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Fluid-based Slotting Optimization for Automated Order Picking System with Multiple Dispenser Types

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Abstract: Slotting strategy heavily influences the throughput and operational cost of automated order picking system with multiple dispenser types, which is called the complex automated order picking system (CAOPS). Existing research either focuses on one aspect of the slotting optimization problem or only considers one part of CAOPS, such as the Low-volume Dispensers, to develop corresponding slotting strategies. In order to provide a comprehensive and systemic approach, a fluid-based slotting strategy is proposed in this paper. The configuration of CAOPS is presented with specific reference to its fast-picking and restocking subsystems. Based on extended fluid model, a nonlinear mathematical programming model is developed to determine the optimal volume allotted to each stock keeping unit (SKU) in a certain mode by minimize the restocking cost of that mode. Conclusion from the allocation model is specified for the storage modules of high-volume dispensers and low-volume dispensers. Optimal allocation of storage resources in the fast-picking area of CAOPS is then discussed with the aim of identifying the optimal space of each picking mode. The SKU assignment problem referring to the total restocking cost of CAOPS is analyzed and a greedy heuristic with low time complexity is developed according to the characteristics of CAOPS. Real life application from the tobacco industry is presented in order to exemplify the proposed slotting strategy and assess the effectiveness of the developed methodology. Entry-item-quantity (EIQ) based experiential solutions and proposed-model-based near-optimal solutions are compared. The comparison results show that the proposed strategy generates a savings of over 18% referring to the total restocking cost over one-year period. The strategy proposed in this paper, which can handle the multiple dispenser types, provides a practical quantitative slotting method for CAOPS and can help picking-system-designers make slotting decisions efficiently and effectively.

Key words: slotting, complex automated order picking system, restocking cost, dispenser

1 Introduction

Among the available solutions for split-case (or single-piece) order picking, automated dispensing systems with high throughput capacity are often preferred in the specialized applications such as tobacco, pharmaceutical or cosmetic distribution, where a high volume of small orders needs to be processed at a very fast pace with a high degree of accuracy and minimum support personnel. Complex automated order picking system (CAOPS) is a kind of these systems with two types of dispensers.

Total automation obviously involves relevant investments justified and, as a consequence, requires a focused managerial attention towards fully exploiting their utilization and operational performances in a cost-effective manner[1]. One of the most important decisions for CAOPS, which is generally referred to as slotting[2], is to decide which stock keeping unit (SKU) should be assigned to which automated picking mode (SKU assignment) and for how many dispensing channels or cartridges each SKU should be allotted in its assigned mode (SKU allocation)[3–4]. The slotting strategy made when setting up CAOPS heavily influences the number of orders the system can fulfill without intervention of the manual operator, thus affecting the order picking automation level which is the main performance measure. Furthermore, it also affects the utilization of the system replenishment operators, thus determining the staffing level and operational costs[5]. Inappropriate slotting strategy may cause an excessive frequency of operational stockouts which will increase the total operational cost and reduce the throughput of the whole system. Therefore, a need arises to develop an optimal slotting strategy for CAOPS to minimize its operational cost and improve the management level.

Although some research has been made about CAOPS

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referring to its picking efficiency optimization[6–9], there is limited research performed for its slotting optimization problem. CAPUTO, et al[5], developed a decision support system (DSS) to slot a CAOPS-like system used in pharmaceutical industry. The significant component of the DSS is the consideration of channel fill-levels which tends to be much higher than appropriate in practice. LIU, et al [3–4], presented an optimal slotting strategy for CAOPS over a single or multiple periods when the volume of the two picking mode is pre-determined. BARTHOLDI, et al[10], developed an optimal SKU allocation strategy, which is part of the slotting strategy, for small parts in the fast-picking area and made a comparison with equal-time and equal-space allocations that are commonly used in industry. LI[10] proposed an optimal SKU assignment strategy, which is another part of the slotting strategy, under the equal-space allocation assumption. As for A-frame Dispenser System which is part of the picking modes of CAOPS, a near-optimal slotting method was presented based on replenishment cost [2, 12] and a slotting heuristic was developed regarding to both labor and infrastructure costs[13]. ZHANG[14] proposed an items assignment optimization method for another CAOPS-like automated sortation system with double picking zones.

Existing research either focuses on one aspect of the CAOPS slotting optimization problem or only considers one part of CAOPS, such as low-volume dispensers, to develop corresponding slotting strategies. To the best of our knowledge, there is no literature addressing the two related aspects of the slotting optimization problem for the whole system, which is critical to the design and operation of CAOPS. Therefore, the problem is examined in this paper based on fluid model, and a more practical slotting strategy is proposed to minimize the total restocking cost of CAOPS with space constraints.

The reminder of this paper is organized as follows. Following a brief introduction of CAOPS in the order fulfillment area of the distribution center, with specific reference to its restocking subsystem, the SKU allocation model is developed based on the extended fluid model to minimize the restocking cost of a certain mode. Optimal allocation of storage resources in the fast-picking area is then discussed with the aim of identifying the optimal space of each picking mode in CAOPS. SKU assignment problem referring to the total restocking cost of CAOPS is analyzed in detail and a greedy heuristic with low time complexity is developed according to the characteristics of CAOPS. A real life application from the tobacco industry is then presented in order to exemplify the proposed slotting strategy and assess the effectiveness of the developed methodology.

