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Adaptive Trajectory Tracking Control for a Nonholonomic Mobile Robot

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Abstract: As one of the core issues of the mobile robot motion control, trajectory tracking has received extensive attention. At present, the solution of the problem only takes kinematic or dynamic model into account separately, so that the presented strategy is difficult to realize satisfactory tracking quality in practical application. Considering the unknown parameters of two models, this paper presents an adaptive controller for solving the trajectory tracking problem of a mobile robot. Firstly, an adaptive kinematic controller utilized to generate the command of velocity is designed based on Backstepping method. Then, in order to make the real velocity of mobile robot reach the desired velocity asymptotically, a dynamic adaptive controller is proposed adopting reference model and Lyapunov stability theory. Finally, through simulating typical trajectories including circular trajectory, fold line and parabola trajectory in normal and perturbed cases, the results illustrate that the control scheme can solve the tracking problem effectively. The proposed control law, which can tune the kinematic and dynamic model parameters online and overcome external disturbances, provides a novel method for improving trajectory tracking performance of the mobile robot.

Key words: nonholonomic mobile robot, trajectory tracking, model reference adaptive

1 Introduction

In recent years, there has been an increasing amount of research on the subject of mobile robotics. As a part of research interests, the trajectory tracking problem, indeed, is particularly relevant in practical applications. Many researchers have worked in this field for a long period. Such studies have been divided into two main portions: one utilizes the kinematic trajectory tracking controller to achieve only tracking issue, while the other one, which combines kinematic and dynamic controller, is being more frequently adopted.

The kinematic tracking control problem of mobile robot, which is controlled by the velocity input, has been widely studied. In Ref. [1], a sliding mode control scheme is proposed to solve the trajectory tracking problem based on the exact discrete time model of the robot. A fuzzy controller[2–3] is designed to realize tracking control for mobile robots. In Ref. [4], a model–predictive tracking control applied to a mobile robot is presented, where the control law primarily minimized quadratic cost function consisting of tracking errors and the control effort. In Ref. [5], an adaptive tracking controller is proposed for the mobile robot based on integrating the analog neural network into the Backstepping technique.

Some researchers focused on solving the tracking control problem considering the dynamic model of mobile robot. In Ref. [6], a state feedback control law based on dynamic model is proposed for wheeled mobile robot to control its two driving motors synchronously. A dynamic control algorithm[7] is presented for realizing the tracking control based on modeling.

Adaptive control has been applied to solving tracking control problem popularly because of the ability of resolving uncertainties. In Refs. [8–11], an adaptive dynamic controller is proposed for mobile robot to attenuate the effects of the uncertainties and disturbances. The adaptive control law is obtained by analyzing Lyapunov function. A discrete-time sliding mode approach[12] is presented for a mobile robot, considering the uncertainties in the dynamical model.

Above methods have played an important theoretical guiding role in trajectory tracking of mobile robots. However, these control schemes do not consider the kinematic and dynamic model of the robot simultaneously, so that the tracking performance may be not satisfactory in practice.

In this paper, an adaptive trajectory-tracking controller is proposed with kinematic and dynamic uncertainties, and its stability is proved by using the Lyapunov theory. The design of the controller is divided in two parts, each part being a controller itself. The first one is an adaptive

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controller, which is presented for the kinematic model with uncertainties, the other one is a model reference adaptive controller that is applied to dynamic model with unknown parameters. The simulation results show that the proposed controller is capable of guiding the robot to track a desired trajectory with a quite small error.

This paper is organized as follows. In section 2, the model for mobile robot with nonholonomic constrains is introduced. In section 3, an adaptive kinematic controller and a model reference adaptive dynamic controller are presented. Simulation results for the proposed strategy are given in section 4. Finally section 5 gives the conclusions.

2 Mobile Robot Model

The mobile robot with two independent driven wheels is shown in Fig.1. O–xy is the world coordinate system. The mass center of robot is C, which is also the middle of the driving wheels. r is the radius of rear wheels and l is the distance of rear wheels. m_b and m_w are the mass of the body and wheel with a motor, I_c, I_w, I_m are the moment of inertia of the body about the vertical axis through C, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively.

