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Comparing the reliability of a discrete-time and a continuous-time Markov chain model in determining credit risk

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This article compares the reliability of a discrete-time and a continuous-time Markov chain model for estimating credit risk and for investigating loans of Chiao Tung Bank in Taiwan. The continuous-time Markov chain model can capture the migration of rare events. The time-varying risk premium was also extracted from the loan value and corresponding risk-free price and the transition matrix was transferred to risk-neutral transition matrix by the time-varying risk premium. Finally, the empirical results indicate that the discrete-time Markov chain model may be underestimating the default probability in both the lowest risk and speculative rating class. Comparing the loss given default and the NPL ratio, the continuous-time Markov chain model is more reliable and effective for gauging the credit risk of bank loans.

I. Introduction

Credit risk management has evolved considerably over the past decade. Early credit risk models focused on using credit scoring to predict the likelihood of default, such as the Altman's Z-score, logit and probit models. These models usually emphasize the cross-sectional rather than the time-series dimension of the sample.

In the last decade, rating-based models in credit risk management became very popular. The popularity was due to the straightforwardness of the then new capital accord, Basel II, which was proposed by the Basel Committee, a regulatory body under the bank of international settlements, on banking supervision. Basel II also allowed banks to base their capital requirements on internal as well as external rating systems.

Reduced-form models were developed by Jarrow and Turnbull (1995), Duffie and Singleton (1997), Jarrow *et al.* (1997) and Lando (1998) using rating systems for estimating the default risk. However, previous studies always analysed the credit risk of bonds with the discrete-time Markov chain and rarely applied it to investigate the credit risk of bank loans. An important feature of the discrete-time Markov chain model is that if a transition from category i to j does not occur in a given period, the estimate of the corresponding rate is zero. Therefore, in this study, the usefulness of both the discrete and the continuous state modelling approach for measuring the credit risk of bank loans in Taiwan were compared from both an analytical and empirical perspective. In particular, the issue whether the continuous-time Markov chain model provide a sufficiently rich description of credit risk outcomes compared to the discrete-time Markov chain model was addressed.

This study is one of the first to adopt both discrete- and continuous-time Markov chain models to estimate the credit risk of banks loans. The credit risk of

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Chiao Tung Bank's loans was analysed. Another contribution is the time-varying risk premium that transfers the transition matrix to a risk-neutral transition matrix. Finally, the empirical findings revealed a rich and diverse perspective on the discrete- and continuous-time Markov chain models.

The article is organized as follows: the following section briefly describes the theoretical framework. Section III describes the data. Section IV discusses the main empirical results. Section V concludes the article.

II. Theoretical Framework

Discrete-time Markov chain model

Estimation in a discrete-time Markov chain is based on the fact that the transition away from a given state i can be viewed as a multinomial approach. Let $N_i(t)$ denote the umber of firms recorded to be in state i at the beginning of year t and $N_{ij}(t,\ t+1)$ denote the number of firms that migrate to state j at time $t+1$. As a result, the estimation of the one-year transition probability is

$$p_{ij} = \frac{N_{ij}(t, t+1)}{N_i(t)} \tag{1}$$

Therefore, Equation 1 is the element of discrete-time transition matrix P.

Continuous-time Markov chain model

Estimation based on a continuous-time Markov chain relies on estimating the generator matrix. Let $\hat{\mathrm{P}}(t)$denote the transition probability matrix of a continuous-time Markov chain with finite state space $\{1, \ldots, K\}$ so that the ijth element of this matrix is $\mathrm{P}_{ij}(s, t) = \mathrm{P}(\eta_t = j |\ \eta_s = i)$, $s < t$. We assume the generator matrix Λ is a $\mathrm{K} \times \mathrm{K}$ matrix, and the diagonal element is $\lambda_{ii}(t) \equiv \lambda_i(t) \equiv -\sum_{j=1, j\neq i}^{K} \lambda_{ij}(t), i \neq j$. The maximum-likelihood estimator of λ_{ij} based on observing realizations of the chain from zero to T is $\hat{\lambda}_{ij} = N_{ij}(T)/\int_0^T Y_i(s)\mathrm{d}s, i \neq j$, where $Y_i(s)$ is the number of firms rated i at time s and so the denominator of $\hat{\lambda}_{ij}$ gives the total time spent in state i by all the firms in the sample. Therefore, the continuous-time transition matrix is estimated as

