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Structural Damage Detection Method Based on Decomposition of the Operating Deflection Shapes*

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Abstract: Full-field measurement techniques such as the scanning laser Doppler vibrometer (LDV) and the electronic speckle pattern interferometry systems can provide a dense and accurate vibration measurement on structural operating deflection shape (ODS) on a relatively short period of time. The possibility of structural damage detection and localization using the ODS looks likely more attractive than when using traditional measurement techniques which address only a small number of discrete points. This paper discusses the decomposition method of the structural ODSs in the time history using principal component analysis to provide a novel approach to the structural health monitoring and damage detection. The damage indicator is proposed through comparison of structural singular vectors of the ODS variation matrixes between the healthy and damaged stages. A plate piece with a fix-free configuration is used as an example to demonstrate the effectiveness of the damage detection and localization using the proposed method. The simulation results show that: (1) the dominated principal components and the corresponding singular vectors obtained from the decomposition of the structural ODSs maintain most of all vibration information of the plate, especially in the case of harmonic force excitations that the 1st principal component and its vectors mostly dominated in the system; （2）the damage indicator can apparently flag out the damage localization in the case of the different sinusoidal excitation frequencies that may not be close to any of structural natural frequencies. The successful simulation indicates that the proposed method for structural damage detection is novel and robust. It also indicates the potentially practical applications in industries.

Key words: damage detection, decomposition, operating deflection shape (ODS)，principal component

1 Introduction

Non-destructive evaluation (NDE) methods such as ultrasound, X-ray, dye penetrates, magnetic particle, and acoustic emission can complement visual inspections for structural damage detection and localization in civil, mechanical, and aeronautical industries. These techniques are often limited to observation in a limited area and rely on a presumption of the likely area of damage. Therefore, they have been commonly used only for local defect detection. A new technique based on pulsed eddy current NDE is recently developed to detect and characterize the defects at the surface and sub-surface of structures[1]. However, the pulsed eddy current NDE needs effective interpretation technique to analyze the structural response to the applied pulsed coil excitation and translate the response signal from eddy current sensors into meaningful information regarding defects and quantitative characterization. LANG, et al[2]. also developed a system identification approach to establish a transfer function model for inspected structures from the measured eddy current sensor response to the pulsed coil excitation in order to use the model parameters to reflect the changes of the structural characteristics. On the other hands, structural damage detection using non-destructive vibration data is able to evaluate the entire structural performance due to the simple setup, and potential automation of data acquisition and processing. It has received considerable attention since early in the decade. The conventional vibration-based

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damage detection techniques are mainly based on the change of modal properties of a structure such as modal frequencies, mode shapes, curvature mode shapes and modal flexibilities[3–5]. One advantage of using modal properties for damage detection is due to the substantial reduction in measured vibration data. However, modal parameters are not always easy to interpret in terms of mathematical modeling of linear vibrating systems and the lower-frequency modal parameters are insensitive. An alternative approach is to use the measured frequency response functions (FRFs)[6, 7] and FRF sensitivities[8]. Normally, damage detection algorithms dealing with FRFs attempt to find discrepancies between the validated FE model (which is usually used as a reference model) and the FRFs of some damaged specimen[9] and are considered as the expansion of model updating applications. PARK et al.[10] proposed a damage detection technique that utilizes an accurate analytical finite model (i.e. validated model) based on the incomplete FRF data in numerical and experimental environments. A major advantage of using FRF data over using modal data in model updating is that much more information can be provided in a desired frequency range [11]. But, the validation of the FE model for comparison purpose of damage detection is not straightforward. It requires several steps such as model verification, correlation, and model updating in validation process in order that the validated model can represent and predict the dynamic characteristic of a structure within the concerned frequency range. Instead of above approaches, the raw time signals measured directly without any signal processing degradation can also be used for damage detection [12-13]. However, the large number of sensors is required for effective damage estimation and there are difficulties, such as in dealing with large volumes of data, inherent measurement noise, and reliable feature extraction tools etc.