The configuration of the mobile robot can be described by five generalized coordinates:

$$\tilde{\boldsymbol{q}} = [x, y, \theta, \phi_r, \phi_l]^T, \tag{1}$$

where (x, y) are the coordinates of C, θ is the orientation angle of robot, and ϕ_r, ϕ_l are the angles of the right and left driving wheels.

The kinematic model for the mobile robot under the nonholonomic constraint of pure rolling and non-slipping is[13]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ \frac{r}{2l} & -\frac{r}{2l} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{2}$$

Assuming all the uncertainties and disturbances a re zero, the dynamic equation of the simple model of the nonholonomic mobile robot is

$$\boldsymbol{M}(\tilde{\boldsymbol{q}})\dot{\boldsymbol{v}} = \boldsymbol{B}\boldsymbol{\tau}, \tag{3}$$

where $\boldsymbol{\tau} = [\tau_1 \;\; \tau_2]^T$ is the torque applied to the right and left wheels, $\boldsymbol{v} = [v_1 \; v_2]^T$ represent the angular velocities of right and left wheels. $\boldsymbol{M}$ and $\boldsymbol{B}$ are selected as

$$\boldsymbol{M} = \begin{bmatrix} \frac{r^2}{4l^2}(ml^2 + I) + I_w & \frac{r^2}{4l^2}(ml^2 - I) \\ \frac{r^2}{4l^2}(ml^2 - I) & \frac{r^2}{4l^2}(ml^2 + I) + I_w \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

where $m = m_b + 2m_w$, $I = 2m_w l^2 + I_c + 2I_m$.

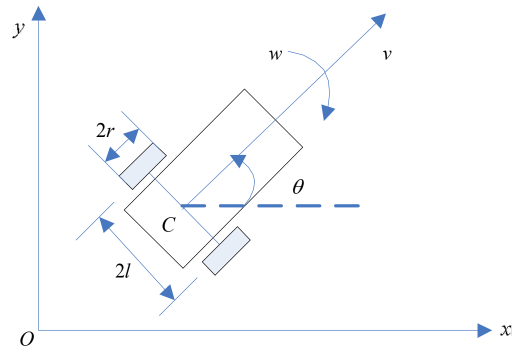

Fig. 1. Mobile robot configuration and its motion coordination

3 Adaptive Controller Design for Mobile Robot

In general, the trajectory tracking problem aims at tracking a reference mobile robot with a known posture $\boldsymbol{q}_r=[x_r, y_r, \theta_r]^T$. Therefore the errors between the actual and desired postures are $\boldsymbol{q}_e=\boldsymbol{q}_r-\boldsymbol{q}=[x_r-x, y_r-y, \theta_r-\theta]^T=[x_e, y_e, \theta_e]^T$.

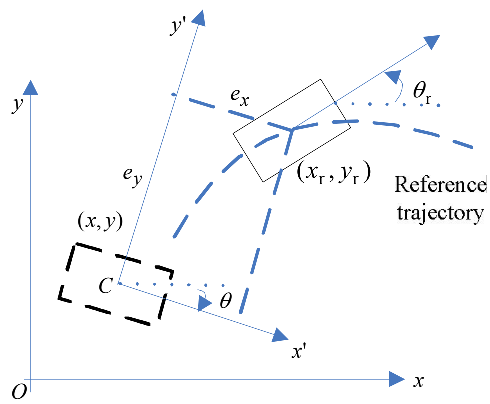

Fig. 2. Robot posture error coordinate

The posture error $\boldsymbol{e}_p$ expressed in the frame C–$x'y'$ of the real robot, as shown in Fig.2, is:

$$\boldsymbol{e}_p = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \boldsymbol{T}_e(\boldsymbol{q}_r - \boldsymbol{q}) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix}, \tag{4}$$

where $\boldsymbol{T}_e$ is transformation matrix.

Therefore, the control algorithm should be designed to force the robot to track the reference trajectory precisely, i.e. $\lim_{t\to\infty} \boldsymbol{e}_p = 0$.