2 Complex Automated Order Picking System

CAOPS, a kind of dispenser-based, conveyor-aided, single-piece order picking system, is mainly composed of three subsystems, namely restocking, fast-picking and packaging. Restocking subsystem ensures that on-hand inventory is always available for the fast-picking subsystem when needed. Fast-picking subsystem dispenses required number of pieces onto the conveyor according to the customer orders while packaging subsystem packs each order into one or more totes. We will briefly introduce the fast-picking subsystem and then focus on the restocking subsystem which is highly related with the slotting decision.

2.1 Fast-picking subsystem of CAOPS

Appropriate number of High-volume Dispensers and Low-volume Dispensers are combined together to serve as the fast-picking subsystem of CAOPS, as it is shown in Fig. 1. The combination configuration is relied heavily on the order structure and the space which is particularly valuable in the distribution center.

Fig. 1. Fast-picking subsystem of CAOPS

Both high-volume dispensers and low-volume dispensers are automated picking modes in which the picking is totally labor-free. However, during one dispensing cycle, a single high-volume dispenser can dispense one to five pieces onto the conveyor while a single low-volume dispenser can only dispense one piece. So the high-volume dispenser has higher picking efficiency than the low-volume dispenser.

2.2 Restocking subsystem of CAOPS

Restocking subsystem is used to maintain the daily operation of CAPOS. As shown in Fig. 2, it is a three-tier, five-mode inventory system[2].

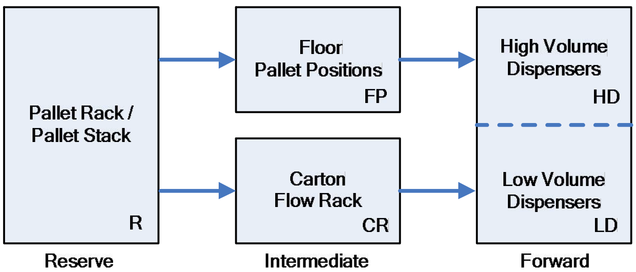

Fig. 2. Restocking subsystem of CAOPS

The first tier is the Reserve mode which provides the bulk stocks for the whole system and is denoted as mode R Pallet rack or pallet stack are often used in this mode.

Intermediate tier is established between the Reserve tier and Forward tier to reduce the cost of restocking the Forward tier and to ensure that restocks are close at hand when needed[2]. There are two separate storage modes in this tier. Floor pallet positions, denoted as mode FP, are the dedicated replenishment area for all high-volume dispensers. Carton flow rack, denoted as mode CR, is the dedicated replenishment area for all low-volume dispensers.

Forward tier is composed of the storage modules of high-volume dispensers and low-volume dispensers. Dispensing channels, denoted as mode HD, is the storage module for high-volume dispensers. Dispensing cartridges, denoted as mode LD, is the storage module for low-volume dispensers. Each SKU can be assigned to at most one mode in this tier. And each SKU should be allotted an integer number of channels or cartridges (at least one) in this tier.

During the picking process, manual replenishment of the channels or cartridges may be performed. Floor pallet positions and carton flow racks are normally located behind the channels and cartridges. Replenishers retrieve appropriate number of products from them to restock the required channels or cartridges. Once the floor pallet positions or carton flow racks are out of stock, replenishers need to retrieve a pallet of appropriate number of products from the pallet rack or pallet stack to restock the required mode in intermediate tier. For simplicity, we will focus on the restocking activity from mode R, mode FP and mode CR, without considering the picking activity from them.

2.3 Characteristics of CAOPS

High-volume dispenser is much more efficient than low-volume dispenser in terms of picking efficiency, storage capacity and restocking efficiency. But the trade-off is that it occupies more space and thus can deal with fewer kinds of SKUs. Therefore, appropriate number of high-volume dispensers and low-volume dispensers need to be combined together to utilize the advantages of both and satisfy the practical constraints such as space.

Another critical characteristic of CAOPS is its labor-intensive manual-restocking. Compared with mode FP and mode CR, the inventory level of each SKU in mode HD and mode LD is relatively low. In order to ensure the continuity of the picking process, which is very important to maintain the efficiency of the whole system, frequent manual restocking is required from intermediate tier to forward tier to sustain CAOPS.

3 Slotting Optimization for CAOPS

3.1 Estimating the restocking cost

From the analysis of section 2, it can be seen that CAOPS is maintained by restocking and the picking cost of it equals to zero. Furthermore, for simplicity, it is assumed that other costs that are related to the restocking and picking activities, such as the cost of equipment, management, support staff, and utilities are fixed with respect to the our decisions. Therefore, the operational cost of CAOPS is the total restocking cost of it.

The cost of restocking a single SKU in its assigned mode depends mainly on the number of times the SKU requires replenishment[12], which is also called the number of restocks, and the cost per restock.

Fluid model, first developed by BARTHOLDI[12], is a theory in which the number of restocks required per period for a SKU is determined by its rates of flow through the warehouse and its quantities stored in the fast-picking area. Bartholdi also concluded that fluid model, which treats each SKU as an incompressible, continuously divisible fluid, is more suitable for small parts (such as cartons, pieces).

Over the long term, the rate at which each SKU is restocked in its assigned mode must equal the rate at which the assigned mode request restocks of that SKU. So the number of restocks is also related with the restocking protocol that is when the restock occurs. Existing research has assumed that a SKU is restocked in a storage mode when the supply of the SKU there has been entirely depleted, with no allowance for safety stock [2, 12].

However, this assumption is very impractical. For example, during the whole order picking process, if one channel of the high-volume dispenser or one cartridge of the low-volume dispenser is stock out with the unfinished dispensing task, the whole system will stop until this channel or cartridge is replenished appropriately. This will definitely reduce the whole throughput of CAOPS. Things are even worse if multiple channels and cartridge are out of stock simultaneously. Same problem exists for floor pallet positions and carton flow racks since they are the dedicated replenishment areas for mode HD and mode LD, respectively.

