ is the dimensionless Barbero−Immirzi parameter. This parameter could be fixed as 0.2375 in order to give the area formula of black hole entropy in loop quantum gravity [19]. The observational constraints due to the quadratic term in (1) are discussed in the literature [21], where it is shown that the model with quadratic correction to density is consistent with the observational tests. Thus we should not worry on the solar system tests in this model.

Due to the corrected term in (1), the big bang singularity is replaced by a quantum bounce happening at ρ_c. The bounce is supposed to happen when the matter−energy density reaches the critical value ρ_c. However the numerical simulations show that modified Friedmann equations are valid in the whole cosmic evolution including the bounce [17].

Another FRW equation is

$$\dot{H} = -\frac{\kappa^2}{2}(\rho + p)\Big(1 - 2\frac{\rho}{\rho_c}\Big). \tag{2}$$

For a spatially flat Universe, the total energy conservation equation is

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{3}$$

where H is the Hubble parameter, ρ is the total energy density and p is the total pressure of the background fluid.

We assume a three component fluid containing matter, dark energy and unparticle having an interaction. The corresponding continuity equations are [16]

$$\begin{aligned} \dot{\rho}_d + 3H(\rho_d + p_d) &= \Gamma_1, \\ \dot{\rho}_m + 3H\rho_m &= \Gamma_2, \\ \dot{\rho}_u + 3H(\rho_u + p_u) &= \Gamma_3, \end{aligned} \tag{4}$$

which satisfy collectively (3) such that $\Gamma_1 + \Gamma_2 + \Gamma_3 = 0$.

We define dimensionless density parameters via

$$x \equiv \frac{\kappa^2\rho_d}{3H^2}, \quad y \equiv \frac{\kappa^2\rho_m}{3H^2}, \quad z \equiv \frac{\kappa^2\rho_u}{3H^2}. \tag{5}$$

Making use of parameters in (5) in the modified Friedmann equation (1) is

$$(x + y + z)\Big(1 - \frac{3H^2}{\kappa^2}\frac{x + y + z}{\rho_c}\Big) = 1. \tag{6}$$

Using (1) and (2), we can write

$$-\frac{\dot{H}}{H^2} = \frac{3}{2}(2 - x - y - z)\Big(1 + \frac{w_d x + w_u z}{x + y + z}\Big). \tag{7}$$

The equation of state parameter of the total fluid is

$$w_{\rm tot} = \frac{p}{\rho} = \frac{w_d x + w_u z}{x + y + z}. \tag{8}$$

The continuity equations (4) in dimensionless variables reduce to

$$
\begin{aligned}
x' &= 3x\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3x - 3w_d x + \frac{\kappa^2}{3H^3}\Gamma_1, \\
y' &= 3y\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3y + \frac{\kappa^2}{3H^3}\Gamma_2, \qquad (9)\\
z' &= 3z\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3z - 3w_u z + \frac{\kappa^2}{3H^3}\Gamma_3.
\end{aligned}
$$

Above the primes denote differentiation with respect to $N = \ln a$. The coupling functions Γ_i, $i = 1, 2, 3$ are in general functions of the energy densities and the Hubble parameter i.e. $\Gamma_i(H\rho_i)$. The system of equations in (9) is analyzed by first equating them to zero to obtain the critical points. Next we perturb (9) up to first order about the critical points and check their stability.

Following [16], one can consider various coupling functions to model the interaction. However all the models described here are phenomenological. There is, at yet, no deep theoretical justification for assuming this interaction in the absence of the quantum gravity. There has been a striking resurgence in modeling dark energy via such interacting models as they explain the astrophysical data with the desired accuracy. Moreover, in such models, one can obtain attractor solutions (stable equilibrium points against perturbation) at which the energy densities come at equilibrium and thereby explain cosmic coincidence. These models have very little fine tuning problems since these contain only one arbitrary coupling parameter. In literature, this parameter has been constrained via astrophysical data from supernovae, cosmic background radiation, baryon acoustic oscillations, gas mass fraction in galaxy clusters, the history of the Hubble function, and the growth function [23]. The signature of the coupling parameter is of central importance: its positive (negative) sign gives the direction of interaction.

