

Criteria to define a pair-ion plasma and the role of electrons in nonlinear dynamics

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A criterion to define a pure pair-ion (PI) plasma is presented. It is suggested that the lighter elements (like H and He) are more suitable to produce PI plasmas. The observation of ion acoustic wave (IAW) in recent experiments with fullerene plasmas clearly indicates the presence of electrons in the system. A set of two coupled non-linear differential equations has been obtained for PI plasma dynamics. In moving frame, it can be reduced to a form similar to Hasegawa-Mima equation but it does not contain drift wave. **Criteria, pair-ion plasma, ion acoustic wave**

I. HISTORY OF THE PROBLEM

Peculiar experimental observations [1] of pair-ion (PI) fullerene C_{60}^{\pm} plasma have invoked a great deal of interest in this topic. It has been reported that a pure PI fullerene plasma can support three kinds of electrostatic waves propagating parallel to the external static magnetic field. These waves are the ion plasma wave (IPW), the ion acoustic wave (IAW), and the third one has been named as the intermediate frequency wave (IFW). In an earlier experiment [2], an alkali-metal-fullerene plasma (K^+, e^-, C_{60}^-) was produced by introducing fullerenes into the potassium plasma to realize a PI plasma. The IAW speed in Ref. [1] has been defined as $c_s = (\frac{\gamma_i T_i}{m_i})^{\frac{1}{2}}$ where $m_+ = m_- = m_i$ and $T_+ = T_- = T_i$ have been used. Here γ_i is the ratio of ion specific heats. The subscripts plus and minus denote the singly charged positive and negative ions, respectively. It is important to note that here $c_s k$ is the frequency of the ion thermal wave and is not the IAW. This thermal mode is similar to the sound wave in neutral fluids.

The electrostatic waves were excited in fullerene plasma externally and following observations were noted[1]. First the IAW has frequency larger than the theoretically calculated frequency $v_{T_i} k$ where $v_{T_i} = (\frac{\gamma_i T_i}{m_i})^{\frac{1}{2}}$ is the ion thermal speed in our notation. Since electron density has been assumed to be zero ($n_{e0} = 0$), therefore the ion acoustic speed, say c_s , has not been defined as a function of electron temperature as $c_s = (\frac{T_e}{m_i})^{\frac{1}{2}}$.

Second, it has been noticed that IFW has a feature that the group velocity is negative but the phase velocity is positive i.e. the mode is like a backward wave. However, the IPW shows no special features in PI plasmas.

After these observations, some theoretical investigations have shown that the acoustic speed becomes larger in a pair-ion plasma if it is not pure and contains significant concentration of electrons [3,4]. The

IAW and some other modes have been investigated in pair-ion-electron plasma and the linear IAW dispersion relation has been obtained using quasi-neutrality [3]. In Ref. [4], the IAW in PIE plasma has been discussed using kinetic approach and it has been pointed out that the quasi-neutrality is not a good approximation for PIE plasmas. As the number density of electrons decreases, the electron Debye length $\lambda_{De} = (\frac{T_e}{4\pi n_{e0} e^2})^{\frac{1}{2}}$ increases and the charge reparation effects become important. Furthermore, it has been shown that the Landau damping of IAW is reduced in the PI plasmas due to the presence of electrons and hence this mode can be excited in the PIE plasmas easily in the limit $1 < \lambda_{De}^2 k^2$. The basic definition of plasma requires $\lambda_{De}^2 k^2 < 1$, but the opposite limit $k \lambda_{De}^2 k^2$ seem to be a possible case in PIE plasmas. The experimental set up may be very reliable to produce PI plasmas, but the counter check is necessary to be sure whether the produced plasma can behave as a pure PI plasma or not. For this one needs to estimate the electron density n_{e0} or the densities of positive n_{+0} and negative n_{-0} ions in the system.