3.1 Design of the adaptive kinematic controller

The design of the kinematic controller is based on the kinematic model of the robot. Linear velocity v and angular velocity w are related to v_1 and v_2 by the transformation:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{l}{r} \\ \frac{1}{r} & -\frac{l}{r} \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}. \quad (5)$$

Substituted Eq. (5) for Eq. (2), the ordinary form of a mobile robot with two actuated wheels can be obtained

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}, \quad (6)$$

where $\boldsymbol{q}=[x, y, \theta]^{\mathrm{T}}$ stands for the pose of mobile robot in inertial coordination O- xy.

Eq. (4) describes the difference of position and direction of the reference robot from the real robot. Using the Backstepping method, the input $\boldsymbol{u} = [v, w]^{\mathrm{T}}$, which makes $\boldsymbol{e}_p$ converge to zero, is given by the following[8]:

$$\begin{cases} v_c = v_r \cos e_\theta + k_1 e_x , \\ w_c = w_r + k_2 v_r e_y + k_3 v_r \sin e_\theta , \end{cases} \quad (7)$$

where k_1, k_2 and k_3 are positive constants, v_r, and w_r are desired linear and angular velocities of robot. The objective of such a controller is to generate the reference velocities of wheels for the dynamic controller based on Eq. (5).

If the parameters r and l in Eq. (2) are unknown, Eq. (7) cannot be chosen as an input because of the relationship Eq. (5). Hence, an adaptive controller is designed to attain the control objective by using the estimates of r and l. Setting $\beta_1=1/r$ and $\beta_2=l/r$, the velocity control input can be expressed as

$$\begin{bmatrix} v_{1c} \\ v_{2c} \end{bmatrix} = \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 \\ \hat{\beta}_1 & -\hat{\beta}_2 \end{bmatrix} \begin{bmatrix} v_c \\ w_c \end{bmatrix}, \quad (8)$$

where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the estimates of β_1 and β_2, respectively, and $\hat{\beta}_1 = \beta_1 + \tilde{\beta}_1$, $\hat{\beta}_2 = \beta_2 + \tilde{\beta}_2$, noticing $\hat{\beta}_1$, $\hat{\beta}_2$>0.

The parameter-updating laws are chosen as[13]

$$\begin{cases} \dot{\hat{\beta}}_1 = \alpha_1 e_x v_c , \\ \dot{\hat{\beta}}_2 = \alpha_2 \dfrac{w_c \sin e_\theta}{k_2} , \end{cases} \quad (9)$$

where α_1 and α_2 are positive constants.

According to Ref. [13], the control system is stable.

3.2 Design of the model reference adaptive dynamic controller

In this section, velocity vector $\boldsymbol{v}_c = [v_{1c}\ v_{2c}]^{\mathrm{T}}$ generated by Eq. (8) are considered as the reference angular velocities of wheels. These values are regarded as the reference inputs for the dynamic controller.

The following control laws are employed to prepare tracking of v_{1c}, and v_{2c}[14]:

$$\boldsymbol{\tau} = k_a \tilde{\boldsymbol{v}} + \boldsymbol{B}^{-1}\boldsymbol{M}\dot{\boldsymbol{v}}_c , \quad (10)$$

where k_a is a positive constant, and $\tilde{\boldsymbol{v}} = \boldsymbol{v}_c - \boldsymbol{v}$ is angular velocity error vector.

It is well known that the parameter variations of the system, such as mass and inertia, are difficult to measure. Therefore, the control laws should be written as follows:

$$\boldsymbol{\tau} = k_a \tilde{\boldsymbol{v}} + \hat{\boldsymbol{P}}\dot{\boldsymbol{v}}_c , \quad (11)$$

where $\boldsymbol{P} = \boldsymbol{B}^{-1}\boldsymbol{M} \in \mathbf{R}^{2\times 2}$, $\hat{\boldsymbol{P}} \in \mathbf{R}^{2\times 2}$ is estimate of $\boldsymbol{P}$.

