Frictional damping in radiative electrodynamics and its scaling to macroscopic systems

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Abstract

Radiation force in Abraham-Lorentz-Dirac equation is revisited for possible signature of irreversible action in the dynamics. The analysis shows that the classical electron can 'dissipate' out a certain fraction of field energy that distinguishes itself from the well known Larmor radiation loss. The dissipation occurs only when the acceleration changes course from its linear evolution. A measure of the dissipative action in the radiative electrodynamics is worked out to compare the two distinctly different modes of energy losses. The measure is shown to be applicable uniquely in all scales of the systems and is used as a common thread to explain frictional contributions of phonons and electrons recently reported.

Key words: Abraham-Lorentz-Dirac equation, radiation force, Unruh radiation, frictional dissipation.
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Accelerated charged particle experiences reaction force having some similarity to frictional damping. This apparent in the radiative electrodynamics, $m_0 c\dot{v}_\mu = (q/c)\ F_{\mu\nu} v^\nu +\ R_\mu$, due to Abraham-Lorentz-Dirac (ALD, for brevity) [1]. m_0 and q respectively are the rest mass and charge of the electron, $F_{\mu\nu}$ is the external electromagnetic field, and v_μ, $\dot{v}_\mu$, and $\ddot{v}_\mu$ are the instant 4-velocity, 4-acceleraton, and 4-jerk respectively. Unlike the other forces involved in the dynamics, the reaction $R_\mu \equiv (\ddot{v}_\mu + \dot{v}^2 v_\mu) 2q^2/3c$ arising out of bound and radiated electromagnetic field momenta does not remain invariant under time inversion. The loss of time invariance in R_μ is not due to the radiation reaction, $(\dot{v}^2 v_\mu) 2q^2/3c$, since this term is compensated out by the 4-velocity components of the relativistic form of Schott term, $\ddot{v}_\mu (2q^2/3c)$, that is, $(2q^2/3c)(\ddot{v}v)v_\mu$, $(\ddot{v}v \equiv \ddot{v}^\alpha v_\alpha = -\dot{v}^2)$. The remaining part of the 4-jerk force is thus responsible for the loss of the time reversibility in ALD electrodynamics. Noting this feature some authors have preferred to recognize R_μ as damping force though very little is talked about its irreversible consequence. This work introspects into the property of R_μ for its irreversible manifestations in electrodynamics. For this, R_μ is rewritten as sum of two orthogonal 4-vectors as $R_\mu \equiv R'_\mu + R''_\mu$, where $R'_\mu = (2q^2/3c)[\ddot{v}_\mu + \dot{v}^2 v_\mu - (\ddot{v}\dot{v})\dot{v}_\mu/\dot{v}^2]$, and $R''_\mu = (2q^2/3c)(\ddot{v}\dot{v})\dot{v}_\mu/\dot{v}^2 \equiv -(2q^2/3c)(\ddot{v}e)e_\mu$, $R'^\mu R''_\mu = 0$ and $e_\mu \equiv \dot{v}_\mu/\sqrt{-\dot{v}^2}$ is unit 4-acceleration. R'_μ can be equivalently expressed as $R'_\mu = (2q^2/3c)[\sqrt{-\dot{v}^2}(de_\mu/d\tau_z) + \dot{v}^2 v_\mu]$, $\sqrt{-\dot{v}^2}(de_\mu/d\tau_z) = [\ddot{v}_\mu - (\ddot{v}\dot{v})\dot{v}_\mu/\dot{v}^2]$, showing that its first term manifesting from the change of acceleration course in motion can upset dynamic reversibility under time reversal. This aspect is considered in the subsequent analysis made from an instant commoving inertial frame ($\bar{\mathrm{v}}$=0 and γ=1), where the 3-force due to radiation reaction is explicitly a null. There, the 3-vector component of $R_\mu \equiv [\mathrm{R}^0, -\bar{\mathrm{R}}]$ is given by $\bar{\mathrm{R}} \equiv (2q^2/3c)(\ddot{\bar{\mathrm{v}}}/\mathrm{c}^2)$ and the local force equation is expressed as $m_0\dot{\bar{\mathrm{v}}} = q\bar{\mathrm{E}}+\bar{\mathrm{R}}$, $\bar{\mathrm{R}} = \bar{\mathrm{R}}' + \bar{\mathrm{R}}''$, $\bar{\mathrm{R}}' \equiv \Gamma\dot{\mathrm{v}}\mathrm{d}\hat{\mathrm{e}}/\mathrm{dt}$ ($\hat{\mathrm{e}} \equiv \dot{\bar{\mathrm{v}}}/\dot{\mathrm{v}}$, and $\mathrm{d}\hat{\mathrm{e}}/\mathrm{dt} \equiv [\ddot{\bar{\mathrm{v}}}\text{-}(\ddot{\bar{\mathrm{v}}}\bullet\hat{\mathrm{e}})\hat{\mathrm{e}}]/\dot{\mathrm{v}}$), and $\bar{\mathrm{R}}'' \equiv \Gamma(\ddot{\bar{\mathrm{v}}}\bullet\hat{\mathrm{e}})\hat{\mathrm{e}}$, $\Gamma \equiv 2q^2/c^3$, and $\bar{\mathrm{R}}'\bullet\bar{\mathrm{R}}'' = 0$. (The 4-vector components involved in ALD equation are $v^\mu \equiv [v_0, \bar{v}]$, $\dot{v}^\mu \equiv [\dot{v}_0, \dot{\bar{v}}]$, and $\ddot{v}^\mu \equiv [\ddot{v}_0, \ddot{\bar{v}}]$ where $v_0 = \gamma$, $\bar{v} = \gamma\bar{\mathrm{v}}/\mathrm{c}$ $\dot{v}_0 \equiv \dot{\gamma}$, $\dot{\gamma} = \gamma^4(\bar{\mathrm{v}}\bullet\dot{\bar{\mathrm{v}}})/c^3$, $\dot{\bar{\mathrm{v}}} \equiv \mathrm{d}\bar{\mathrm{v}}/\mathrm{dt}$, $\dot{\bar{v}} \equiv \dot{\gamma}\,\bar{\mathrm{v}}/\mathrm{c} + \gamma^2\,\dot{\bar{\mathrm{v}}}/\mathrm{c}^2$, $\ddot{v}_0 \equiv \ddot{\gamma} = 4\gamma^7(\dot{\bar{\mathrm{v}}}\bullet\bar{\mathrm{v}})^2/c^6 +$ $\gamma^5(\ddot{\bar{\mathrm{v}}}\bullet\bar{\mathrm{v}} + \dot{\bar{\mathrm{v}}}\bullet\dot{\bar{\mathrm{v}}})/\mathrm{c}^4$, $\ddot{\bar{\mathrm{v}}} \equiv \mathrm{d}\dot{\bar{\mathrm{v}}}/\mathrm{dt} = \mathrm{d}^2\bar{\mathrm{v}}/\mathrm{dt}^2$ and $\ddot{\bar{v}} = \ddot{\gamma}\,\bar{\mathrm{v}}/\mathrm{c} + 3\gamma^5(\dot{\bar{\mathrm{v}}}\bullet\bar{\mathrm{v}})\dot{\bar{\mathrm{v}}}/c^5 + \gamma^3\,\ddot{\bar{\mathrm{v}}}/\mathrm{c}^3$)). If $\bar{\mathrm{E}}_e$ and $\bar{\mathrm{E}}_\varepsilon$ respectively are the two components of the external field, the dynamics is expressed by the two component equations: $\mathrm{q}\bar{\mathrm{E}}_e \equiv (q\bar{\mathrm{E}}\bullet\hat{\mathrm{e}})\hat{\mathrm{e}} = m_0\dot{\bar{\mathrm{v}}}\text{ -}\Gamma(\ddot{\bar{\mathrm{v}}}\bullet\hat{\mathrm{e}})\hat{\mathrm{e}}$ and $\mathrm{q}\bar{\mathrm{E}}_\varepsilon \equiv (q\bar{\mathrm{E}}\bullet\bar{\varepsilon})\bar{\varepsilon} = -\Gamma\dot{\mathrm{v}}\mathrm{d}\hat{\mathrm{e}}/\mathrm{dt}$, $\bar{\varepsilon}$ is the unit vector of $\mathrm{d}\hat{\mathrm{e}}/\mathrm{dt}$ ($\bar{\varepsilon}\bullet\hat{\mathrm{e}} = 0$).