Therefore, we set up safety stock for each SKU in its assigned mode and assume that the restocking activity for a SKU in a certain mode is triggered if the inventory level of that SKU there has decreased under the safety stock level. To be both usefully general and simple, it is also assumed that, when restock occurs, the required restocking quantity of a SKU equals to the maximum stock of that SKU minus the current stock of that SKU and can be carried in one trip from the corresponding predecessor tier instantaneously.

Based on the above analyses and assumptions, the classical fluid model can be extended and the number of restocks over a certain period is

$$\frac{f_i}{v_i^{\mathrm{M}} - s_i^{\mathrm{M}}},$$

where i—Index for SKU i, $i = 1, 2, \cdots, n$,

f_i—Flow of SKU i over a certain period,

$f_i = (\text{\# pieces sold / period}) \times \text{volume per piece}$,

v_i^{M} —Volume allotted to SKU i in mode M,

$\mathrm{M} = \{\mathrm{HD, LD, FP, CR}\}$,

s_i^{M} —Safety stock set up for SKU i in mode M,

$s_i^{\mathrm{M}} > 0$.

And f_i, v_i^{M} and s_i^{M} are measured by cubic meters.

Assume that the cost per restock is the same for all SKUs in the same mode, and let C_{M} denote this value for mode M. Then the cost of restocking a SKU in its assigned mode is the product of the cost per restock of that mode and the number of restocks required per period for that SKU. The cost of restocking a certain mode is sum of the cost of restocking all SKUs in that mode. And the total restocking cost of CAOPS is the sum of restocking cost for all SKUs in all modes.

3.2 Allotting space to each SKU

It is straightforward to think of allotting one channel to each SKU in mode HD and one cartridge to each SKU in mode LD. However, BARTHOLDI had proved that the Equal Space Allocation is not optimal[12]. Moreover, SKU is related with each other. As more channels are assigned to a certain SKU, number of restocks for that SKU will be reduced but there will be less available space for others at the same time.

Then what is the optimal number of channels or cartridges each SKU should be allotted in its assigned mode? The above question can be answered in two stages: the general SKU allocation model for mode M is proposed first; the conclusion is then specified for mode HD and LD.

Assuming that a certain set of SKUs has been assigned to mode M over a certain period, the fluid-based mathematical programming model minimizing the restocking cost of mode M with respect to v_i^{M} is as follows:

$$\begin{cases} \min \sum_{i\in S(\mathrm{M})} C_{\mathrm{M}} \dfrac{f_i}{v_i^{\mathrm{M}} - s_i^{\mathrm{M}}}, \\ \text{s.t.} \quad \sum_{i\in S(\mathrm{M})} v_i^{\mathrm{M}} \le V^{\mathrm{M}}, \\ \qquad v_i^{\mathrm{M}} > s_i^{\mathrm{M}},\ \forall i \in S(\mathrm{M}), \end{cases} \tag{1}$$

where $S(\mathrm{M})$ —Set of SKUs assigned to mode M,

V^{M} —Volume of mode M, V^{M} is measured by cubic meters.

Theorem 1. To minimize the total restocking cost of all the SKUs in mode M, the optimal volume allotted to each SKU $i \in S(\mathrm{M})$ is

$$v_i^{\mathrm{M}} = \left(\frac{\sqrt{f_i}}{\sum_{i\in S(\mathrm{M})}\sqrt{f_i}}\right)(V^{\mathrm{M}} - \sum_{i\in S(\mathrm{M})} s_i^{\mathrm{M}}) + s_i^{\mathrm{M}}.$$

Proof. We prove the first part by induction.

When $i=1$, there is only one SKU stored in and picked from mode M. In this case, $S(\mathrm{M}) = \{1\}$ and the result is straightforward.

Assuming that the result holds for k SKUs, we now consider the problem

$$\begin{cases} \min \sum_{i=1}^{k+1} C_{\mathrm{M}} \dfrac{f_i}{v_i^{\mathrm{M}} - s_i^{\mathrm{M}}} \\ \text{s.t.} \quad \sum_{i=1}^{k+1} v_i^{\mathrm{M}} \le V^{\mathrm{M}}, \\ \qquad v_i^{\mathrm{M}} > s_i^{\mathrm{M}},\ i = 1, 2, \cdots, k+1 \end{cases}$$

to see whether we can get

$$v_{k+1}^{\mathrm{M}} = \left(\frac{\sqrt{f_{k+1}}}{\sum_{i=1}^{k+1}\sqrt{f_i}}\right)\left(V^{\mathrm{M}} - \sum_{i=1}^{k+1} s_i^{\mathrm{M}}\right) + s_{k+1}^{\mathrm{M}}.$$

Let $v_i^{\mathrm{M}*}$ denote the optimal volume for SKU $i \in \{1, 2, \cdots, k\}$ and let $v = \sum_{i=1}^{k} v_i^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}}$. According to the inductive hypothesis, we have

$$v_i^{\mathrm{M}*} - s_i^{\mathrm{M}} = \frac{v\sqrt{f_i}}{\sum_{i=1}^{k}\sqrt{f_i}},\ \forall i \in \{1, 2, \cdots, k\}.$$

So the number of restocks of SKU $i \in \{1, 2, \cdots, k\}$ is

$$\frac{f_i}{(v_i^{\mathrm{M}*} - s_i^{\mathrm{M}})} = \frac{f_i}{v\sqrt{f_i} \Big/ \left(\sum_{i=1}^{k}\sqrt{f_i}\right)} = \frac{\sqrt{f_i}\left(\sum_{i=1}^{k}\sqrt{f_i}\right)}{v}.$$

And the total number of restocks of k SKUs is $\left(\sum_{i=1}^{k}\sqrt{f_i}\right)^2 \Big/ v$.