Substituting Eq. (3) in Eq. (11), the following closed loop equation of velocities is obtained:

$$\boldsymbol{P}\dot{\boldsymbol{v}} = \hat{\boldsymbol{P}}\dot{\boldsymbol{v}}_c + k_d \tilde{\boldsymbol{v}} . \quad (12)$$

Define $\boldsymbol{P}^{-1}\hat{\boldsymbol{P}} = \boldsymbol{M}' \in \mathbf{R}^{2\times 2}$, $\boldsymbol{P}^{-1}k_d = \boldsymbol{N} \in \mathbf{R}^{2\times 2}$, the Eq. (12) can be rewritten as[14]

$$\dot{\boldsymbol{v}} = \boldsymbol{M}'\dot{\boldsymbol{v}}_c + \boldsymbol{N}\tilde{\boldsymbol{v}} . \quad (13)$$

Reference model for angular velocity error is selected as follows:

$$\dot{\boldsymbol{v}}_e + T\boldsymbol{v}_e = 0 , \quad (14)$$

where T is the positive time constant of error damping. $\boldsymbol{v}_e = \boldsymbol{v}_c - \boldsymbol{v}_m$, $\boldsymbol{v}_m$ is the velocity of the reference model. Then Eq. (15) can be obtained:

$$\dot{\boldsymbol{v}}_m = \dot{\boldsymbol{v}}_c + T\boldsymbol{v}_c - T\boldsymbol{v}_m , \quad (15)$$

The adaptation error, which is the difference between $\boldsymbol{v}$ and the velocity of the reference model $\boldsymbol{v}_{\rm m}$, is $\boldsymbol{e}=\boldsymbol{v}-\boldsymbol{v}_{\rm m}$.

The derivative of $\boldsymbol{e}$ is

$$\begin{aligned}\dot{\boldsymbol{e}} &= \dot{\boldsymbol{v}}-\dot{\boldsymbol{v}}_{\rm m}\\ &= \boldsymbol{M}'\dot{\boldsymbol{v}}_{\rm c}+\boldsymbol{N}\boldsymbol{v}_{\rm c}-\boldsymbol{N}\boldsymbol{v}-\dot{\boldsymbol{v}}_{\rm c}-T\boldsymbol{v}_{\rm c}+T\boldsymbol{v}_{\rm m}\\ &= -T\boldsymbol{e}+(\boldsymbol{M}'-\boldsymbol{E})\dot{\boldsymbol{v}}_{\rm c}+(\boldsymbol{N}-T\boldsymbol{E})\boldsymbol{v}_{\rm c}-(\boldsymbol{N}-T\boldsymbol{E})\boldsymbol{v}\end{aligned}, \quad (16)$$

where $\boldsymbol{E}\in\mathbf{R}^{2\times2}$ is unit matrix.

Defining

$$\boldsymbol{M}'=\begin{bmatrix}\boldsymbol{M}_1\\ \boldsymbol{M}_2\end{bmatrix}=\begin{bmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{bmatrix},\quad \boldsymbol{N}=\begin{bmatrix}\boldsymbol{N}_1\\ \boldsymbol{N}_2\end{bmatrix}=\begin{bmatrix}n_{11} & n_{12}\\ n_{21} & n_{22}\end{bmatrix},$$

$$\boldsymbol{e}=\begin{bmatrix}e_1\\ e_2\end{bmatrix},\quad \boldsymbol{E}=\begin{bmatrix}\boldsymbol{E}_1\\ \boldsymbol{E}_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

and considering the following Lyapunov function candidate

$$\begin{aligned}V_1 &= \frac{1}{2}e_1^{\,2}+\frac{1}{\lambda_1}(\boldsymbol{M}_1-\boldsymbol{E}_1)(\boldsymbol{M}_1-\boldsymbol{E}_1)^{\rm T}+\\ &\quad \frac{1}{\lambda_2}(\boldsymbol{N}_1-T\boldsymbol{E}_1)(\boldsymbol{N}_1-T\boldsymbol{E}_1)^{\rm T}\end{aligned}, \quad (17)$$

with λ_1, λ_2>0, its derivative obtained by using Eq. (16) is

$$\begin{aligned}\dot{V}_1 &= e_1\dot{e}_1+\frac{1}{\lambda_1}(\boldsymbol{M}_1-\boldsymbol{E}_1)\dot{\boldsymbol{M}}_1^{\rm T}+\frac{1}{\lambda_2}(\boldsymbol{N}_1-T\boldsymbol{E}_1)\dot{\boldsymbol{N}}_1^{\rm T}\\ &= -Te_1^{\,2}+(m_{11}-1)(\frac{1}{\lambda_1}\dot{m}_{11}+e_1\dot{v}_{1\rm c})+\\ &\quad m_{12}(\frac{1}{\lambda_1}\dot{m}_{12}+e_1\dot{v}_{2\rm c})+\\ &\quad (m_{21}-T)(\frac{1}{\lambda_2}\dot{m}_{21}+e_1v_{1\rm c}-e_1v_1)+\\ &\quad m_{22}(\frac{1}{\lambda_2}\dot{m}_{22}+e_1v_{2\rm c}-e_1v_2).\end{aligned} \quad (18)$$