One cannot say with certainty that a plasma has exactly zero electron density. As the electron density is reduced in a system, the electron plasma frequency $\omega_{pe} = (\frac{4\pi n_{e0} e^2}{m_e})^{\frac{1}{2}}$ may become smaller than the frequency of oscillations of positive ions viz,

$$\omega_{pe} \ll \omega_{pi+} \quad (1)$$

$$\text{where, } \omega_{pi+} = (\frac{4\pi n_{+0} e^2}{m_i})^{\frac{1}{2}}$$

We need to find out a criterion for a PI plasma. It is the quantitative limit on the ratio of electron density to positive ion density ($\frac{n_{e0}}{n_{+0}}$) which can decide if the role of electrons is negligible in a plasma. Several authors [5,10] have theoretically studied various aspects of linear and nonlinear waves and instabilities in pure PI plasmas after the above mentioned experimental observations. In an investigation, the behavior of the so called backward mode (IFW) has been attributed to the nonlinear dynamics of PI plasma [5]. Recently two more experimental research papers have

appeared in the literature on PI plasmas [11,12]. In one of these works [11], an effort has been made to produce PI plasma with positive and negative Hydrogen ions ($H^+ + H^-$). On the other hand, already a recent theoretical work [13], it has been pointed out that a plasma of lighter ions (of low - Z materials) can behave as a pure PI plasma with relative larger electron density n_{e0} compared to plasmas of heavier ions like fullerenes. Therefore, it has been suggested that it is more suitable to try to produce PI plasmas using low- Z materials if other physical conditions like the electron attachment cross-section (to produce negative ions), ionization and recombination rates can be controlled. Some linear and nonlinear waves of PI and PIE plasmas are also discussed in this work.

II. IAW IN PIE PLASMAS

Here we derive the linear dispersion relation of ion acoustic wave (IAW) in pair-ion-electron (PIE) plasma for the simplest case, $T_+ = T_- = T_i \ll T_e$ using fluid equations. The wave is assumed to propagate along the external magnetic field to compare the theoretical result with the experimental observation [1].

Let the constant external magnetic field be along z -axis, i.e. $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ and the wave be propagating along the field lines with wave vector $\mathbf{k} = k \hat{\mathbf{z}}$. The equations of motion for singly charged positive and negative ions become, respectively,

$$\partial_t v_{+z} \simeq -\frac{e}{m_i} \partial_z \varphi \quad (2)$$

and

$$\partial_t v_{-z} \simeq +\frac{e}{m_i} \partial_z \varphi \quad (3)$$

The continuity equations yield,

$$\partial_t (n_+ - n_-) + n_{+0} \partial_z v_{+z} - n_{-0} \partial_z v_{-z} = 0 \quad (4)$$

Assuming electrons to follow the Boltzmann density distribution in electrostatic field $E = -\Delta \varphi$ as,

$$n_e \simeq n_{e0} e^{\frac{e\varphi}{T_e}} \simeq n_{e0} \left(1 + \frac{e\varphi}{T_e}\right) \quad (5)$$

the Poisson equation gives,

$$(n_+ - n_-) \simeq -\frac{\nabla^2 \varphi}{4\pi e} + n_{e0} \frac{e\varphi}{T_e} \quad (6)$$

Using the above set of equations, we can obtain the linear dispersion relation as,

$$\omega_s^2 = N_0 \frac{c_s^2 k^2}{1 + \lambda_{De}^2 k^2} \quad (7)$$

where $N_0 = \left(\frac{n_{+0} + n_{-0}}{n_{e0}}\right)$. This is the same as Eq. [5] of Ref.[4]. Note that $c_s k < \omega_s$ when $n_{e0} < n_{+0}$ because $1 < N_0$. The IAW can be excited easily in PIE plasma because the Landau damping rate decreases in the limit $1 < \lambda_{De}^2 k^2$ [4].

III. CRITERIA FOR PAIR-ION PLASMA

To decide whether the produced plasma can be called a pure PI plasma or not, one needs to estimate the ratio $\frac{n_{e0}}{n_{+i}}$. If the electrons pressure is not unusually high due to some external heating mechanism, then the situation $\omega_{pe}^2 \ll \omega_{pi+}^2$ implies that their role in plasma dynamics can be neglected. Such a system can behave as a pure PI plasma. Let us consider the plasma wave dispersion relation using kinetic approach written as,

$$1 + \sum_j \frac{1}{k^2 \lambda_{Dj}^2} \{1 + \iota \sqrt{\pi} Z_j W(Z_j)\} = 0 \quad (8)$$