If the first k SKUs occupy the volume $v + \sum_{i=1}^{k} s_i^{\mathrm{M}}$, then the $(k+1)$th SKU must occupy the remaining space $v_{k+1}^{\mathrm{M}} = V^{\mathrm{M}} - v - \sum_{i=1}^{k} s_i^{\mathrm{M}}$, otherwise the solution could be improved. So the problem we considered formerly can be rewrote to

$$\begin{cases} \min C_{\mathrm{M}} \dfrac{\left(\sum_{i=1}^{k}\sqrt{f_i}\right)^2}{v} + C_{\mathrm{M}} \dfrac{f_{k+1}}{V^{\mathrm{M}} - v - \sum_{i=1}^{k} s_i^{\mathrm{M}} - s_{k+1}^{\mathrm{M}}}, \\ \text{s.t.} \quad v + \sum_{i=1}^{k} s_i^{\mathrm{M}} + v_{k+1}^{\mathrm{M}} = V^{\mathrm{M}},\ v > 0,\ v_{k+1}^{\mathrm{M}} > s_{k+1}^{\mathrm{M}}. \end{cases}$$

And we can further convert the above problem to

$$\begin{cases} \min C_{\mathrm{M}} \dfrac{\left(\sum_{i=1}^{k}\sqrt{f_i}\right)^2}{V^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}} - v_{k+1}^{\mathrm{M}}} + C_{\mathrm{M}} \dfrac{f_{k+1}}{v_{k+1}^{\mathrm{M}} - s_{k+1}^{\mathrm{M}}}, \\ \text{s.t.} \quad s_{k+1}^{\mathrm{M}} < v_{k+1}^{\mathrm{M}} < V^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}}. \end{cases}$$

Differentiating the objective function with respect to v_{k+1}^{M}, setting the derivative to 0, we have

$$\frac{\left(\sum_{i=1}^{k}\sqrt{f_i}\right)^2}{\left(V^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}} - v_{k+1}^{\mathrm{M}}\right)^2} - \frac{f_{k+1}}{(v_{k+1}^{\mathrm{M}} - s_{k+1}^{\mathrm{M}})^2} = 0,$$

$$\frac{\sum_{i=1}^{k}\sqrt{f_i}}{V^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}} - v_{k+1}^{\mathrm{M}}} = \frac{\sqrt{f_{k+1}}}{v_{k+1}^{\mathrm{M}} - s_{k+1}^{\mathrm{M}}},$$

$$v_{k+1}^{\mathrm{M}}\left(\sum_{i=1}^{k}\sqrt{f_i} + \sqrt{f_{k+1}}\right) = \left(\sum_{i=1}^{k}\sqrt{f_i}\right)s_{k+1}^{\mathrm{M}} + \sqrt{f_{k+1}}\left(V^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}}\right),$$

$$v_{k+1}^{\mathrm{M}}\left(\sum_{i=1}^{k+1}\sqrt{f_i}\right) = \left(\sum_{i=1}^{k}\sqrt{f_i}\right)s_{k+1}^{\mathrm{M}} + (\sqrt{f_{k+1}})s_{k+1}^{\mathrm{M}} + \sqrt{f_{k+1}}\left(V^{\mathrm{M}} - \sum_{i=1}^{k} s_i^{\mathrm{M}}\right) - (\sqrt{f_{k+1}})s_{k+1}^{\mathrm{M}},$$

$$v_{k+1}^{\mathrm{M}}\left(\sum_{i=1}^{k+1}\sqrt{f_i}\right) = \left(\sum_{i=1}^{k+1}\sqrt{f_i}\right)s_{k+1}^{\mathrm{M}} + \sqrt{f_{k+1}}\left(V^{\mathrm{M}} - \sum_{i=1}^{k+1} s_i^{\mathrm{M}}\right),$$

$$v_{k+1}^{\mathrm{M}} = \left(\frac{\sqrt{f_{k+1}}}{\sum_{i=1}^{k+1}\sqrt{f_i}}\right)\left(V^{\mathrm{M}} - \sum_{i=1}^{k+1} s_i^{\mathrm{M}}\right) + s_{k+1}^{\mathrm{M}}.$$

Corollary 1. The minimum total restocking cost of mode M is

$$C_{\mathrm{M}} \frac{(\sum_{i\in S(\mathrm{M})}\sqrt{f_i})^2}{V^{\mathrm{M}} - \sum_{i\in S(\mathrm{M})} s_i^{\mathrm{M}}}.$$

Proof. Substituting the result of theorem 1 into the objective function of (1), we can easily come to the conclusion.

Corollary 2. To minimize the total restocking cost of mode HD, the optimal number of channels allotted to SKU $i \in S(\mathrm{HD})$ in mode HD is $round(v_i^{\mathrm{HD}}/vc^{\mathrm{HD}})$, where the $round(x)$ function rounds x to the nearest integer and vc^{HD} is the volume per channel of the High-volume Dispenser. The adjusted volume allotted to each SKU $i \in S(\mathrm{HD})$ is then $round(v_i^{\mathrm{HD}}/vc^{\mathrm{HD}}) * vc^{\mathrm{HD}}$.