Choose

$$\begin{cases}\dot{m}_{11}=-\lambda_1e_1\dot{v}_{1\rm c},\\ \dot{m}_{12}=-\lambda_1e_1\dot{v}_{2\rm c},\\ \dot{m}_{21}=-\lambda_2e_1(v_{1\rm c}-v_1),\\ \dot{m}_{21}=-\lambda_2e_1(v_{2\rm c}-v_2),\end{cases} \quad (19)$$

then Eq. (18) becomes

$$\dot{V}_1=-Te_1^{\,2}\leqslant 0. \quad (20)$$

Similarly for the next part:

$$\begin{cases}\dot{n}_{11}=-\lambda_3e_2\dot{v}_{1\rm c},\\ \dot{n}_{12}=-\lambda_3e_2\dot{v}_{2\rm c},\\ \dot{n}_{21}=-\lambda_4e_2(v_{1\rm c}-v_1),\\ \dot{n}_{21}=-\lambda_4e_2(v_{2\rm c}-v_2),\end{cases} \quad (21)$$

where parameters λ_i (i=1,2,3,4)>0, and will be tuned to reach the desired performance. According to the Lyapunov theory, the control system is asymptotically stable.

4 Simulation Results

To observe universality of the approach, three kinds of typical reference trajectories for the simulation are chosen: the first one is a circular trajectory, the second one is fold line trajectory and the other one is a parabola trajectory. System parameters of the robot are selected as follows r=0.15 m, l=0.75 m, $m_{\rm b}$=30 kg, $m_{\rm w}$=1 kg, $I_{\rm c}$=15.625 kg•m^2, $I_{\rm w}$=0.005 kg•m^2, $I_{\rm m}$=0.0025 kg•m^2, α_1=α_2=5, T=10, k_1=4.0, k_2=6.0, k_3=5.5, λ_i (i=1,2,3,4)=5.

The resulting mobile robot trajectory tracking, obtained by the proposed adaptive controller, is shown in Fig.3-Fig.14 including trajectory tracking and tracking errors. In these results, the sampling period was set to T_0=0.01 s, the ‘ideal’ and ‘real’ in Figs. 3, 5, 7, 9, 11 stand for ideal trajectory and real trajectory of mobile robot, respectively.

4.1 Nominal Case

In this case, there are no external disturbances. Fig.3-Fig.8 shows the tracking results.

(1) The circular trajectory was generated form the desired velocity $v_{\rm r}$=1 m/s and angular velocity $w_{\rm r}$=1 rad/s.