where $\lambda_{Dj}^2 = \frac{T_j}{4\pi n_{0j} e^2}$; $Z_j = \frac{\omega}{\sqrt{2} k v_{Tj}}$; $v_{Tj} = \left(\frac{T_j}{m_j}\right)^{\frac{1}{2}}$ and $W(Z_j)$ is the plasma dispersion function for the j th species ($j = \pm, e$). In equilibrium, the quasineutrality demands $n_{0e} + n_{-0} = n_{+0}$ where both the positive and negative ions are assumed to be singly charged. This is the case of Ref. [1] as well. For the time being, let us assume $T_i < T_e$ and $v_{Te} \ll \frac{\omega}{k}$ such that $1 \ll |Z_e| \ll |Z_i|$ which allows us to use an asymptotic expansion of $W(Z_j)$ to study the system analytically. The case of IAW with $v_{Ti} \ll \frac{\omega}{k} \ll v_{Te}$ has been discussed in Ref. [4] in detail.

Assuming $m_+ = m_- = m_i$, Eq. [8] can be expressed as,

$$\begin{aligned} &\omega^2 - P_n^0 \omega_{pi+}^2 - \frac{3}{\omega^2} \{k^2 v_{Te}^2 \omega_{pe}^2 + k^2 v_{Ti}^2 \omega_{pi-}^2 + k^2 v_{Ti+}^2 \omega_{pi+}^2\} \\ &+ \iota \sqrt{\pi} \omega^2 \left\{ \frac{Z_e}{k^2 \lambda_{De}^2} e^{-Z_e^2} + \frac{Z_{i-}}{k^2 \lambda_{Di-}^2} e^{-Z_{i-}^2} + \frac{Z_{i+}}{k^2 \lambda_{Di+}^2} e^{-Z_{i+}^2} \right\} \\ &= 0 \end{aligned} \quad (9)$$

where $P_n^0 = \left(1 + \frac{n_{-0}}{n_{+0}} + \frac{n_{e0}}{n_{+0}} \frac{m_i}{m_e}\right)$. For $\omega = \omega_r - i\gamma$, the real and imaginary parts of Eq. [9] become, respectively,

$$\begin{aligned} \omega_r^2 \simeq &P_n^0 \omega_{pi\pm}^2 + \frac{3}{P_n^0} [\nu_{Ti\pm}^2 k^2 + \left(\frac{\omega_{pi-}^2}{\omega_{pi+}^2} k^2 \lambda_{Di-}^2\right) \omega_{pi-}^2 \\ &+ \left(\frac{\omega_{pe}^2}{\omega_{pi+}^2} k^2 \lambda_{De}^2\right) \omega_{pe}^2] \end{aligned} \quad (10)$$

and

$$\gamma \simeq \sqrt{\frac{\pi}{4}} \left[\frac{Z_{i+}}{k^2 \lambda_{Di+}^2} \exp\{-Z_{i+}^2\} + \frac{Z_{i-}}{k^2 \lambda_{Di-}^2} \exp\{-Z_{i-}^2\} \right. \\ \left. + \frac{Z_e}{k^2 \lambda_{De}^2} \exp\{-Z_e^2\} \right] \omega_r \quad (11)$$

The important point to note is that in the limit $\omega_{pe} < \omega_{pi+}$ and $v_{Te} k < \omega$ the electron plasma wave turns into the ion plasma wave. Furthermore in the limit $(\frac{n_{0e}}{n_{i+}}) \rightarrow 0$, we need not to use the perturbed electron density n_{e1} in the Poisson equation.

The comparison between the two terms of Eq. [10], i.e., $k^2 v_{Te}^2 \omega_{pe}^2$ and $k^2 v_{Ti+}^2 \omega_{pi+}^2$ can be very important to decide if the plasma can be called a pure (PI) system. Note that if $T_i = T_{i+} = T_{i-}$ and $m_{i+} = m_{i-} = m_i$, then $k^2 v_{Te}^2 \omega_{pe}^2 \ll k^2 v_{Ti+}^2 \omega_{pi+}^2$ provided that

$$\frac{n_{0e}}{n_{+0}} \ll \frac{T_i}{T_e} \left(\frac{m_e}{m_i} \right)^2 \quad (12)$$

In this case the electron contribution to the plasma dynamics can be neglected only for a very small value of the ratio $\frac{n_{0e}}{n_{i+}}$. Then the plasma can be called a pure (PI) plasma.