The two component equations of electrodynamics can be rewritten as $(\mathrm{q}/m_0)\bar{\mathrm{E}}_e = \dot{\bar{\mathrm{v}}}\text{ -}(\Gamma/m_0)(\ddot{\bar{\mathrm{v}}}\bullet\hat{\mathrm{e}})\hat{\mathrm{e}} \equiv \dot{\bar{\mathrm{v}}} - \delta\dot{\bar{\mathrm{v}}}$, and $(\mathrm{q}/\mathrm{m}_0)\bar{\mathrm{E}}_\varepsilon = -(\Gamma/\mathrm{m}_0)\dot{\mathrm{v}}\mathrm{d}\hat{\mathrm{e}}/\mathrm{dt}$ respectively, where $\dot{\bar{\mathrm{v}}} - \delta\dot{\bar{\mathrm{v}}}$ is the acceleration contributed by the field component $\bar{\mathrm{E}}_e$ directly and $\delta\dot{\bar{\mathrm{v}}} \equiv (\Gamma/\mathrm{m}_0)(\ddot{\bar{\mathrm{v}}}\bullet\hat{\mathrm{e}})\hat{\mathrm{e}}$ is that contributed indirectly via the jerk's action over the characteristic time Γ/m_0. The first equation suggests that the power, $\Gamma(\dot{\mathrm{v}}\text{-}\delta\dot{\mathrm{v}})^2 \equiv \mathrm{P}_e$ representing a major part of the Larmor radiation loss ($\Gamma\dot{\mathrm{v}}^2$) is made up of the energy flux component, $8\pi\Gamma(\mathrm{q}/\mathrm{m}_0)^2(\mathrm{E_e}^2/8\pi)$ of the external field. The flux P_e would fully account for the observed radiation power when the jerk is either disappearing or orthogonal to acceleration to result in null $\delta\dot{\bar{\mathrm{v}}}$. The second equation $(\mathrm{q}/\mathrm{m}_0)\bar{\mathrm{E}}_\varepsilon = -(\Gamma/\mathrm{m}_0)\dot{\mathrm{v}}\mathrm{d}\hat{\mathrm{e}}/\mathrm{dt}$ would in that case account for the energy flux of $8\pi\Gamma(\mathrm{q}/\mathrm{m}_0)^2(\mathrm{E}_\varepsilon{}^2/8\pi) \equiv \mathrm{P}_\varepsilon$ from the other component of the field. The component equation suggests that P_ε should manifest dynamically as $(1/m_0)^2\Gamma^3\dot{v}^2(d\hat{e}/dt)^2$. P_e and P_ε appear to be different kind of power manifestations out of the field intensity $(E^2/8\pi)c$. P_ε would be nonzero when the motion involves change in the acceleration course ($d\hat{e}/dt \neq 0$). P_ε is distinguishably different from the electrical power spent in the displacement of the external field for the manifestation of ordered form of energies, namely, Larmor radiation and particle's kinetic energy corrected for the contribution from jerk action in the time period of (Γ/m_0). The field displacement to be expressed as $q\bar{E}\bullet\bar{v} = \Gamma\dot{v}^2 + dK/dt - \Gamma d(\bar{\dot{v}}\bullet\bar{v})/dt$ shows that the manifested power has linear connection with Γ unlike the cubic connection seen in $P_\varepsilon \equiv (1/m_0)^2\Gamma^3\dot{v}^2(d\hat{e}/dt)^2$. Evidently, P_ε manifestation has no displacement signature of a force and it can be attributed to power flow in disordered form having no work potential. The power partitioning in the disordered mode and Larmor radiation mode bear the ratio of $(\Gamma/m_0)^2(d\hat{e}/dt)^2 \equiv \Omega$ (say).

Replacing $8\pi\,\Gamma(q/m_0)^2$ by $(4/3)\,[4\pi c(q^2/m_0c^2)^2]$ in the power expression, $P_\varepsilon \equiv 8\pi\Gamma(q/m_0)^2(E_\varepsilon{}^2/8\pi)$, one finds that the field of intensity $(E^2/8\pi)c$ is sweeping across the surface area, $(4/3)\,[4\pi(q^2/m_0c^2)^2] \equiv \Sigma$ (say), enclosing the charge. The enclosed space is that of a sphere with radius, $\sqrt{\delta_x^2+\delta_y^2+\delta_z^2}$, $\sqrt{\delta_x^2} = \sqrt{\delta_y^2} = \sqrt{\delta_z^2} \equiv 2q^2/3m_0c^2$, which is statistically significant from the point of view that the electron enjoys freedom on the dynamically relevant dimension, $2q^2/3m_0c^2$ equally in the three directions. For subsequent analysis, Σ, will be recalled as dynamically relevant surface.