$$\hat{\mathrm{P}} = \exp(\Lambda t) \tag{2}$$

Time-varying risk premium

The risk premium was extracted from the bank loan value and the corresponding risk-free price. First, let $V_0(t,\ T)$ be the time-t price of a risk-free bond maturing at time T, and $V_i(t, T)$ be its higher risk, i.e. risky, counterpart for the rating class, i. Then, let δ be the proportions of the loan's principle and interest, which is collectable on default, $0 < \delta \leq 1$, where in general δ will be referred to as the recovery rate.[1]

The risk premium was estimated in the same way as Kijima and Komoribayashi (1998) did in order to transfer the discrete-time transition matrix, P and continuous-time transition matrix, $\hat{\mathrm{P}}$, to a risk-neutral transition matrix.[2] Consequently, the relation between loan value and risk-free price under risk-neutral probability measure is

$$\begin{aligned} V_i(t,T) &= V_0(t,T)\left\{\tilde{Q}_t^i(\tau > T) + \delta\left[1 - \tilde{Q}_t^i(\tau > T)\right]\right\} \\ &= V_0(t,T)\left\{\delta + (1-\delta)\tilde{Q}_t^i(\tau > T)\right\} \end{aligned} \tag{3}$$

where $\tilde{Q}_t^i(\tau > T)$ is the probability under the risk-neutral probability measure that the loan with rating class i will not be in default before time T. As a result, the probability of default occurs before time T is

$$\tilde{Q}_t^i(\tau \leq T) = \frac{V_0(t,T) - V_i(t,T)}{(1-\delta)V_0(t,T)}, \quad \text{for } i = 1, \ldots, K \text{ and } T = 1, 2, \ldots, \tag{4}$$

According to Basel II, the loss given default (LGD) was defined as

$$\mathrm{LGD} = \text{The probability of default} \times (1-\delta) \tag{5}$$

Therefore, the probability of default and LGD of a bank's loans in Taiwan were estimated.

III. Data

In this study, the credit risk for Chiao Tung Bank was estimated. Chiao Tung Bank was established in 1907 and is now Taiwan's second largest investment bank. Chiao Tung Bank also has significant and expanding business in pioneer and venture capital investments.

The data are from two databases, Taiwan Corporate Risk Index (TCRI) and short-and long-term bank loans as reported in the Taiwan Economic Journal (TEJ). Both these TEJ databases contain issuer credit ratings from Quarter 1, 1998 to Quarter

[1] If there is no collateral or asset backing, then $\delta = 0$.
[2] There are two approaches to gauge the risk premium, namely Jarrow *et al.* (1997) and Kijima and Komoribayashi (1998). However, Jarrow *et al.*'s procedure will cause the risk premium to explode. Consequently, Kijima and Komoribayashi's (1998) method was adopted to extract the risk premium, which is time-varying.

4, 2003. The assigned credit rating represents TEJ's assessment on the likelihood of each issuer honouring any type of future debt payment.

The TCRI database uses a numerical rating from 1 to 9 and D for each classification. Obligations rated 1 are generally considered to be the lowest in terms of default risk, which is similar to the investment grade for Standard & Poor's and Moody. Obligations rated 9 are the most risky and rating class D denotes a default borrower.

Subsequently, the credit risk of bank loans was investigated according to the debt contract of each borrower based on the short- and long-term bank database. The yield of government bonds, as published by the Central Bank in Taiwan, was taken as proxy for the risk-free rate. Furthermore, the yield of a government bond whose maturity was the closest was interpolated and taken as a risk-free rate to make the yield of a government bond and loan consistent.