In the case of fullerene plasma, $m_i = 720m_p$ (where m_p is mass of the proton) and $\frac{m_e}{m_p} \sim \frac{1}{1836}$. Therefore we have $\frac{m_e}{m_i} = 7.56 \times 10^{-7}$. If $T_i < T_e$ is assumed, then the fullerene plasma discussed in Ref. [1] can be called a pure pair-ion plasma only if the following limit holds:

$$\frac{n_{0e}}{n_i} \ll (7.56 \times 10^{-7})^2 \frac{T_i}{T_e} \quad (13)$$

The plasma density in Ref. [1] is $n_i \sim 10^7 \text{cm}^{-3}$. It means that this system can become a pure (PI) plasma only if there is no electron in the system which is very unlikely physically.

Fortunately, we have a better condition than (12) to call the plasma a pure (PI) plasma. Our main requirement is the limit $\omega_{pe}^2 \ll \omega_{\pm}^2$, which replaces the relation (12) by a new limit as follows:

$$\frac{n_{0e}}{n_{\pm}^0} \ll \alpha \frac{m_e}{m_i} \quad (14)$$

where α must satisfy the condition $\alpha \ll 1$. Correspondingly the condition on thermal correction term is,

$$k^2 v_{Te}^2 \frac{\omega_{pe}^2}{\omega_{pi+}^2} \leq \omega_{pi+}^2 \quad (15)$$

Since $\frac{\omega_{pe}^2}{\omega_{pi+}^2} \simeq \alpha$ therefore (15) suggests the following maximum limit on the value of α to call a plasma as PI

plasma:

$$c_s k \ll \alpha k v_{Te} \ll \omega_{pi+} \quad (16)$$

If we choose $m_e/m_i \ll \alpha \ll 1$ the condition (16) is satisfied and it is then in agreement with the fact that the ion acoustic wave should not appear in the pure (PI) plasma. However, a smaller value of α is preferable. If n_{e0} is so small that α is almost zero, then we will have,

$$\alpha v_{Te}^2 k^2 \ll c_s^2 k^2 \ll \omega_{pi+}^2 \quad (17)$$

In this case the IAW remains almost non-existent. That is n_{e0} is too small and electron pressure does not contribute to plasma dynamics. The only normal mode of the system with $\mathbf{k} \parallel \mathbf{B}_0$ is the ion plasma wave which may have a negligible contribution from the small number of hot electrons. For the case of Helium (He) plasma $m_e/m_p \simeq 10^{-4}$ and if $T_i < T_e$ is assumed, then it will become almost a pure (PI) plasma if $n_{0e}/n_i \ll 10^{-5}$ holds. Therefore we conclude that it is more suitable to try to produce (PI) plasma of lighter atoms (or molecules) if other physical conditions like the ionization/recombination rates, and the electron attachment cross section can be controlled. Therefore it is suggested that the Hydrogen and Helium systems can be very useful to achieve a pure PI plasma.

IV. VORTICES IN PIE PLASMAS

If $T_i \neq 0$, then the perpendicular drift velocities for ions can be written as,

$$\mathbf{v}_{j\perp} = \frac{c}{B_0} \mathbf{E}_{\perp} \times \mathbf{z} - \frac{\nabla_{pj} \times \mathbf{z}}{\Omega_j m_j n_j} - \frac{1}{\Omega_i} (\nabla_i + \mathbf{v}_j \cdot \nabla) \mathbf{v}_j \times \mathbf{z} \\ = \mathbf{v}_E + \mathbf{v}_{Dj} + \mathbf{v}_{pj} \quad (18)$$