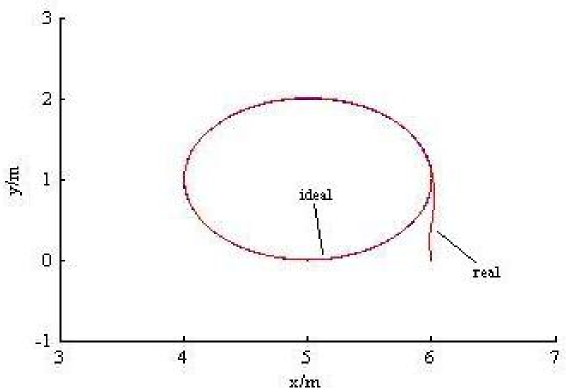

Fig. 3. Circular trajectory tracking

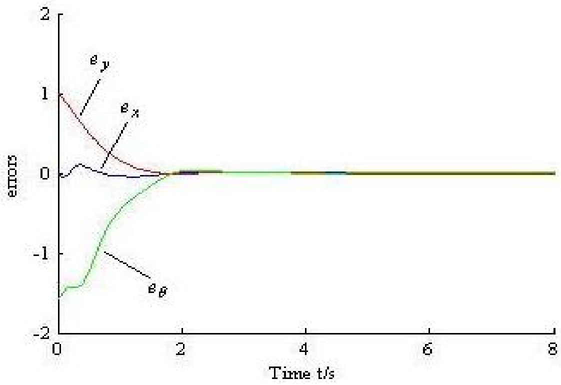

Fig. 4. Tracking errors in circular trajectory tracking

The initial posture of the reference trajectory is set at $\boldsymbol{q}_r(0)=[5,\ 0,\ 0]^T$ while the actual initial posture of robot is $\boldsymbol{q}(0)=[6,\ 0,\ \pi/2]^T$. Circular trajectory tracking simulation and error curve are seen in Fig .3 and Fig. 4, respectively, where adjustment time t= 2.0 s.

(2) The fold line trajectory was generated form the desired velocity in time:

$$v_r=\begin{cases}1.5\ \text{m/s}, & 0<t\le 5s\\ 1.2\ \text{m/s}, & 5s<t\le 10s\\ 1.5\ \text{m/s}, & 10s<t\le 15s\end{cases},$$

and angular velocity is w_r=0 rad/s.

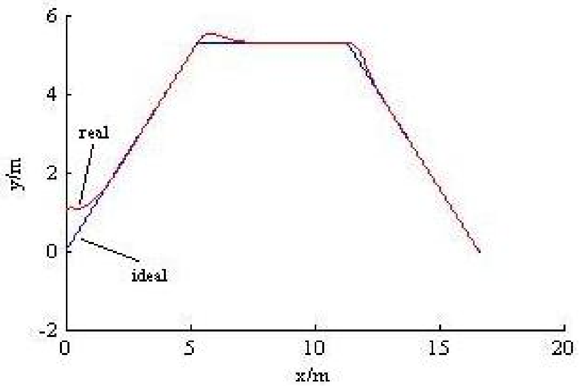

Fig. 5. Fold line trajectory tracking

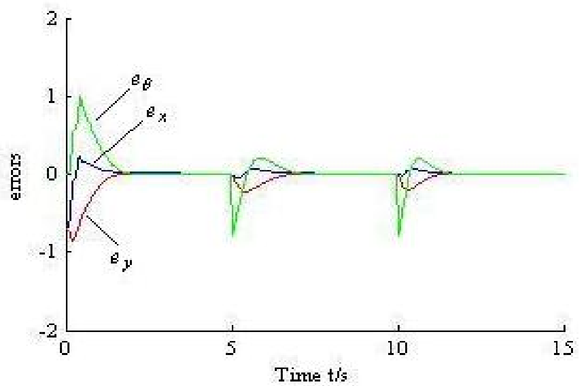

Fig. 6. Tracking errors in fold line trajectory tracking

The initial posture of the reference trajectory is set at $\boldsymbol{q}_r(0)=[0,\ 0,\ \pi/4]^T$ while the actual initial posture of the robot is $\boldsymbol{q}(0)=[0,\ 1,\ \pi/4]^T$. Fold line trajectory tracking simulation and error curve are shown in Fig. 5 and Fig. 6, respectively. In Fig. 5, the reference orientation alters suddenly form $\pi/4$ to zero at 5s and changes from zero to $-\pi/4$ at 10s, so two pulses can be observed in error curve as shown in Fig. 6. Moreover, after each reference orientation changing, the tracking errors converge to zero quickly.

(3) The parabola trajectory was produced by the desired velocity v_r=0.8 m/s and angular velocity w_r=–0.16 rad/s. The initial posture of the reference trajectory is set at $\boldsymbol{q}_r(0)=[0,\ 0,\ \pi/4]^T$ while the actual initial posture of robot is $\boldsymbol{q}(0)=[1,\ 0,\ \pi/2]^T$. Parabola trajectory tracking simulation and error curve are depicted in Fig .7 and Fig. 8, respectively, where adjustment time t= 2.5 s.