Proof. As mentioned in section 2.2, each SKU $i \in S(\mathrm{HD})$ should be allotted an integer number of channels in practice. So base on the result of theorem 1, it is reasonable to employ the $round(v_i^{\mathrm{HD}}/vc^{\mathrm{HD}})$ function, which rounds $(v_i^{\mathrm{HD}}/vc^{\mathrm{HD}})$ to the nearest integer, to estimate the optimal number of channels allotted to each SKU $i \in S(\mathrm{HD})$ in mode HD. And the actual volume for each SKU $i \in S(\mathrm{HD})$ is then adjusted to $round(v_i^{\mathrm{HD}}/vc^{\mathrm{HD}}) * vc^{\mathrm{HD}}$.

Following the same idea, we have Corollary 3 for low-volume dispensers.

Corollary 3 To minimize the total restocking cost of mode LD, the optimal number of cartridges allotted to SKU $i \in S(\mathrm{LD})$ in mode LD is $round(v_i^{\mathrm{LD}}/vc^{\mathrm{LD}})$, where the $round(x)$ function rounds x to the nearest integer and vc^{LD} is the volume per cartridge of the Low-volume Dispenser. The adjusted volume allotted to each SKU $i \in S(\mathrm{LD})$ is then $round(v_i^{\mathrm{LD}}/vc^{\mathrm{LD}}) * vc^{\mathrm{LD}}$.

3.3 Determining the optimal allocation of storage resources in the fast-picking area

According to the result of theorem 1, V^{HD} and V^{LD} should be known to determine the value of v_i^{HD} for all SKUs assigned to mode HD and v_i^{LD} for all SKUs assigned to mode LD. But in practice, appropriate number of high-volume dispensers and low-volume dispensers are combined together and share the total space of the fast-picking area. Therefore, we can treat the fast-picking area as a virtual mode HD+LD with the volume of $V^{\mathrm{HD+LD}}$, which equals to V^{HD} and V^{LD}, and determine the optimal allocation of total space in this mode to minimize the total restocking cost.

Let the variable α represent the proportion of space in mode HD+LD dedicated to SKUs assigned to mode HD. And then $1-\alpha$ denotes the proportion of space in mode HD+LD dedicated to SKUs assigned to mode LD. Fig. 3 shows the allocation of the total space in mode HD+LD and the corresponding volume, cost per restock of each mode in the restocking subsystem of CAOPS.

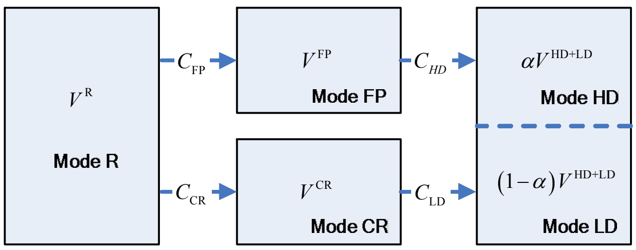

Fig. 3.　Volume and cost per restock of each mode

Assuming that all SKUs have been assigned to each mode, we minimize the total cost of restocking all SKUs in the system if we minimize the cost of restocking each mode in the system, independent of all other modes[2]. Furthermore, for simplicity, it is assumed that we do not consider picks from mode R, mode FP and mode CR. By the result of theorem 1 and corollary 1, the total cost of restocking CAOPS is then

$$C_{\text{FP}}\frac{(\sum_{i\in S(\text{FP})}\sqrt{f_i})^2}{V^{\text{FP}}-\sum_{i\in S(\text{FP})}s_i^{\text{FP}}}+C_{\text{HD}}\frac{(\sum_{i\in S(\text{HD})}\sqrt{f_i})^2}{\alpha V^{\text{HD+LD}}-\sum_{i\in S(\text{HD})}s_i^{\text{HD}}}+$$
$$C_{\text{CR}}\frac{(\sum_{i\in S(\text{CR})}\sqrt{f_i})^2}{V^{\text{CR}}-\sum_{i\in S(\text{CR})}s_i^{\text{CR}}}+C_{\text{LD}}\frac{(\sum_{i\in S(\text{LD})}\sqrt{f_i})^2}{(1-\alpha)V^{\text{HD+LD}}-\sum_{i\in S(\text{LD})}s_i^{\text{LD}}}. \tag{2}$$

The problem is then converted to finding the optimal value of α which is denoted as α^* to minimize the above expression. Take the derivative of Eq. (2) with respect to α, set this equals to 0 and then we have

$$\alpha^*=\frac{1}{1+\frac{\sqrt{C_{\text{LD}}}(\sum_{i\in S(\text{LD})}\sqrt{f_i})}{\sqrt{C_{\text{HD}}}(\sum_{i\in S(\text{HD})}\sqrt{f_i})}}+\frac{\sum_{i\in S(\text{HD})}s_i^{\text{HD}}}{V^{\text{HD+LD}}\left[\frac{\sqrt{C_{\text{HD}}}(\sum_{i\in S(\text{HD})}\sqrt{f_i})}{\sqrt{C_{\text{LD}}}(\sum_{i\in S(\text{LD})}\sqrt{f_i})}+1\right]}-\frac{\sum_{i\in S(\text{LD})}s_i^{\text{LD}}}{V^{\text{HD+LD}}\left[1+\frac{\sqrt{C_{\text{LD}}}(\sum_{i\in S(\text{LD})}\sqrt{f_i})}{\sqrt{C_{\text{HD}}}(\sum_{i\in S(\text{HD})}\sqrt{f_i})}\right]}. \tag{3}$$