Here $j = \pm$ and $\Omega_j = \frac{q_j B_0}{m_j c}$. For electrons, we have

$$\mathbf{v}_{e\perp} = \frac{c}{B_0} \mathbf{E}_{\perp} \times \mathbf{z} - \frac{\nabla_{pe} \times \mathbf{z}}{\Omega_e m_e n_e} = \mathbf{v}_E + \mathbf{v}_{De} \quad (19)$$

where $|\partial_t| \ll \Omega_e = \frac{e B_0}{m_e c}$ has been used. The continuity equations of the ions yield,

$$\partial_t (n_{+} - n_{-}) + \frac{c}{B_0} \nabla n_{e0} \cdot (\mathbf{Z} \times \nabla_{\perp} \phi) - \frac{c}{B_0 \Omega_i} (n_{+0} + n_{-0}) \\ \times (\partial_t + \mathbf{v}_E \cdot \nabla) \nabla^2 \phi = n_{-0} \partial_z v_{z-} - n_{+0} \partial_i v_{z+} \quad (20)$$

and the parallel equation of motion becomes,

$$(\partial_t + \mathbf{v}_E \cdot \nabla) = n_{-0} \partial_z v_{z-} - n_{+0} \partial_i v_{z+}$$

$$= \frac{e}{m_i}(n_{+0} + n_{-0})\partial_z\phi \quad (21)$$

Assuming Boltzmann density distribution for electrons $n_e \sim n_{e0}e^{\frac{e\phi}{T_e}}$ and using the Poisson equation.

$$\nabla \cdot (E) = 4\pi e(n_+ - n_- - n_e) \quad (22)$$

the nonlinear Eqs. [19] and [20] can be written, respectively, as:

$$\begin{aligned} \partial_t\{-\lambda_{De}^2\nabla^2\Phi\} + D_e\kappa_{ne}\cdot(\mathbf{z} \times \nabla_{\perp}\Phi) - N_0\rho_s^2 \\ \times (\partial_t + D_e\mathbf{z} \times \nabla_{\perp}\Phi \cdot \nabla)\nabla_{\perp}^2\Phi = \partial_z V \end{aligned} \quad (23)$$

and

$$(\partial_t + D_e\mathbf{z} \times \nabla_{\perp}\Phi \cdot \nabla)V = c_s^2 N_0 \partial_z \Phi \quad (24)$$

where $V = \frac{(n_{-0}v_{-z} - n_{+0}v_{+z})}{n_{e0}}$, $N_0 = \frac{(n_{+0} + n_{-0})}{n_{e0}} = \frac{e\phi}{T_e}$, $D_e = \frac{eB_0}{cT_e}$, $\rho_s^2 = \frac{c_s^2}{\Omega_i^2}$ and $c_s = (\frac{T_e}{m_i})^{\frac{1}{2}}$.

Equation [23] is the Hasegawa-Mima (HM) equation for (PIE) plasma if the RHS is ignored (for $v_{iz} \rightarrow 0$) and $\lambda_{De}^2 k^2 \ll 1$ is assumed.

These equations give the coupled linear dispersion relation of drift wave and IAW in (PIE) plasmas as,

$$G_0\omega^2 - \omega_e^* - N_0c_s^2k_0^2 = 0 \quad (25)$$

where $G_0 = (1 + \lambda_{De}^2k^2 + N_0\rho_s^2k_{\perp}^2)$ and $\omega_e^* = \mathbf{v}_0^* \cdot \mathbf{k}$,

If $N_0c_s^2k_{\perp}^2 \ll \omega_e^*$ holds, then we obtain only the drift wave dispersion relation as,

$$\omega = \frac{\omega_e^*}{(1 + \lambda_{De}^2k^2 + N_0\rho_s^2k_{\perp}^2)} \quad (26)$$

In (PIE) plasma the quasi-neutrality can break down for IAW in the limit $1 \ll \lambda_{De}^2k^2$ because n_{e0} can be very small. On the other hand the inequality $\lambda_{De} < \rho_s$ always holds. Since $\rho_s^2k^2 < 1$ in the fluid model, therefore $\lambda_{De}^2k^2$ should not be much larger than 1. This means in magnetized plasmas, the IAW cannot have wavelengths shorter than λ_{De} within fluid theory framework because of the limit,

$$\lambda_{De}^2k^2 < \rho_s^2k_{\perp}^2 < 1 \quad (27)$$

It is important to note that as n_{e0} decreases, N_0 increases to have $\lambda_{De}^2k^2 \ll N_0\rho_s^2k_{\perp}^2$ and hence Eqs. [23] and [24] give the pair plasma convective cell (PPCC) mode,

$$\omega^2 = \frac{k_s^2}{k_{\perp}^2}\Omega_i^2$$

As n_{e0} decreases, the IAW converts into the PPCC mode. In between these two limits, the electron drift wave couples with IAW and PPCC.