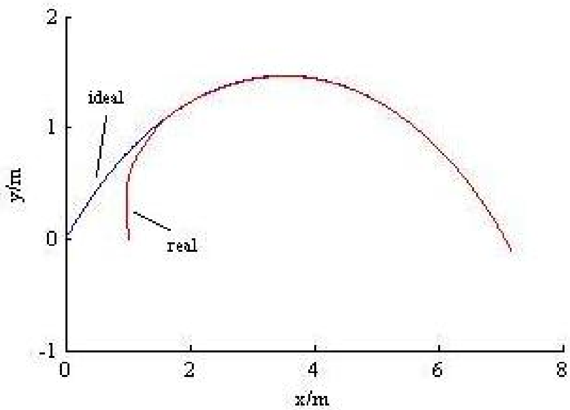

Fig. 7. Parabola trajectory tracking

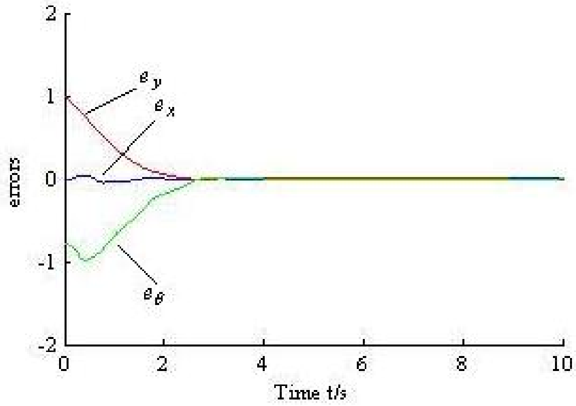

Fig. 8. Tracking errors in parabola trajectory tracking

Evidently, this controller has a perfect tracking ability as shown in above results. It can be seen that the controller can correct deviations quickly and there are little tracking errors.

4.2 Perturbed Case:

To test capacity of resisting disturbance of controller, white noise $randn(t)$ is added based on above reference trajectories in tracking process, respectively. *randn*(t) stands for white noise with average value 0 and variance 1. $x_r^{'}(t)$, $y_r^{'}(t)$ and $\theta_r^{'}(t)$ are respectively measured values of $x_r(t)$, $y_r(t)$ and $\theta_r(t)$. Fig.9-Fig.14 demonstrate the tracking results.

(1) Supposing $x_r^{'}(t) = x_r(t) + 0.01 randn(t)$, the circular trajectory tracking process and tracking error are shown in Figs. 9, 10, respectively, where adjustment time t= 2.3 s.

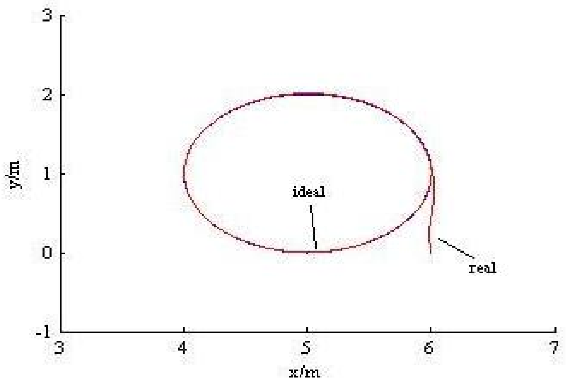

Fig. 9. Circular trajectory tracking in the perturbed case

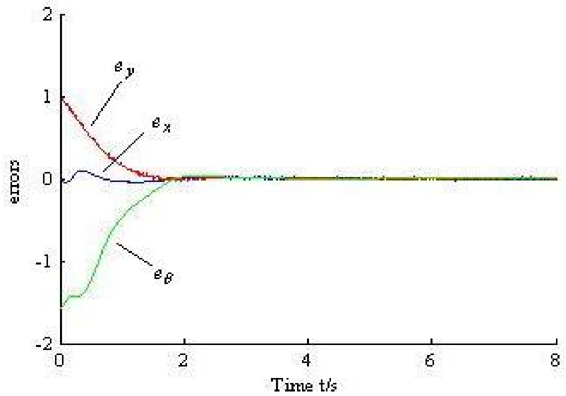

Fig. 10. Tracking errors in circular trajectory tracking

(2) Supposing $y_r^{'}(t) = y_r(t) + 0.005 randn(t)$, the fold line trajectory tracking process and tracking error are described in Fig. 11 and Fig. 12, respectively.