Substituting the value of α^* into Eq. (2), the total cost of restocking CAOPS can be rewritten as

$$\frac{\left[\sqrt{C_{\text{FP}}}(\sum_{i\in S(\text{FP})}\sqrt{f_i})\right]^2}{V^{\text{FP}}-\sum_{i\in S(\text{FP})}s_i^{\text{FP}}}+\frac{\left[\sqrt{C_{\text{HD}}}(\sum_{i\in S(\text{HD})}\sqrt{f_i})+\sqrt{C_{\text{LD}}}(\sum_{i\in S(\text{LD})}\sqrt{f_i})\right]^2}{V^{\text{HD+LD}}-\sum_{i\in S(\text{HD})}s_i^{\text{HD}}-\sum_{i\in S(\text{LD})}s_i^{\text{LD}}}+\frac{\left[\sqrt{C_{\text{CR}}}(\sum_{i\in S(\text{CR})}\sqrt{f_i})\right]^2}{V^{\text{CR}}-\sum_{i\in S(\text{CR})}s_i^{\text{CR}}}. \tag{4}$$

3.4 Assigning SKUs to each mode

In the previous sections, when allotting space to SKUs, it is always assumed that all SKUs have already been assigned to a certain mode. Then we need to determine how to assign each SKU to the four given modes in the restocking subsystem of CAOPS optimally and efficiently.

This question can be formulated as a 0-1 multidimensional knapsack problem since there is a net benefit of including a SKU in the given mode as well as a cost based on the amount of space consumed in the mode. BARTHOLDI[12] and MELLER[13] developed two different greedy heuristics for the similar problem with only one forward mode. JERNIGAN[2] presented another greedy heuristic based on the optimal total rootflow in three-mode scenario. However, given the four storage modes, and the safety stock to be accounted for each SKU in each mode, the existing methods prove to be unfeasible for our problem which is more complex in practice. Moreover, suitable analytical optimization models for CAOPS are not yet available in Ref. [5]. Therefore, a fluid-based greedy heuristic is developed in this section.

Since mode FP and CR are the dedicated replenishment areas for mode HD and mode LD respectively, we then have $S(\text{FP})=S(\text{HD})$ and $S(\text{CR})=S(\text{LD})$. According to Eq. (4), for a given assignment of SKUs to mode HD and mode LD, the total cost of restocking CAOPS is

$$\frac{\left[\sqrt{C_{\text{FP}}}(\sum_{i\in S(\text{HD})}\sqrt{f_i})\right]^2}{V^{\text{FP}}-\sum_{i\in S(\text{HD})}s_i^{\text{FP}}}+\frac{\left[\sqrt{C_{\text{HD}}}(\sum_{i\in S(\text{HD})}\sqrt{f_i})+\sqrt{C_{\text{LD}}}(\sum_{i\in S(\text{LD})}\sqrt{f_i})\right]^2}{V^{\text{HD+LD}}-\sum_{i\in S(\text{HD})}s_i^{\text{HD}}-\sum_{i\in S(\text{LD})}s_i^{\text{LD}}}+\frac{\left[\sqrt{C_{\text{CR}}}(\sum_{i\in S(\text{LD})}\sqrt{f_i})\right]^2}{V^{\text{CR}}-\sum_{i\in S(\text{LD})}s_i^{\text{CR}}}. \tag{5}$$

As mentioned in section 2.2, each SKU can be assigned to either mode HD or mode LD, that is $S(\text{HD})\cap S(\text{LD})=\varnothing$. Therefore, we have $S=S(\text{HD})\cup S(\text{LD})$, where S denotes the set composed of all SKUs. And the SKU assignment problem can be stated as partitioning S into two parts, one for mode HD and another for mode LD, to minimize the total restocking cost of CAOPS. If there are totally n SKUs in set S, we need to search over all 2^n subsets of S and calculate the value of Eq. (5) for each subset to find the best SKU assignment strategy.

For a typical distribution center using CAOPS, n is often large and the above procedure is very time consuming which makes it impractical for the decision maker to solve the real life SKUs assignment problem. However, the whole procedure can be sped up according to the characteristics of CAOPS. From the difference between High-volume Dispensers and Low-volume Dispensers in terms of storage capacity, picking and restocking efficiency, it is reasonable to conclude that fast-moving SKUs with larger picking quantities should have the priority to be stored in and picked from mode HD compared to those with smaller picking quantity. Moreover, the flow f_i of each SKU $i\in S$, defined in section 3.1, can be used as a measure for picking quantity. Therefore, we can form a list by sorting all SKUs from the largest flow to the smallest and only need to consider n ways of partitioning this ranked list, instead of searching over all 2^n subsets.

The greedy heuristic for the SKUs assignment problem is as follows

Step 1: Sort all SKUs by the flow f_i of each SKU $i\in S$ in descending order.

Step 2: Successively evaluate the total restocking cost of CAOPS by calculating the value of Eq. (5) of assigning no SKUs to mode HD; assigning the first SKU in mode HD;

only the first two SKUs; only the first three; and so on. Choose the strategy that minimizes the total restocking cost of CAOPS.

Step 3: Suppose that the chosen strategy selects the first k SKU in the ranked list of n SKUs. Assign the remaining $n-k$ SKUs to mode LD.

The algorithm shown above has the time complexity of $O(n)$.

3.5 Procedure of the optimal slotting strategy

The optimization problem of slotting CAOPS regarding the total cost of restocking is now solvable. Here is the procedure.

Step 1: After getting the historical sales data or predicted customer orders over a certain period of time for the decision, determine the flow f_i of each SKU $i \in S$ according to its definition in section 3.1.