Generally, banks will set a recovery rate according to kind, liquidity and value of collaterals before lending. Fons (1987), Longstaff and Schwartz (1995), Carty and Lieberman (1996) and Briys and de Varenne (1997) assumed a constant recovery rate. Copeland and Jones (2001) assumed that the recovery rate is equal to zero in all sample years. There is no clear definition of recovery rate. Consequently, the recovery rate was assumed to be exogenous variables from 0.1 to 0.9 in this study.

In conclusion, credit risk for at least a one-year horizon was analysed, thus excluding observations of short-term loans, incomplete data and loans that have an overly low rate because they are likely to have resulted from aggressive accounting politics and would have biased the results. Consequently, the credit risk of mid- and long-term loans with posting collaterals for Chiao Tung Bank's loans were analysed.

IV. Empirical Results

First, the discrete-time transition matrix was calculated with Equation 1. Furthermore, the continuous-time transition matrix was calculated using Equation 2. The average transition matrix of Chiao Tung Bank from 1998 to 2003 is shown in Table 1. It is interesting to note that the default probability of the discrete-time Markov chain model was lower in the speculative grade than that of the continuous-time Markov chain model.

Secondly, the risk premium was estimated to transform the transition matrix into a risk-neutral transition matrix using the same approach as Kijima and Komoribayashi (1998), as listed in Table 2. Thirdly, due to risk-neutral probability measure, the transition matrix was modified to a risk-neutral transition matrix and the results are shown in

Table 1. Average transition matrix, 1998–2003

Rating at the end of year										
Initial rating	1	2	3	4	5	6	7	8	9	D
Panel A. Discrete time										
1	0.739	0.161	0.050	0.017	0.000	0.017	0.000	0.016	0.000	0.000
2	0.038	0.689	0.160	0.049	0.044	0.005	0.015	0.000	0.000	0.000
3	0.000	0.026	0.662	0.206	0.072	0.017	0.017	0.000	0.000	0.000
4	0.000	0.007	0.029	0.701	0.157	0.083	0.017	0.000	0.006	0.000
5	0.000	0.000	0.000	0.045	0.713	0.156	0.062	0.017	0.000	0.007
6	0.000	0.000	0.000	0.010	0.109	0.606	0.184	0.049	0.024	0.018
7	0.000	0.000	0.000	0.002	0.013	0.137	0.647	0.118	0.055	0.028
8	0.000	0.000	0.000	0.000	0.005	0.044	0.132	0.593	0.173	0.053
9	0.000	0.000	0.000	0.000	0.000	0.020	0.058	0.087	0.731	0.104
Panel B. Continuous time										
1	0.808	0.088	0.045	0.023	0.016	0.011	0.006	0.001	0.001	0.001
2	0.021	0.721	0.146	0.044	0.040	0.015	0.012	0.001	0.000	0.000
3	0.002	0.030	0.658	0.208	0.067	0.021	0.012	0.001	0.001	0.000
4	0.000	0.004	0.027	0.691	0.185	0.059	0.022	0.006	0.004	0.002
5	0.000	0.000	0.001	0.059	0.699	0.162	0.056	0.010	0.006	0.007
6	0.000	0.000	0.000	0.010	0.115	0.618	0.180	0.036	0.021	0.020
7	0.000	0.000	0.000	0.003	0.011	0.119	0.613	0.147	0.061	0.046
8	0.000	0.000	0.000	0.000	0.001	0.025	0.155	0.541	0.186	0.092
9	0.000	0.000	0.000	0.000	0.000	0.018	0.066	0.076	0.656	0.184