There are some drawbacks as well for the interacting dark energy models: they are not clearly distinguishable from the non-interacting ones, in the light of the observational data [24]. Another drawback is that there is arbitrariness in the choice of the interaction term (i.e. products of Hubble parameter with the energy densities); there are models in which only the coupling parameter or the dark energy state parameter is taken as a function of scale factor to model interaction without employing the densities or the Hubble parameter [25].

III. ANALYSIS OF STABILITY IN PHASE SPACE

In this section, we will construct four models by choosing different coupling forms Γ_i and analyze the stability of the corresponding dynamical systems about the critical points. We shall plot the phase and evolutionary diagrams accordingly.

A. Interacting Model I

We consider the model with the following interaction terms

$$\Gamma_1 = -6bH\rho_x, \quad \Gamma_2 = \Gamma_3 = 3bH\rho_x, \qquad (10)$$

where b is a coupling parameter and we assume it to be a positive real number of order unity. Thus (10) says that both matter and unparticles have increase in energy density with time, while dark energy loses its energy density. Therefore, it is a decay of dark energy into matter and unparticle.

Using (10), the system (9) takes the form

$$
\begin{aligned}
x' &= 3x\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3x - 3w_d x - 6bx, \\
y' &= 3y\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3y + 3bx, \qquad (11)\\
z' &= 3z\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3z - 3w_u z + 3bx.
\end{aligned}
$$

There are three critical points:

- Point $A_1:\ (0,0,1)$,
- Point $B_1:\ (0,1,0)$,
- Point $C_1 = \Big(\frac{(w_d+2b)(w_d+2b-w_u)}{w_d(w_d+2b-w_u)+w_u b},$ $-\frac{b(w_d+2b-w_u)}{w_d(w_d+2b-w_u)-w_u b}, -\frac{b(w_d+2b)}{w_d(w_d+2b-w_u)-w_u b}\Big)$.

The eigenvalues of the Jacobian matrix for these critical points are:

- Point $A_1 : \lambda_1 = w_u - w_d - 2b, \lambda_2 = w_u,$ $\lambda_3 = -1 - w_u$,
- Point $B_1 : \lambda_1 = -(w_d + 2b), \lambda_2 = -1, \lambda_3 = -w_u$,
- Point $C_1 : \lambda_1 = -(1 + w_d + 2b)$, $\lambda_2 = -w_d\Big(\frac{(w_d+2b)(w_d+2b-w_u)}{w_d(w_d+2b-w_u)+w_u b}\Big)$, $\lambda_3 = -\Big(\frac{(w_d+b)(w_d-w_u)}{w_d} + b\Big)$.

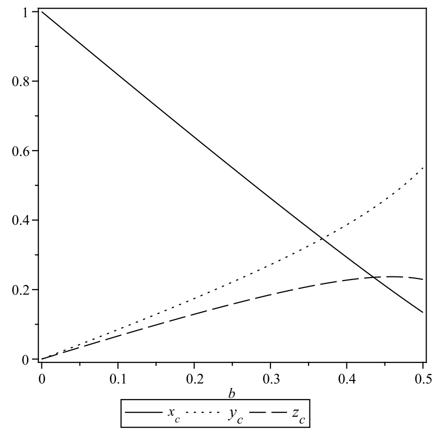

FIG. 1: Variety of x_c, y_c and z_c with b at the critical point C_1 for fixed $w_d = -1.2$ and $w_u = 0.28$. Here the coupling constant b is located in the region $w_d < -b$.

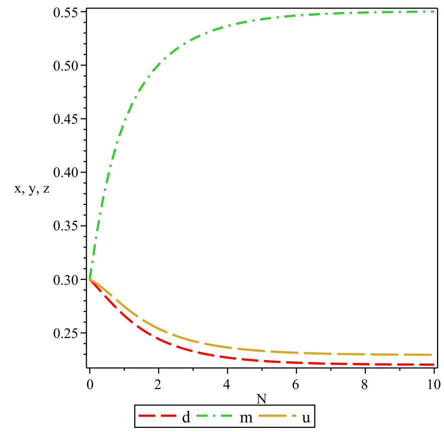

FIG. 2: Variety of x, y, z as a function of the $N = \ln(a)$. The initial conditions chosen are $x(0) = y(0) = z(0) = 0.3$, $w_u = 0.28$ and $w_d = -1.2$ and $b = 0.5$. We observe that the dominant part of in Model I is matter.