Step 2: Apply the greedy heuristic algorithm developed in section 3.3 to solve the SKU assignment problem for mode HD and LD.

Step 3: Calculate the optimal proportion of $V^{\mathrm{HD+LD}}$ according to Eq. (3) and get the value of V^{HD} and V^{LD}.

Step 4: Based on theorem 1, corollary 2 and corollary 3, calculate the optimal number of channels assigned to each SKU $i \in S$ to solve the SKU allocation problem for mode HD and LD.

4 Case Study from the Tobacco Industry

In order to present an application example of the proposed methodology for slotting CAOPS, the case of a typical cigarette distribution center is considered in this section. This case study is also the representative of the performance improvement potential resulting from utilizing the proposed slotting strategy in a real life application.

4.1 Profile of the distribution center

The period considered is from January, 2008 to January, 2009. During this one-year sales campaign, the distribution center processed orders for 93 SKUs, with total sales of nearly 14 million pieces.

The highest-selling SKUs during the campaign were much more popular than the lowest-selling SKUs, as shown in Fig. 4, where the volume of one piece is 0.001 21 m^3 and all 93 SKUs are ranked according to their flow from largest to smallest. The first 43 SKUs in the ranked list accounted for over 90% of the total flow while the remaining 50 SKUs accounted for only nearly 10%.

CAOPS has been configured in the order fulfillment area of the distribution center. An EIQ analysis together with the four-level ABC classification, which is based on the flow of each SKU and the designer's experience, is adopted to slot CAOPS now. The current configuration (slotting strategy) is as follows. The first 43 SKUs in the ranked list are assigned to the High-volume Dispensers. The first 3 SKUs are classified as super A class and allotted 3, 2, 2 channels respectively. The next 40 SKUs are classified as A class and allotted one channel for each. The remaining 50 SKUs in the ranked list, classified as B and C class, are assigned to the Low-volume Dispensers and allotted one channel for each. As for the replenishment area, there are 24 pallet positions for 47 high-volume dispensers and 50 bays of 2-carton-deep carton flow racks for 50 Low-volume Dispensers. Each pallet contains 30 cartons and each carton contains 50 pieces.

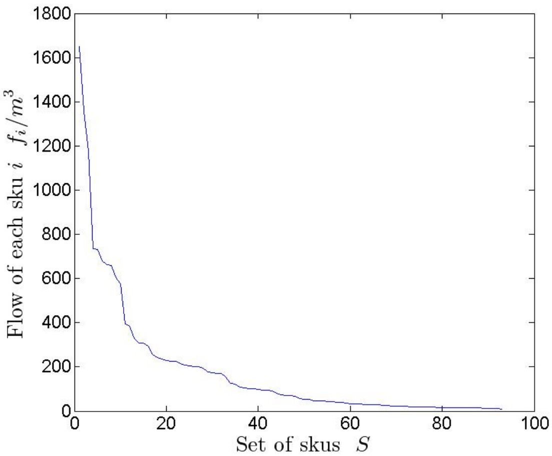

Fig. 4. Distribution of flows of 93 SKUs, ranked from largest to smallest

4.2 Estimating model parameters

To apply the proposed methodology for slotting CAOPS in this distribution center, the model parameters are estimated as follows.

4.2.1 Volume of each storage mode

The physical volume available for storage in each mode is estimated according to the measurements taken during the distribution center visits. Volume of mode HD+LD ($V^{\mathrm{HD+LD}}$) is 7.865 m^3. Volume of mode FP (V^{FP}) is 45.375 m^3. Volume of mode CR (V^{CR}) is 7.26 m^3.

4.2.2 Volume of one channel in mode HD and LD

Based on the shape and available space of the order fulfillment area in the considered distribution center, CAOPS is customized. The volume of one channel for high-volume dispenser (vc^{HD}) and low-volume dispenser (vc^{LD}) are 0.121 m^3 and 0.0303 m^3, respectively.

4.2.3 Volume of safety stock carried for each SKU in each storage mode

During our visits to the distribution center, we found that in each storage mode, the volume of safety stock is the same for all SKUs assigned to that mode, regardless of the different characteristics of each SKU. To be simple and practical, the safety stock parameter is estimated as mentioned above. But we should note that improvement could be made if safety stock for each SKU in each storage mode is carefully calculated.

The volume of safety stock for all SKUs in mode

HD (s_i^{HD}), FP (s_i^{FP}), LD (s_i^{LD}) and CR (s_i^{CR}) are 0.060 5 m^3, 0.060 5 m^3, 0.012 1 m^3, and 0.060 5 m^3, respectively.

4.2.4 Cost per restock for each storage mode

The maximum numbers of labors used per restock (L) and the average time consumed per restock (T) are measured based on the current configuration (slotting strategy) of CAOPS.

As for low-volume dispensers, the actual manual restocking cycle is 5 pieces within 10 s on average. But the time per restock based on the extended fluid model equals to three times of it since the restocking protocol mentioned in section 3.1 requires the amount per restock up to 15 pieces. So the average time consumed per restock for mode LD is adjusted to 30 s.

In the distribution center we consider, the monthly wages of one labor is 2 000 Yuan and working-day per month is 21 days while the working-hour per day is 4 h. Therefore, the cost per labor per second (c) is 0.006 614 Yuan. The cost per restock (Cr) for each mode can then be estimated as $L \times T \times c$, as shown in Table 1.