Table 2. Average risk premium

Rating class	Year 1998	1999	2000	2001	2002	2003
Panel A. Discrete time						
1	1.000	1.000	1.000	0.973	1.000	1.000
2	1.000	1.000	1.000	0.964	0.946	1.000
3	1.000	1.000	1.000	1.000	1.000	0.915
4	0.996	1.000	1.000	0.999	0.979	0.952
5	1.000	1.000	1.000	0.986	0.986	0.977
6	1.000	1.000	1.000	0.942	0.932	0.861
7	0.994	1.004	1.000	1.000	0.982	1.000
8	1.002	1.011	0.968	0.649	0.309	0.889
9	0.984	1.000	1.000	1.000	1.016	1.011
Panel B. Continuous time						
1	1.000	1.000	1.000	0.977	1.000	1.000
2	1.000	1.000	1.000	0.962	0.943	1.000
3	1.000	1.000	1.000	1.000	1.000	0.946
4	0.996	1.000	1.000	0.988	0.960	0.948
5	1.000	1.000	1.000	0.983	0.987	0.991
6	1.000	1.000	1.000	0.987	1.007	0.898
7	0.994	1.016	1.000	1.000	0.977	1.000
8	1.042	0.979	0.967	0.856	0.836	0.675
9	0.984	1.033	1.037	1.013	1.047	1.066

Table 3. Risk-neutral transition matrix, 1998–2003

Initial rating	Rating at the end of year 1	2	3	4	5	6	7	8	9	D
Panel A. Discrete time										
1	0.736	0.160	0.054	0.017	0.000	0.017	0.000	0.016	0.000	0.000
2	0.038	0.678	0.158	0.048	0.043	0.005	0.014	0.000	0.000	0.016
3	0.000	0.026	0.656	0.204	0.071	0.017	0.017	0.000	0.000	0.009
4	0.000	0.007	0.028	0.688	0.154	0.081	0.016	0.000	0.008	0.018
5	0.000	0.000	0.000	0.045	0.709	0.155	0.062	0.017	0.000	0.012
6	0.000	0.000	0.000	0.010	0.107	0.595	0.180	0.048	0.024	0.036
7	0.000	0.000	0.000	0.002	0.013	0.137	0.645	0.117	0.055	0.031
8	0.000	0.000	0.000	0.000	0.004	0.040	0.118	0.529	0.154	0.155
9	0.000	0.000	0.000	0.000	0.000	0.020	0.059	0.090	0.753	0.078
Panel B. Continuous time										
1	0.803	0.088	0.045	0.023	0.016	0.011	0.006	0.001	0.002	0.005
2	0.021	0.710	0.144	0.043	0.039	0.014	0.012	0.001	0.000	0.016
3	0.001	0.030	0.648	0.205	0.066	0.021	0.012	0.001	0.000	0.016
4	0.000	0.004	0.027	0.683	0.183	0.058	0.022	0.006	0.000	0.017
5	0.000	0.000	0.000	0.058	0.693	0.161	0.055	0.010	0.005	0.018
6	0.000	0.000	0.000	0.009	0.110	0.610	0.173	0.034	0.020	0.044
7	0.000	0.000	0.000	0.003	0.011	0.118	0.611	0.147	0.060	0.050
8	0.000	0.000	0.000	0.000	0.001	0.020	0.124	0.524	0.149	0.182
9	0.000	0.000	0.000	0.000	0.000	0.018	0.066	0.076	0.646	0.194

Table 3. For Table 3, the speculative rating class has a lower default probability in a discrete-time Markov chain model than that in a continuous-time model. On the other hand, the default probability of a lowest risk rating class is 0.005 in the continuous-time Markov chain model compared to zero in the discrete-time Markov chain model. That is, these probabilities may be underestimated in the discrete-time Markov chain model both in lowest risk and speculative rating class.