V. NONLINEAR EQUATIONS FOR PI PLASMA DYNAMICS

The set of nonlinear Eqs. [23] and [24] can be transformed into HM equation in a moving frame which admits monopolar and dipolar vortex solutions [13]. The nonlinear dynamics of (PI) plasmas are described by Eqs. [23] and [24] in the limit $1 \ll N_0$ and they reduce, respectively, to the following equations:

$$(\partial_t + D_i\mathbf{z} \times \nabla_{\perp}\Phi \cdot \nabla)\nabla_{\perp}^2\Phi = -\frac{1}{2\rho_i^2}\partial_z V \quad (28)$$

and

$$(\partial_t + D_i\mathbf{z} \times \nabla_{\perp}\Phi \cdot \nabla)V = 2v_{Ti}^2\partial_z\Phi \quad (29)$$

where $\Phi = \frac{e\phi}{T_i}$, $D_i = \frac{cT_i}{eB_0}$, $\rho_i = \frac{v_{Ti}}{\Omega_i}$ and $V = (v_{z-} - v_{z+})$

In the stationary (η, x) frame, the coupled Eqs. [28] and [29] can be written as,

$$C_1d_{\eta}\nabla_{\perp}^2\Phi + V_0d_{\eta}\Phi + \{\nabla_{\perp}^2\Phi, \Phi\} = 0 \quad (30)$$

where $C_1 = \frac{-u}{D_i}$, $V_0 = \mu L_0 2\rho_i^2 D_i$, $L_0 = (C_0 - \frac{2\mu\nu_{Ti}^2}{u})$ and C_0 is an arbitrary constant. The important point to note is that the form of Eq. (28) is similar to Hasegawa-Mima equation but the physics of the equation is completely different. The set of nonlinear Eqs. [28, 29] is valid as well for electron-position plasmas in the classical limit. But these equations do not contain the drift wave and the ion acoustic mode. They describe the nonlinear dynamics of (PI) plasmas in the quasi-neutrality approximation.

VI. PI PLASMA FOR FUSION

The drift waves are the fundamental source of instabilities in tokamak fusion plasmas. Several kinds of reactive and dissipative drift instabilities appear in Tokamak plasmas. The presence of electrons in laser-fusion is also problematic for laser absorption, heat conductions and uniform compression. The PI-plasmas can be very suitable fuel for fusion, in principle because drift waves cannot exist in such plasmas. However, it does not seem easy, at least at present times, first to produce PI plasma of Hydrogen or Helium at high densities and high temperatures for fusion. Second the confinement for a longer time can also be a problem because of recombination and production of electrons and neutrals as a result of collisions between positive and negative ions as well as with neutrals.

VII. SUMMARY

A criteria to define a pure pair-ion (PI) plasma has been discussed. The condition (14) must be satisfied along with (16) to call a plasma as a pure PI plasma. But a very small value of α such that $\alpha \ll \frac{m_e}{m_i}$ is preferable. It is suggested that the lighter elements are preferable to produce pure PI plasma if electron attachment cross-section and recombination rate can be controlled for desirable results. In the plasma of lighter elements, the condition (14) can be satisfied even for relatively larger values of α .

It has also been stressed that the observation of ion

acoustic wave (IAW) in the experiment, itself is an indication of the existence of significant concentration of electrons in the produced pair-ion fullerene plasma and hence it cannot behave as a pure PI plasma. Furthermore, the frequency of IAW increases in a pair-ion plasma in the presence of electrons because we have $1 \ll N_0$.

A set of two coupled nonlinear differential equations has also been obtained for the pure PI plasma. In a moving frame, these equations can be reduced to a single equation similar to Hasegawa-Mima equation but it does not contain drift wave.

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