In recent years, laser-based measurement techniques have been applied to many fields including automotive, aeronautics, turbine machinery engineering, civil engineering etc. Applications for vibration measurement have shown a significant growth in structural dynamics. These techniques can be divided into two main categories: (1) laser Doppler instruments and (2) full field techniques such as holographic techniques and electronic speckle pattern interferometry (ESPI). A Scanning Laser Doppler Vibrometer (SLDV) is used as a velocity transducer in which a laser beam can be continuously scanned over a structure that is vibrating sinusoidally at any frequency; the measured LDV output signal can be quickly post-processed to obtain the Operating Deflection Shape (ODS) of the structure. Full-field measurement techniques can provide instantaneous maps of deformation of the vibrating structure by means of imaging techniques. Traditional holographic techniques used for displacement and shape measurement in the early days were based on a camera to capture patterns on photographic films and then these photos were developed for visual comparison. The recent developments of Electronic Speckle Pattern Interferometry (ESPI) systems enable us to provide the measured properties in a digital form and further to analyze with image processing tools. Laser-based measurement techniques for structural health monitoring and damage detection have several advantages. One of main features is non-physical contact with the structure to be measured and greater spatial resolution than accelerometer techniques in the vibration measurement. It is also adequate for high frequency analysis and suitable for use in operational conditions. Such features make it possible to measure vibrations of a static or moving structure with no modification to its properties and also to provide a dense and accurate measurement on degree-of-freedoms（DOFs） in a relatively short period of time. The dense ODS of a structure is considered as the actual vibration displacement or velocity patterns of the structure that includes the contribution of all vibration mode shapes in the steady-state condition due to a specific structural excitation force. Such a feature increases the potential for structural damage detection and localization and looks more attractive than when using traditional measurement techniques which address only a small number of discrete points. Nevertheless, it has to be noticed that the current SLDV system is expensive and it is still not widely used in practice.

This paper discusses the decomposition method of structural ODS for the purpose of structural damage detection. It is arranged as follows. Firstly, the methodology of damage detection using ODS is introduced in section 2. It includes the basic ODS theory, the decomposition of ODS using principal component analysis, and damage indicator through comparison of ODS variation matrixes between the healthy and damaged stages in a structure. Secondly, in section 3, a plate piece with a fix-free configuration is used as an example to demonstrate the effectiveness of the structural damage detection using the approach of decomposition of ODSs. Finally, conclusions are summarized in the last part of this paper.

2 Methodology of Damage Detection Using Structural ODS

2.1 Basic theory of ODS

Generally speaking, if we consider the basic equation of motion for a vibration structure which was excited by a force vector acting on a structure, then responses of a multiple-degrees-of-freedom（MDOF）system can be expressed as

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=f(t), \tag{1}$$

where vector $x(t)$ consists of the spatial coordinates used in describing the motion of the structure. The coefficient matrix ***M***, ***K***, ***C*** are used to represent the mass, stiffness, and damping in the structure. The dimension of these matrices is n×n, where n corresponds to the number of degrees of freedom of the structure. The vector $f(t)$ is used to represent generalized forces acting on the structure. The excitation force and responses are functions of time, *t*.

For the force subject to the particular case of harmonic excitation, the force function can be written as

$$f(t) = F\exp(\mathrm{i}\omega t) \tag{2}$$

Thus, the corresponding harmonic response of a structure is expressed as

$$x(t) = X\exp(\mathrm{i}\omega t) \tag{3}$$

The forced response solution can be directly written as

$$X = \left(K - \omega M + \mathrm{i}\omega C\right)^{-1} = H(\omega)F \tag{4}$$

A more explicit form of this solution may be derived as:

$$X = \sum_{r=1}^{N} \frac{\phi_r^{\mathrm{T}} F \phi_r}{\omega_r^2 - \omega^2 + \mathrm{i}2\zeta_r\omega_r\omega} \tag{5}$$

where ζ_r is the critical damping ratio for the *r*th mode. The vector of responses in Equation (5) is referred to as an operating deflection shape (ODS). It can be seen that the ODSs are functions of the modal properties (natural frequencies, damping, and mode shapes) and the excitation forces (amplitudes and locations acting on a structure). It includes contributions of all vibration modes that can be possibly excited by the force. For excitations close to the *r*th modal frequencies of a structure, the operating deflection shapes will closely approximate to the *r*th mode shape.