For the point A_1, the eigenvalue λ_2 is non-negative, which indicates that A_1 is not a stable point. For the point B_1, we also find that both of the eigenvalues λ_1 and λ_3 are negative when $w_d > -2b$. Therefore the point B_1 is the stable point. For the point C_1 , when $w_d < -b$ and $w_d < w_u$, all of the eigenvalues λ_1, λ_2 and $\lambda_3 2$ are negative, which means that C_1 is a stable point. As figure 1 shows below, the stable critical point C_1 gives us the intuitive picture where the x_c (representing DE) decreases while y_c (representing matter) and z_c (representing unparticle) increases. Moreover, the situation arises where matter dominates asymptotically and unparticle density comes after matter. Since C_1 is an attractor solution, it implies that the dynamical equations (9) yield this behavior for generic initial conditions. It is interesting to note that similar situation arises in Einstein's gravity as well [16]. The stable region of C_1 is not affected by the EoS of the unparticle, but the position of C_1 in the phase space is decided together by w_d, w_u and the coupling constant b. From (8), we also learn that the effective total EoS at point C_1 is

$$\begin{aligned} w_{\rm tot} &= \Big[\frac{w_d(2\,b(w_d+2\,b-w_u)+w_d(w_d+2\,b-w_u))}{w_d(w_d+2\,b-w_u)+bw_u} \\ &\quad -\frac{w_u b(2\,b+w_d)}{w_d(w_d+2\,b-w_u)-bw_u}\Big] \\ &\quad \times\Big[\frac{2\,b(w_d+2\,b-w_u)+w_d(w_d+2\,b-w_u)}{w_d(w_d+2\,b-w_u)+bw_u} \\ &\quad -\frac{b(w_d+2\,b-w_u)}{w_d(w_d+2\,b-w_u)-bw_u} \\ &\quad -\frac{b(2\,b+w_d)}{w_d(w_d+2\,b-w_u)-bw_u}\Big]^{-1} \end{aligned} \tag{12}$$

The critical point C_1 denotes that the DE, DM and unparticle can be coexisted in the late-times of the Universe. Figure (2) shows the evolution of the functions (x, y, z) for a variety set of parameters . As we observe, for a set of density parameters $w_d = -1.2, w_u = 0.28, b = 0.5$, matter is the dominant portion and the unparticle's density always remain below the dark energy. One interesting feature is that for large values of scale factor (the smaller values of the redshift z) the two different parts of the matters in the Universe (unparticle and dark energy) tend to the same value and after reaching to this point, the dominant part is the matter field.

Now we change the parameters to a new set $w_d = -1.7$, $w_u = 0.35$, $b = 0.25$. The behavior of the functions (densities) are very different. Now, if the evolution begins from a very large negative red shift, the dominant part of the model for all times is the dark energy. As the previous case, the unparticle's portion remains under dark energy and matter. For some values of large scale factor, the three different parts of the matters fields (dark energy, matter, unparticle) reach to the same asymptotic's value. These evolutionary scheme has been shown in figure (3) As we observe in figure (4), for a set of density parameters $-1 < w_d < -\frac{1}{3}, w_u > 0, b = 0.5, w_{tot} > 0$, matter is the dominant portion and the unparticle's density always remain below the dark energy. It is consistent with the result shown in figure (2) in which as the coupling is strong enough the Universe will be dominated by DM. Figure (5) shows the phase diagram of interacting dark energy with DM and unparticle in loop quantum cosmology through the coupling terms. The point C_1 is the critical point. Here we choose the values $w_d = -1.7, w_u = 0.28, b = 0.5$ in the stable region $w_d < -b$ and $w_d < w_u$.

B. Interacting Model II

We study another model with the choice of the interaction terms

$$\Gamma_1 = -3bH\rho_d,\ \ \Gamma_2 = 3bH(\rho_d - \rho_m),\ \ \Gamma_3 = 3bH\rho_m. \tag{13}$$

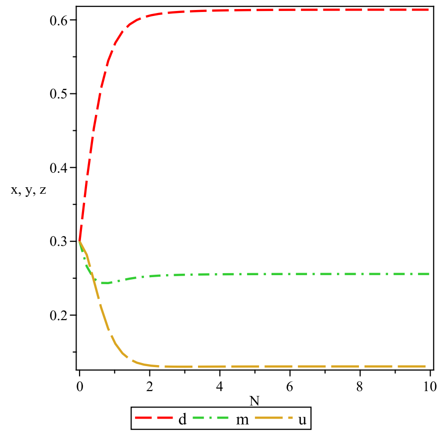

FIG. 3: Variety of x, y, z as a function of the $N = \ln(a)$. As we observe, the dominant part of the model is dark energy. The initial conditions chosen are $x(0) = y(0) = z(0) = 0.3$, $w_u = 0.28$ and $w_d = -1.2$ and $b = 0.5$.