Table 4. The estimated default probability, LGD and NPL ratio of Chiao Tung Bank

	1998	1999	2000	2001	2002	2003
Panel A. Discrete time						
PD	0.0217	0.0207	0.0232	0.0488	0.0501	0.0750
LGD	0.0098	0.0104	0.0108	0.0207	0.0199	0.0291
Panel B. Continuous time						
PD	0.0413	0.0375	0.0423	0.0893	0.1265	0.0661
LGD	0.0197	0.0188	0.0201	0.0396	0.0601	0.0289
NPL ratio	0.0160	0.0160	0.0156	0.0436	0.0307	0.0240

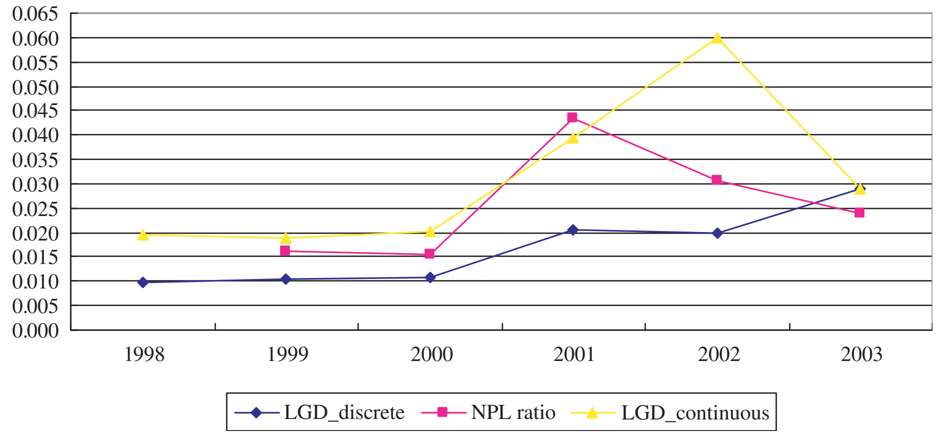

Fig. 1. Chiao Tung Bank's LGD and NPL ratio

Finally, the nonperforming loan[3] (NPL) ratio was compared with LGD as seen in Table 4 and Fig. 1. The LGD is an *ex-ante* perspective. Oppositely, the NPL ratio is an *ex-post* view.[4] Therefore, the LGD should be higher than the NPL ratio except for 2001. Since Taiwan's government is asking banks to write-off their bad debts by selling the bad debts to asset management companies (AMC), the NPL ratio is higher than LGD in 2001. Consequently, as can be seen from Table 4 and Fig. 1, the estimated results for the continuous-time Markov chain model are more reliable in determining the credit risk of bank loans.

V. Concluding Remarks

An important feature of the discrete-time Markov chain model is that if a transition from category i to j does not occur in a given period, the estimate of the corresponding rate is zero. However, for the lowest risk grade, such as AAA in Standard and Poor's rating, to default in a given period is a rare event. These rare events are ignored by the discrete-time Markov chain model. Although there are no transitions of AAA to default, there are transitions from AAA to AA and from AA to default. As a result, the estimator for transitions from AAA to default should be nonzero. Therefore, the discrete-time Markov chain model has to be modified to capture these rare events.

In order to avoid the embedded problem for discrete-time observations, the continuous-time Markov chain model was adopted to make a meaningful estimate of the credit risk, especially for rare events. In this article, the credit risk of a bank's loans was estimated and compared using discrete- and continuous-time Markov chains. The time-varying risk premium was also estimated in order to transfer the transition matrix to a risk-neutral transition matrix. These issues have never been discussed in previous studies. Finally, comparing the LGD and NPL ratio, it was found that the continuous-time Markov chain model is more reliable

[3] The NPL refers to loan accounts, the principle and/or interest of which are past due or exceed the due date. Therefore, the NPL ratio is NPL divided by the total loan portfolio. In this study, the NPL ratio was obtained from TEJ's database.
[4] Since the NPL ratio was concerned, as loans were uncollected after all collection option, such as the realization of collateral or the institution of legal proceedings, has been exhausted.

for measuring the credit risk of bank loans. Therefore, we recommend that the continuous-time Markov chain rather than the discrete-time Markov chain be used for gauging the credit risk of bank loans. In conclusion, the proposed approach is expected to be helpful for banks in facing the coming Basel Capital Accord.

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