2.2 ODS decomposition

Suppose that the number of measured responses (displacement, velocity, or acceleration，etc) at spatial locations is *N*. The operating deflection shapes of a structure at a given time t_i are labeled as $O_{t_i} = \left(x_1(t_i), x_2(t_i), \cdots, x_N(t_i)\right)$. If the responses are sampled *M* times, the response history of a structure can be expressed as follows,

$$O = \left(O_{t_1}, O_{t_2}, \cdots, O_{t_m}\right)^{\mathrm{T}} = \begin{pmatrix} x_1(t_1) & x_2(t_1) & \cdots & x_n(t_1) \\ x_1(t_2) & x_2(t_2) & \cdots & x_n(t_2) \\ \vdots & \vdots & & \vdots \\ x_1(t_m) & x_2(t_m) & \cdots & x_n(t_m) \end{pmatrix}_{m\times n} \tag{6}$$

For each column of the matrix, by subtracting the mean of time histories individually, that is, $\tilde{x}_j(t_i) = x_j(t_i) - \bar{x}_j$, the zero-mean matrix of responses can be rewritten as

$$O = \begin{pmatrix} \tilde{x}_1(t_1) & \tilde{x}_2(t_1) & \cdots & \tilde{x}_n(t_1) \\ \tilde{x}_1(t_2) & \tilde{x}_2(t_2) & \cdots & \tilde{x}_n(t_2) \\ \vdots & \vdots & & \vdots \\ \tilde{x}_1(t_m) & \tilde{x}_2(t_m) & \cdots & \tilde{x}_n(t_m) \end{pmatrix}_{m\times n}. \tag{7}$$

It can be seen from Eq. (7), each row of the matrix represents the spatial locations of the recorded measurement points of a structure at a particular instant in time. Each column represents the zero-mean time histories of a single point. The covariance matrix can be formed as

$$R_{n\times n} = \frac{1}{M} O^{\mathrm{T}} O. \tag{8}$$

The principal components of the covariance matrix R and the corresponding principal component vectors are the eigenvalues and associated eigenvectors, that is,

$$R\psi_i = \lambda_i \psi_i, \tag{9}$$

where *i* is the principal component index which is less than the number N. ψ is the singular vector matrix ($\psi^{\mathrm{T}}\psi = I$), representing the spatial distribution of the amplitudes from eigenvalues. Therefore, the response history of a structure can be expressed as the combination of principal components and their corresponding singular vector matrix.

2.3 Information change with the number of principal components

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the ordered eigenvalues of the covariance matrix R, that is, $\lambda_1 \geqslant \lambda_2 \geqslant, \cdots, \lambda_n$. A measure of the maximum variation from the mean value can be computed as

$$J_v = \sum_{i=1}^{N} \lambda_i. \tag{10}$$

If the first p ($p \leqslant N$) principal components are used, then the distribution of information (energy) change in the system can be measured by

$$J_e = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{P} \lambda_i = \sum_{i=P+1}^{N} \lambda_i. \tag{11}$$

The information (energy) loss in the system can be expressed as

$$E = \frac{\sum_{i=P+1}^{N} \lambda_i}{\sum_{i=1}^{N} \lambda_i}. \tag{12}$$

2.4 Indicators of damage

From Eqs. (8) to (12), we can see that the response history of a structure is alternatively expressed by the principal components of the covariance matrix and the corresponding principal component vectors. Any structure's change will cause the variation of the principal components and their vectors. It indicates that the presence of damage and its location is likely revealed by comparing both the principal components and the corresponding vectors between the healthy and the damaged structures. Therefore, the indicators of damage can be defined as the subtractions of the singular vectors between the damaged and the healthy stages, that is,

$$\eta(i) = \psi_i^d - \psi_i^h, \tag{13}$$

where $\eta(i)$ denotes the indicator based on the ith principal component. ψ_i^d and ψ_i^h are the damaged stages and the corresponding ith principal vectors at the healthy. In particular conditions, if the excitation force is the sinusoidal in nature, the harmonic responses of the structure are significantly dominated by a single principal component and its principal vectors.