Table 1. Estimation of cost per restock for each mode

Mode	Maximum labors used per restock L	Average time consumed per restock T/s	Cost per labor per second c/Yuan	Cost per restock for each mode Cr/Yuan
HD	5	20	0.006 614	0.661 376
FP	2	30	0.006 614	0.396 825
LD	3	30	0.006 614	0.595 238
CR	2	35	0.006 614	0.462 963

4.3 Model implementation and results analysis

Based on the profile of the distribution center and the estimated model parameters, optimal slotting strategy for CAOPS regarding the total cost of restocking is obtained by applying the procedure mentioned in section 3.4.

As the number of SKUs assigned to mode HD increases, larger number of restocks is needed for the high-volume dispensers while smaller number of restocks is needed for the low-volume dispensers. Therefore, the cost of restocking mode HD+LD as well as mode FP increases while the cost of restocking mode CR decreases. The cumulative total cost of restocking CAOPS (C) is then a concave curve that reaches its minimal value when 33 SKUs are assigned to mode HD, as can be seen in Fig. 5.

As for the most cost-efficient alternative of the case data, where 33 SKUs are assigned to mode HD and 60 SKUs are assigned to mode LD, the optimal proportion of V^{HD+AD} dedicated to SKUs assigned to mode HD (α^*) is 0.677 5 according to Eq. (3). And the value of V^{HD} and V^{LD} is then 5.328 5 m^3, 2.536 5 m^3, respectively.

Based on theorem 1, corollary 2 and corollary 3, optimal volume (v_i), optimal number of channels (m_i) and adjusted volume (v'_i) are calculated for each SKU. The result of optimal assignment and allocation for each SKU in the ranked list is shown in Table 2.

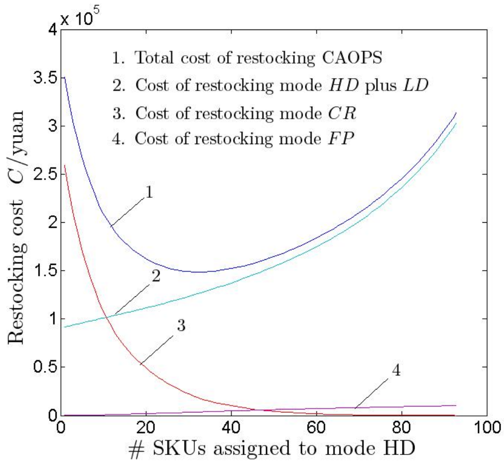

Fig. 5. Restocking cost structure and their relations with the number of SKUs assigned to mode HD

Table 2. Optimal SKU assignment and allocation result

SKU	Flow f_i/m^3	Assigned mode	Optimal volume v_i/m^3	Optimal channels m_i	Adjusted volume v'_i/m^3
1	1 651.015 7	HD	0.270 5	2	0.242
2	1 356.364 6	HD	0.250 8	2	0.242
3	1 177.019 3	HD	0.237 8	2	0.242
4	733.746 4	HD	0.200 5	2	0.242
⋮	⋮	⋮	⋮	⋮	⋮
32	169.233 3	HD	0.127 8	1	0.121
33	154.801 7	HD	0.124 8	1	0.121
34	124.344 1	LD	0.066 8	2	0.0605
35	123.626 3	LD	0.066 6	2	0.0605
⋮	⋮	⋮	⋮	⋮	⋮
90	12.308 7	LD	0.029 3	1	0.0303
91	12.205 9	LD	0.029 2	1	0.0303
92	12.024 7	LD	0.029 1	1	0.0303
93	9.789 2	LD	0.027 4	1	0.0303

The optimal solution result, which is based on the proposed slotting strategy, is compared with the current experiential solution to assess its effectiveness, as listed in Table 3. Number of restocks for corresponding modes is estimated by extended fluid model and the total restocking cost (C) is the sum of restocking cost for all 93 SKUs in all modes.

Compared with the current solution, space utilizations of all modes are increased significantly in the optimal solution. Moreover, number of restocks of mode HD and FP are greatly reduced since fewer SKUs are assigned to these modes under the proposed slotting strategy. Although number of restocks of mode LD and CR are increased accordingly, the optimal solution still generates a savings of over 18% referring to the total restocking cost over one-year period.

5 Conclusions

(1) Complex Automated Order Picking System (CAOPS) in the order fulfillment area of the distribution center is presented with specific reference to its fast-picking and restocking subsystems.

Table 3. Comparison between current and optimal solution

Metrix	Current solution	Optimal solution
SKU assignment		
SKUs assigned to mode HD/FP	43	33
SKUs assigned to mode LD/CR	50	60
SKU allocation		
Channels of mode HD	47	43
Channels of mode LD	50	79
Floor pallet positions	24	25
Bays of carton flow racks	50	60
Space utilization		
of mode HD plus LD	91.54%	97%
of mode FP	94%	100%
of mode CR	83%	100%
Restocks		
of mode HD	204 452	139 644
of mode LD	85 339	87 153
of mode FP	15 281	9 582
of mode CR	25 602	37 567
Total restocking cost C/Yuan	203 933	165 429

(2) Based on extended fluid model, a nonlinear mathematical programming model is developed to determine the optimal volume allotted to each SKU in a certain mode by minimize the restocking cost of that mode. The conclusion from the allocation model is then specified for the storage modules of high-volume dispensers and low-volume dispensers.

(3) Optimal allocation of storage resources in the fast-picking area of CAOPS is then discussed with the aim of identifying the optimal space of each picking mode.

(4) SKU assignment problem referring to the total restocking cost of CAOPS is analyzed in detail and a greedy heuristic with low time complexity is developed according to the characteristics of CAOPS.

(5) A real life application from the tobacco industry is then presented in order to exemplify the proposed slotting strategy and assess the effectiveness of the developed methodology.

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