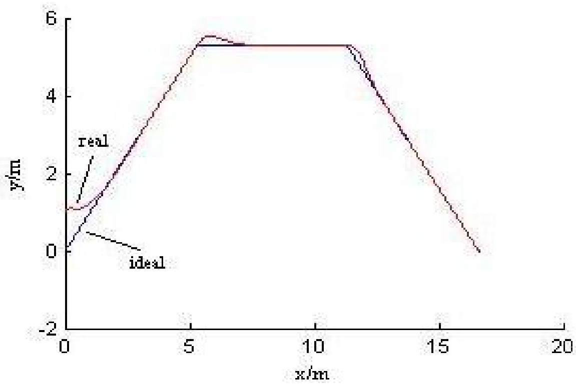

Fig. 11. Fold line trajectory tracking in the perturbed case

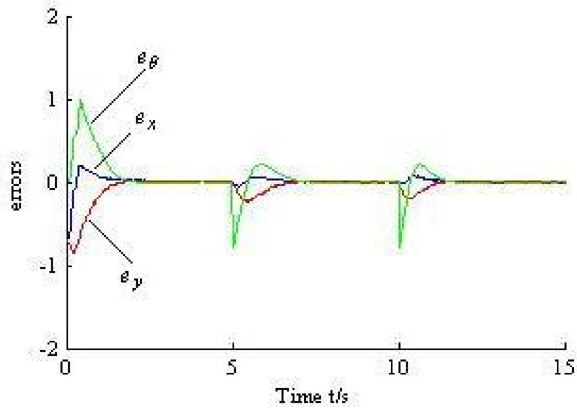

Fig. 12. Tracking errors in fold line trajectory tracking

(3) Supposing $\theta_r^{'}(t) = \theta_r(t) + 0.01 randn(t)$, the parabola trajectory tracking process and tracking error are shown in Figs. 13, 14, respectively, where adjustment time t= 2.4 s.

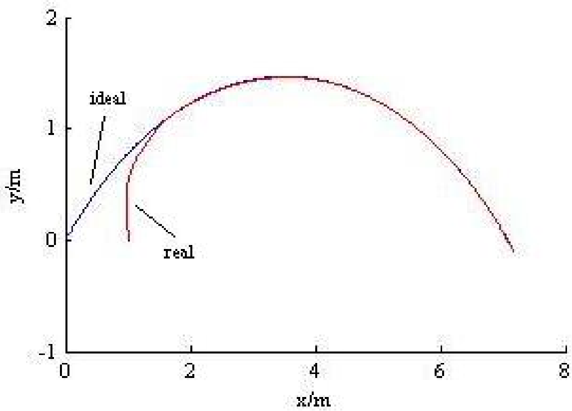

Fig.13. Parabola trajectory tracking in the perturbed case

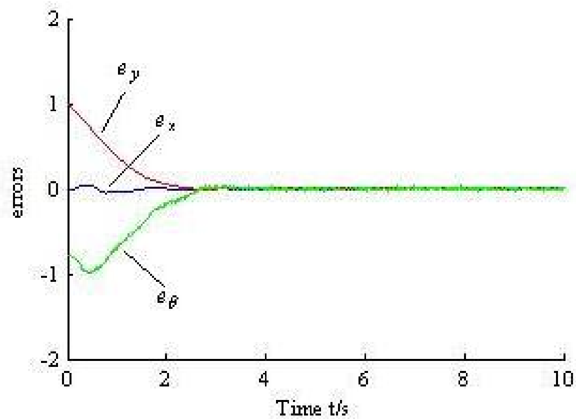

Fig.14. Tracking errors in parabola trajectory tracking

All the simulation results indicate that posture errors converge to a small neighborhood of zero in case of external disturbances. In other words, the results verify that the control method is robust enough to resist the external disturbances.

5 Conclusions

(1) Considering uncertainties in kinematic model of the mobile robot, an adaptive kinematic controller is proposed based on Backstepping method.

(2) A model reference adaptive dynamic controller is presented for the unknown dynamic parameters.

(3) The simulation of the control law is implemented in nominal and perturbed cases. All the simulations indicate that the proposed scheme is indeed feasible and effective.

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