3 Case Study: Damage Detection of a Plate Structure

3.1 Preliminary

To demonstrate the effectiveness of the method, a finite element model of a simple plate structure is selected for simulation investigation. A plate model was created using 24 shell elements and 39 nodes shown in Fig. 1. The Young's modulus of the material property was set at 210 GPa and the material density was 7 840 kg/m^3. The plate was fixed at the left side (nodes 1, 14, and 27) and excited at the middle of the right side (node 26) in the z direction by the input force that is sinusoidal in nature. The damage of a plate was simulated with the Young's modulus reduced with respect to the original values in some elements. For the illustration purpose, the damaged case in this paper was simulated with the 50% reduction of values of the Young's modulus in the elements 7 and 19. The first 6 modes were calculated and listed in Table 1 under Fix-Free conditions both for the healthy (undamaged or original) structure and for the damaged one. Obviously, the damage of the plate causes the decrease of the natural frequencies to some extent. The responses of the plate in the time histories were calculated. Fig. 2 a) and b) are overlays of time histories of the plate model on all of 39 nodes of 24 elements with sinusoidal excitation at the frequency of 24.0 Hz in the healthy and damaged cases respectively. The ODSs of the plate in the time history can vary at any given time. Fig. 3 a) and b) overlay the ODSs of the plate both in the healthy stage and the damaged stage respectively at 4 different times such as t_1=0.304 s, t_2=0.360 s, t_3=0.520 s, and t_4=1.216 s, with the same sinusoidal excitation at the frequency 24.0Hz in both cases. Clearly, from Fig. 3, it is not possible to find the difference by simply comparing the ODSs of both the healthy and the damaged stages at any given time. It is necessary to find other approaches to detect and localize the damage of the plate.

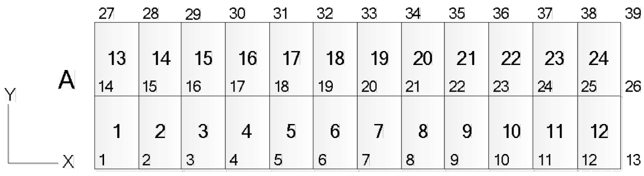
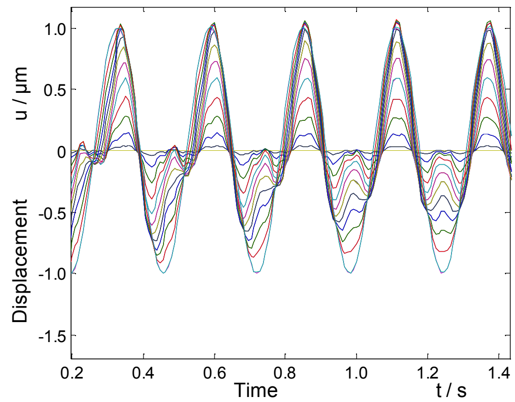


Fig. 1. Plate model with the mesh of 24 elements and 39 nodes

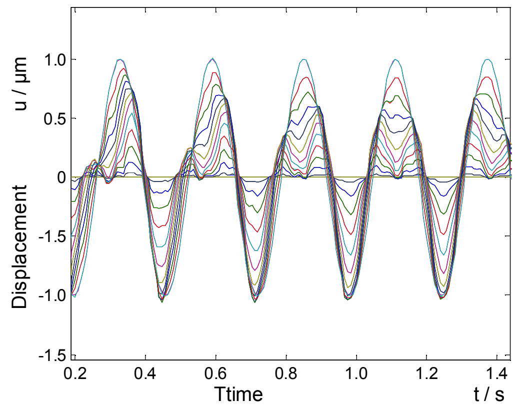
Table 1. The first 6 modes of a plate

Mode No.	Mode shape	Natural frequency f/Hz	
		Healthy case	Damaged case
Mode 1	1st bending	1.802	1.777
Mode 2	1st torsion	2.832	2.785
Mode 3	1st bending (y)	5.768	5.684
Mode 4	2nd bending	11.39	10.55
Mode 5	2nd torsion	12.88	12.14
Mode 6	3rd bending	32.47	31.01