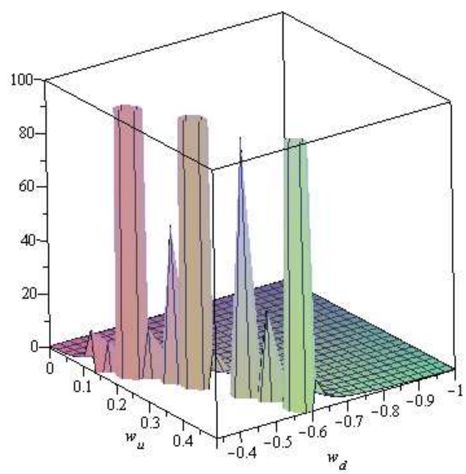

FIG. 4: Variety of w_{tot} as a function of the w_d, w_u. AS we observe, $w_{\rm tot} > 0$.

This model effectively describes the situation when dark energy loses energy density to matter while the unparticle density increases due to interaction with the matter.

$$\begin{aligned} x' &= 3x\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\ &\quad -3x - 3w_d x - 3bx, \\ y' &= 3y\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\ &\quad -3y + 3b(x-y), \qquad (14) \\ z' &= 3z\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\ &\quad -3z - 3w_u z + 3by. \end{aligned}$$

There are three critical points:

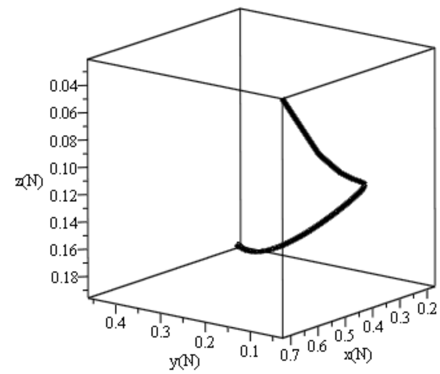

FIG. 5: The phase diagram of interacting dark energy with DM and unparticle in loop quantum cosmology through the coupling terms. Here we choose the values $w_d = -1.7, w_u = 0.28, b = 0.5$ in the stable region $w_d < -b$ and $w_d < w_u$.

- Point A_2 : $(0, 0, 1)$,
- Point B_2 : $(0, 1 - \frac{b}{w_u}, \frac{b}{w_u})$,
- Point $C_2 = \Big(\frac{w_d(b+w_d-w_u)}{w_d(w_d-w_u)+bw_u}, -\frac{b(w_d-b-w_u)}{w_d(w_d-w_u)+bw_u}, \frac{b^2}{w_d(w_d-w_u)+bw_u}\Big)$.

The eigenvalues of the Jacobian matrix for these critical points are:

- Point $A_2 : \lambda_1 = w_u - w_d - b, \lambda_2 = w_u - b$, $\lambda_3 = -(1 + w_u)$,
- Point $B_2 : \lambda_1 = -w_d, \lambda_2 = -(1 + b)$, $\lambda_3 = \frac{b}{w_u}(w_u - 1 - 2b) + b$,
- Point $C_2 : \lambda_1 = -(1 + w_d + b)$, $\lambda_2 = -w_d\Big(\frac{w_d(b+w_d-w_u)}{w_d(w_d-w_u)+bw_u}\Big)$, $\lambda_3 = -\Big(\frac{(w_d-b)(w_d-w_u)}{w_d}\Big)$.