Note: bending (y) represents bending in y direction

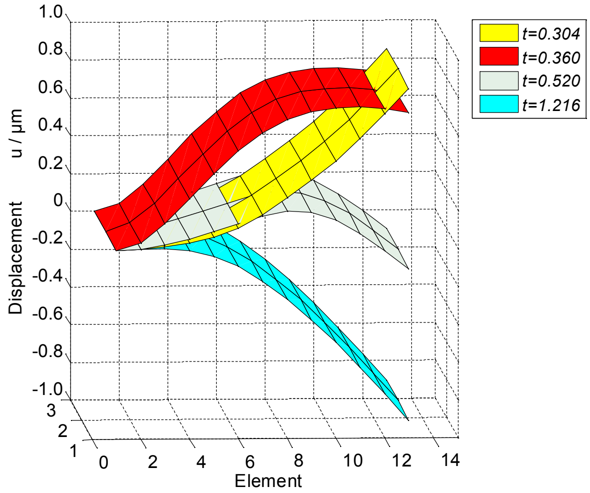


(a) Healthy case

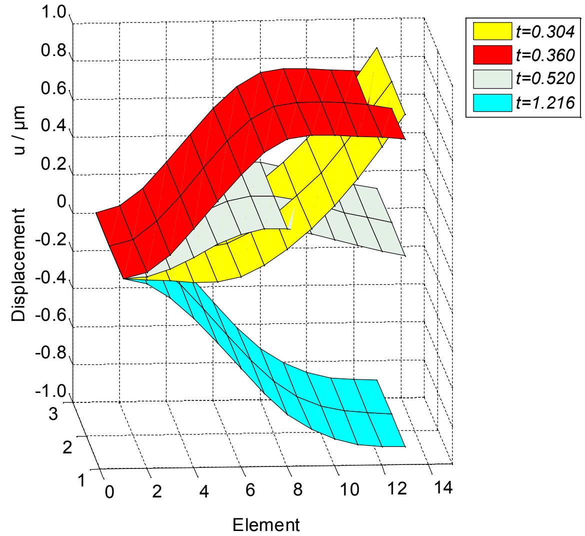


(b) Damaged case

Fig. 2. Overlay of time histories of the plate with sinusoidal excitation at the frequency 24.0 Hz.



(a) The healthy case



(b) The damaged case

Fig. 3. Overlay of ODSs at a certain time with sinusoidal excitation at the frequency 24.0Hz.

3.2 Damage detection of a plate structure

3.2.1 Decomposition of the ODSs

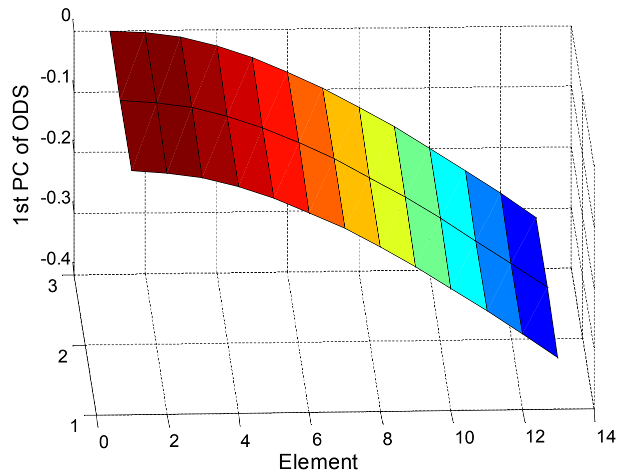
Suppose the sinusoidal excitation frequency is 2.3Hz which is between the first mode and the second mode, more closely to the first mode. The time responses at all 39 nodes of the model were recorded 500 times for the healthy and the damaged stages. Therefore, the total zero-mean responses are at the size of 39×500 for both stages respectively. The covariance matrix of ***R*** is a (39×39) squire matrix with 39 principal components and singular vectors. Table 2 lists the first 3 principal components and the cumulative percentages of the information. It can be seen the first principal component mostly dominated in the system with 99.95% in the healthy stage and 99.96% in the damaged stage. The first 2 principal components contain 99.999% information in both stages. Fig. 4 and Fig. 5 plot the ODSs decomposition corresponding to the 1st and 2nd principal components in the healthy and damaged stages. By visualizing the plots, any differences in any position/location of a plate between the healthy and the damaged stages cannot be detected straightforwardly.

3.2.2 Indicator of damage

Considering the first principal component which mostly dominated the responses, the corresponding indicator was calculated by subtraction of the singular vectors between the damaged and the healthy stages. The indicators with the sinusoidal excitation at the frequencies 2.3 Hz and 24.0Hz respectively were plotted in Fig. 6 (a) and (b). It can be seen that the damaged elements 7 and 19 of the plate were clearly shown in both pictures.

Table 2. Principal components (pcs) and the cumulative percentage with sinusoidal exciting force at frequency 2.3Hz

No.	Healthy case: Ratio of *i*th pc with sum of pcs	Healthy case: Percentage in sum of all pcs / %	Damaged case: Ratio of *i*th pc with sum of pcs	Damaged case: Percentage in sum of all pcs / %
1	0.9995	99.950 63	0.999 6	99.956 32
2	$0.486\,2\times10^{-3}$	99.999 26	$0.429\,8\times10^{-3}$	99.999 30
3	$0.680\,3\times10^{-5}$	99.999 94	$0.647\,5\times10^{-5}$	99.999 95



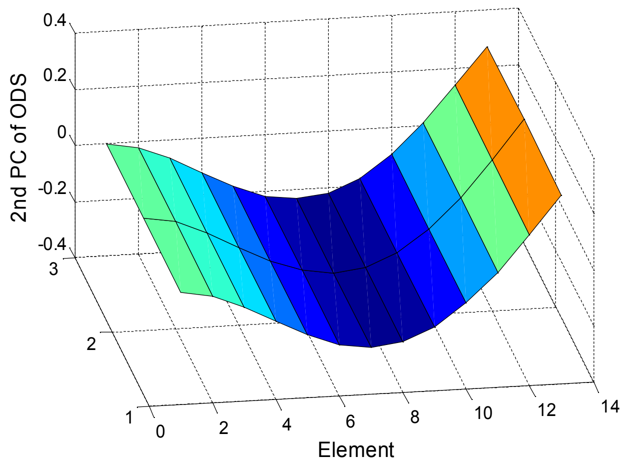


Fig. 4. The 1st & 2nd PCs of ODSs with sinusoidal excitation at the frequency 2.3Hz: healthy case

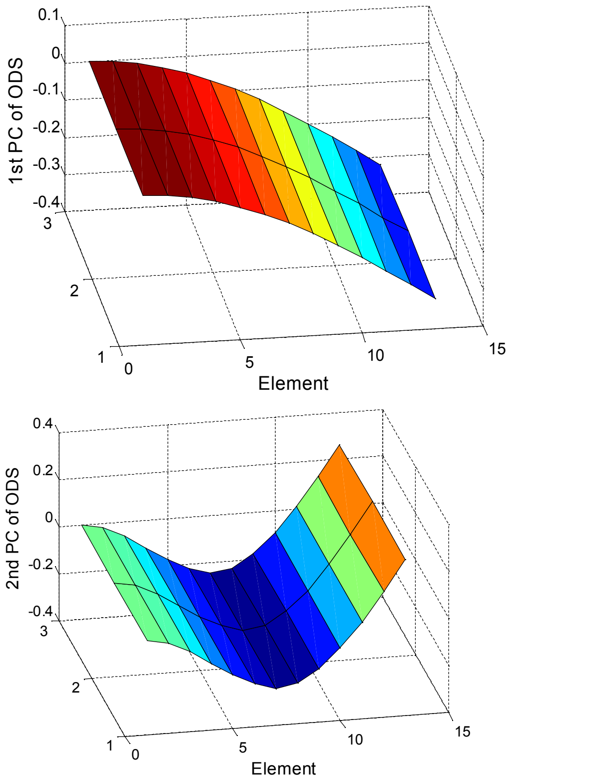
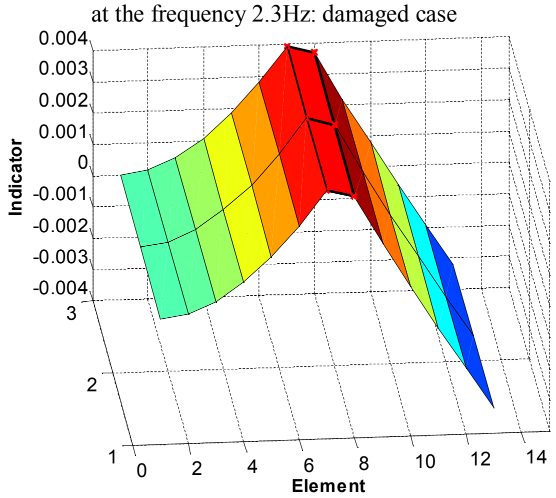
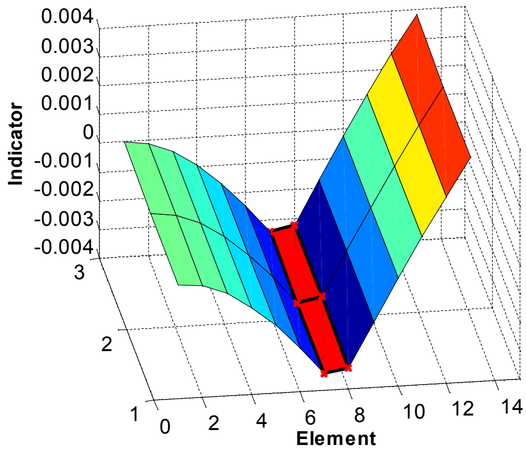