For point A_2, the eigenvalue $\lambda_i < 0$ if $b > w_u - w_d$ for $w_d < 0$, hence point A_2 is a stable critical point. The second critical point B_2 is unstable since $\lambda_1 > 0$. It is easy to see that the third critical point C_2 is stable when we have the next set of inequalities

$$-(1 + w_d) < b < w_d \qquad (15)$$

$$w_u - w_d < b, b < \frac{w_d}{w_u}(w_u - w_d) \qquad (16)$$

Figure (3) shows that all the three components start with equal energy densities, they evolve differently. Here the dark energy dominates over matter and unparticle at late times. The energy density of dark energy evolves

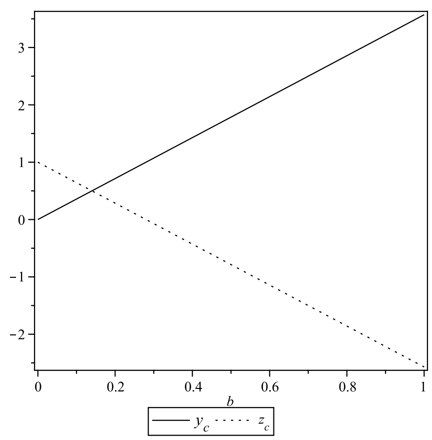

FIG. 6: Variety of x_c, y_c and z_c with b at the critical point B_2 for fixed $w_d = -1.2$ and $w_u = 0.28$.

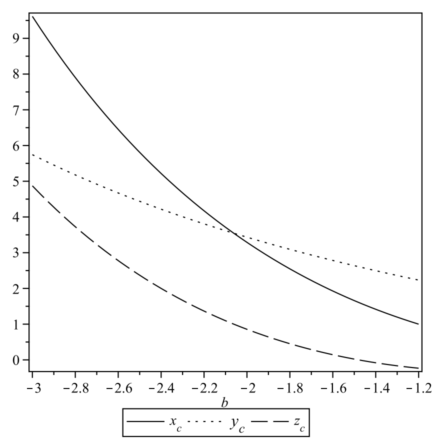

FIG. 7: Variety of x_c, y_c and z_c with b at the critical point C_2 for fixed $w_d = -1.2$ and $w_u = 0.28$. Here the coupling constant b is located in the region $b < w_d, w_d < w_u$.

to $x \sim 0.64$ and $y \sim 0.23$ compatible with the observations, while the unparticle density falls to zero at an early epoch $N \sim 2$. Clearly this phenomenological model of dynamical interaction explains the present state of the Universe. The next figure (8), shows, the phase diagram of interacting dark energy with DM and unparticle in loop quantum cosmology through the coupling terms . The point C_2 is the stable critical point. Here we choose the values $w_d = -1.7, w_u = 0.28, b = 0.5$ in the stable region $-(1+w_d) < b < w_d, w_u - w_d < b, b < \frac{w_d}{w_u}(w_u - w_d)$.

C. Interacting Model - III

The coupling terms containing the product of energy densities have been studied previously with the intent to reveal some new behavior of the dynamical systems [20].

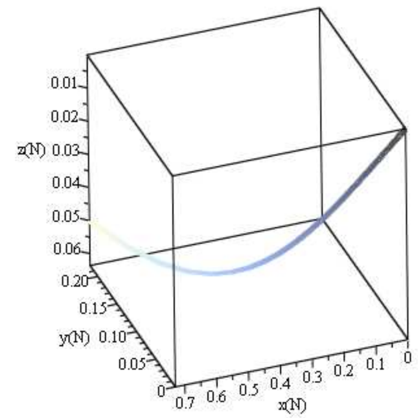

FIG. 8: The phase diagram of interacting dark energy with DM and unparticle in loop quantum cosmology through the coupling terms. Here we choose the values $w_d = -1.7, w_u = 0.28, b = 0.5$ in the stable region $-(1+w_d) < b < w_d, w_u - w_d < b, b < \frac{w_d}{w_u}(w_u - w_d)$.

Let us take the interaction terms [16]

$$\Gamma_1 = -6b\kappa^2 H^{-1}\rho_d\rho_u, \quad \Gamma_2 = \Gamma_3 = 3b\kappa^2 H^{-1}\rho_d\rho_u. \quad (17)$$

The system in (9) takes the form

$$\begin{aligned} x' &= 3x\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\ &\quad -3x - 3w_d x - 18bxz, \\ y' &= 3y\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\ &\quad -3y + 9bxz, \\ z' &= 3z\Big(1+\frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\ &\quad -3z - 3w_u z + 9bxz. \end{aligned} \quad (18)$$

There are five critical points:

- Point A_3 : $(1, 0, 0)$,
- Point B_3 : $(0, 1, 0)$,
- Point C_3 : $(0, 0, 1)$,
- Point D_3 : $\Big(\frac{1}{3}\frac{(1+w_u)}{b}, -\frac{1}{6}\frac{1+w_u+w_d+w_d w_u}{b}, -\frac{1}{6}\frac{1+w_d}{b}\Big)$,
- Point E_3 : $\Big(\frac{1}{3}\frac{w_u(w_d-w_u+6b)}{b(2w_d-w_u+6b)}$, $\frac{1}{3}\frac{-9w_u b + w_u^2 + 18b^2 + 9w_d b - 2w_d w_u + w_u^2}{b(2w_d-w_u+6b)}$, $-\frac{1}{3}\frac{w_d(w_d-w_u+3b)}{b(2w_d-w_u+6b)}\Big)$.

Now we must diagonalize the Jacobian matrix near these critical points. For points A_3, B_3, C_3 we have

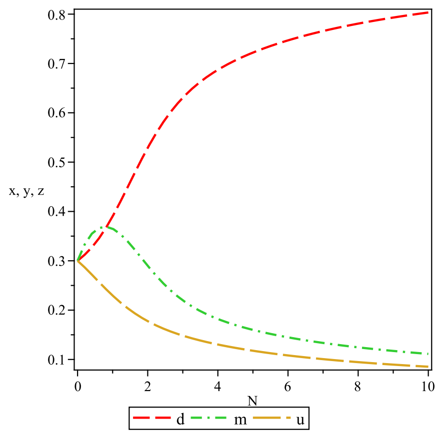

FIG. 9: Variety of x, y, z as a function of the $N = \ln(a)$. The initial conditions chosen are $x(0) = y(0) = z(0) = 0.3$, $w_u = 0.28$, $w_d = -1.2$ and $b = 0.5$.

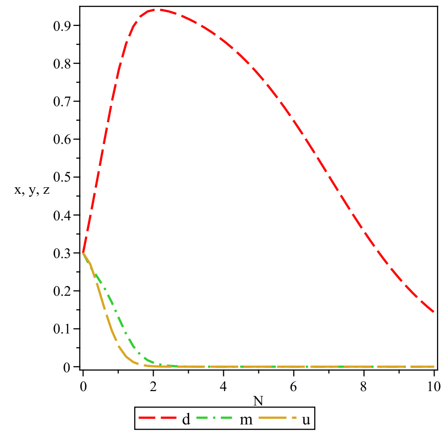

FIG. 10: Variety of x, y, z as a function of the $N = \ln(a)$. The initial conditions chosen are $x(0) = y(0) = z(0) = 0.3$, $w_u = 0.28$, $w_d = -1.2$ and $b = 0.5$.

- Point A_3 : $\lambda_1 = -3(1+w_d), \lambda_2 = 3w_d, \lambda_3 = 3(3b - w_u + w_d)$,
- Point B_3 : $\lambda_1 = -3, \lambda_2 = -3w_u, \lambda_3 = -3w_d$,
- Point C_3 : $\lambda_1 = -3(1 + w_u), \lambda_2 = 3w_u, \lambda_3 = 3(-6b + w_u - w_d)$,

For point A_3 since always $w_u > 0, w_d \leq -1$ thus this point is un stable. Similarly, the point B_3, C_3 both are unstable. The analysis of stability for points D_3, E_3 are so complicated. Indeed , the Jacobian matric in these cases, are not diagonal and the behaviors of the eigenvalues are not trivial. Theoretically, we cannot distinguish between these points as the stable or un stable points. Thus, it is computationally and analytically impossible to analyze these cases. Using the same initial conditions,Figure (9) shows that dark energy density rises while matter density dominates over dark energy and unparticle till $N \sim 1$. For $N > 1$, dark energy density rises indefinitely (behaving like phantom energy) while matter and unparticle density tends to zero.

D. Interacting Model - IV

Consider another model with the interaction terms [16]

$$\begin{aligned}
\Gamma_1 &= -3b\kappa^2 H^{-1}\rho_x\rho_u, \\
\Gamma_2 &= 3b\kappa^2 H^{-1}(\rho_x\rho_u - \rho_m\rho_u), \\
\Gamma_3 &= 3b\kappa^2 H^{-1}\rho_m\rho_u. \qquad (19)
\end{aligned}$$

The system in (9) takes the form

$$\begin{aligned}
x' &= 3x\Big(1 + \frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3x - 3w_d x - 9bxz, \\
y' &= 3y\Big(1 + \frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3y + 9b(xz - yz), \qquad (20) \\
z' &= 3z\Big(1 + \frac{w_d x + w_u z}{x+y+z}\Big)(2-x-y-z) \\
&\quad -3z - 3w_u z + 9byz.
\end{aligned}$$

There are six critical points:

- Point A_4 : $(1, 0, 0)$,
- Point B_4 : $(0, 1, 0)$,
- Point C_4 : $(0, 0, 1)$,
- Point D_4 : $\left(0, \frac{1}{3}\frac{(1+w_u)}{b}, -\frac{1}{3b}\right)$,
- Point E_4 : $\left(\frac{1}{3}\frac{-w_u+3b+w_d}{b}, \frac{1}{3}\frac{-2w_d w_u + w_u{}^2 + 3w_d b + w_d{}^2 - 3w_u b}{b(-w_u+3\,b)}, \frac{1}{3}\frac{w_d(w_d - w_u)}{b(-w_u+3b)}\right)$,
- Point F_4 : $\left(\frac{1}{3}\frac{w_d(1+w_u)}{b(1+w_d)}, \frac{1}{3}\frac{1+w_u}{b}, -\frac{1}{3}\frac{1+w_d}{b}\right)$.

Now we must diagonalize the Jacobian matrix near these critical points. For points A_4, B_4, C_4 we have

- Point A_4 : $\lambda_1 = -3(1 + w_d), \lambda_2 = 3w_d$, $\lambda_3 = 3(w_d - w_u)$,
- Point B_4 : $\lambda_1 = -3, \lambda_2 = -3w_d, \lambda_3 = 3(3b - w_u)$,
- Point C_4 : $\lambda_1 = -3(1 + w_u), \lambda_2 = 3(w_u - 3b)$, $\lambda_3 = 3(-3b + w_u - w_d)$.

For point A_4 since always $w_u > 0, w_d \leq -1$ thus this point is un stable, specially for the cross line $w_d = -1$ which in this case, the system is unstable near A_4. Similarly, the point B_4 is unstable. But the point C_4 is a stable point if $w_d < -1$, and $b > (w_u - w_d)/3$. The analyses of stability for points D_4 and E_4 are as complicated as the Model III. Indeed, the Jacobian matric in these cases, are not diagonal and the behaviors of the eigenvalues are not trivial. Theoretically, we cannot distinguish between these points as stable or unstable points. Thus, it is computationally and analytically impossible to analyze these cases. However, the behavior of the dynamical equations in (20) is plotted in Figure (10) which shows that dark energy density rises rapidly till $N \sim 2.5$ after it decreases sharply (behaving like quintessence), while matter and unparticle density decrease and approach to zero at $N \geq 2$. It is clear that dark energy does not decay into matter or unparticle, otherwise their densities would have increased. Hence the decay of dark energy into some mysterious component is not clear.

IV. DISCUSSION

In this paper, we have studied the dynamical behaviors when dark energy is coupled to dark matter and the unparticle in the background of flat FRW spacetime. The analysis was performed in the theoretical framework of loop quantum cosmology which is one of the emerging fields of quantum cosmology. It was assumed that the correction terms due to LQC remain benign and become effective near the Planck density. To address the coincidence problem, we introduced a phenomenological interaction between dark energy, dark matter and unparticle. We constructed four toy models: here Model I describes a stable attractor solution for a dark matter dominated Universe; Model II also contains a stable attractor solution for a dark energy dominated Universe; Model III and IV are dynamically unstable since either they give completely dark energy filled Universe or completely devoid of it.

Comparison of our LQC dynamical models with those of Einstein cosmology [16] reveals the following differences: the form of most of the critical points (and hence eigenvalues) are different, consequently the conditions and regions of stability of these critical points also differs from Einstein cosmology. Also a comparison of Fig. 1 above with Fig.2 of [16] shows that the evolution of energy densities is much slower in LQC than in Einstein's case, however, the asymptotic evolution is the same. Finally, unlike [16] where all the considered models were stable, we have only two stable scaling solutions, namely model I and II. In other words, LQC results are very different from Einstein's relativistic cosmology.

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