Fig. 5. The 1st & 2nd PCs of ODSs with sinusoidal excitation at the frequency 2.3Hz: damaged case



(a) With the excitation frequency 2.3Hz



(b) With the excitation frequency 24.0Hz

Fig. 6. Indicators of the damage location with sinusoidal excitation at the frequencies 2.3Hz.and 24.0Hz respectively

3 Conclusions

(1)The decomposition of the structural ODSs in the time history provides a novel approach to the structural health monitoring and damage detection. The structural ODS in the time domain is considered as the actual vibration displacement or velocity pattern of the structure including the contribution of all vibration mode shapes excited by the specific force acting on the structure.

(2)The dominated principal components and the corresponding singular vectors obtained from the decomposition of the structural ODSs maintain most of all information of the system. When the structure is excited with the harmonic force, the 1st principal component and its vectors mostly dominated in the system. Such a feature will be rather helpful for potentially practical applications in industries.

(3)The excitation frequency is not required to be close to any of the natural frequencies for structural health monitoring and damage detection. Such a feature increases the robustness of damage detection methods.

(4)There are still several issues that require further exploration. For instance, the influence of excitation positions on the structural health monitoring and localization of damage, the influence of smoothness of the structural ODS that determines how many number of measurement points are required, the experimental test to validate the method and make further comparisons between the novel method of damage detection and the traditional damage detection techniques，etc. The research on those issues is currently underway and their results will be reported in the future.

References

[1] TIAN G Y, SOPHIAN A. Defect classification using a new feature for pulsed eddy current sensors[J]. *NDT & E Int*., 2005, 38: 77−78.

[2] LANG Z Q, AGURTO A, TIAN G Y, et al. A system identification based approach for pulsed eddy current non-destructive evaluation[J]. *Measurement Science and Technology*, 2007, 18: 2 083−2 091.

[3] CONTURSI T, MESSINA A, WILLIAMS E J. A multiple damage location assurance criterion based on natural frequency changes[J]. *Journal of Vibration and Control*, 1998, 4(5): 619−633.

[4] RIZOS P F, ASPRAGATHOS N, DIMAROGONAS A D. Identification of crack location and magnitude in a cantilever beam from the vibration modes[J]. *Journal of Sound and Vibration,* 1990, 138(3): 381−38.

[5] LAM H F, KO J M, WONG C W. Localization of damaged structural connections based on experimental modal and sensitivity analysis[J]., *Journal of Sound and Vibration,* 1998, 210(1): 91−115.

[6] MAIA N M M, SILVA J M M, RIBEIRO A M R, et al. On the use of frequency-response-functions for damage detection[C]//*The 2nd International Conference on Identification in Engineering Systems*, Swansea, UK, March 29-31, 1999, 460−471.

[7] ZANG Chaoping, IMREGUN M. Structural damage detection using artificial neural networks and measured FRF data reduced via principal component projection[J]. *Journal of Sound and Vibration,* 2001, 242(5): 813−827.
[8] ARAUJO DOS SANTOS J V, MOTA SOARES C M, MAIA N M M. Structural damage identification in laminated structures using FRF data[J]. *Composite Structure*, 2005, 67 : 239−249.
[9] ZANG Chaoping, IMREGUN M. Structural damage detection and localization using FRF-based model updating approach [J]. *Key Engineering Materials*, 2003, 245−246: 191−202.
[10] PARK N G, PARK Y S. Damage detection using spatially incomplete frequency response functions[J]. *Machine System and Signal Processing,* 2003, 17(3): 519−532.
[11] LIN R M, EWINS D J. Model updating using FRF data[C]// *Proceedings of the 15th international seminar on modal analysis,* Leuven, Belgium, 1990.
[12] LEW J S, SATHANANTHAN S, GU Y. Comparison of damage detection algorithms using time domain data[C]// *The 15th International Modal Analysis Conference*, Orlando, Florida, February 3-6, 1997, 645−651.
[13] ZANG chaoping, FRISWELL M I, IMREGUN M. Structural damage detection using independent component analysis[J]. *Structural Health Monitoring: An International Journal,* 2004, 3(1): 69−84.

Biographical notes

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