Noting that the magnitude $|d\hat{e}/dt|$ is the angular speed ($\dot{\theta}$) of the unit vector $\hat{e}$, the ratio Ω represents the square of angular displacement of acceleration in the characteristic time period of

$(\Gamma/m_0) \approx 6.2\text{x}10^{-24}$ s. The partitioned power P_ε is generally insignificant compared to the Larmor power loss since the characteristic angular displacement $6.2\text{x}10^{-24}\dot{\theta}$ is negligibly small. An ultra relativistic electron (1GeV) in synchrotron of radius 10 m will have $\dot{\theta}$ value of about $3\text{x}10^7$ s^{-1}, so that the said angular displacement is about $1.8\text{x}10^{-16}$ radian. On event horizon of a black hole, electrons and positrons are produced in pairs and with c as escape velocity, g/c is the magnitude of their angular velocities ($\hat{\dot{\theta}}$) in opposite directions (g being the gravitational acceleration at event horizon of the spherical black hole). There, the quantities Ω and P_ε can be respectively expressed as $\Omega = (\Gamma g/m_0 c)^2$ and $P_\varepsilon \equiv \Gamma(\Gamma/m_0 c)^2 g^4$. If one considers that that the energy flux P_ε is lost from the dynamically relevant surface of area $\Sigma \equiv (4/3)\,[4\pi(q^2/m_0 c^2)^2]$, the flux intensity can be expressed as $\Phi \equiv P_\varepsilon/\Sigma$, $\Phi = (160\pi\alpha/3)[\sigma(\hbar g/2\pi c k_B)^4]$, where $\alpha = (q^2/c\hbar)$, is the fine structure constant and $\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$, is the Stefan-Boltzmann constant. The factor $(160\pi\alpha/3)$ having approximate value of $(1.05)^4$, the evaluated energy flux is seen to be quite comparable with reported thermal property of event horizon of a black hole [2].

From the treatment made so far, it is apparent that the dissipative power loss P_ε is present only when the acceleration deviates from its linear evolution ($|d\hat{e}/dt| \equiv \dot{\theta} \neq 0$). The linearly accelerated charge constituting a Rindler frame however encounters thermally excited quantized vacuum field of temperature $[\dot{v}\hbar/(2\pi c k_B)] \equiv T_U$ (say) [3], and as the result it's acceleration course can undergo fluctuation from its linear course by the transverse action of the field. Within the characteristic time (Γ/m_0), the acceleration will undergo change of its course with the r.m.s. value of $(\Gamma/m_0)\dot{\theta}$ radian, where $\dot{\theta} = 2\pi k_B T_U/\hbar$. Accordingly, the power dissipation is given by , $P_\varepsilon \equiv (1/m_0)^2\Gamma^3\dot{v}^2\dot{\theta}^2$. In P_ε, substituting $\dot{v}$ as $\dot{v} \equiv (2\pi c k_B T_U)/\hbar$ and using the expression for the r.m.s. value of the acceleration course, P_ε can be simplified as $P_\varepsilon \equiv \Gamma(\Gamma/m_0 c)^2 (2\pi c k_B T_U/\hbar)^4$. If one consider that P_ε is lost from the dynamically relevant surface area, Σ, then the intensity of energy flux is given by $(1.05)^4 \sigma T_U{}^4$. The flux intensity arrived is somewhat an overestimate of the ultimate value ($\sigma T_U{}^4$) expected for the surface to attain as the consequence of thermal scattering from the surrounding heat bath of temperature T_U. Nevertheless, the above exercise shows that the Rindler frame dynamically attains the thermal state much the way a subsystem inside a heat bath attains in the Laboratory. The overestimating factor, $160\pi\alpha/3 \approx (1.05)^4$ for the flux intensity stems from the presumption of equality of the radiating surface with the dynamically relevant surface Σ.

From the analysis of radiative motion in electrodynamics it is seen that dissipative loss P_ε is encountered whenever the charged particle driven by transverse force, $q\bar{E}_\varepsilon = -\Gamma\dot{v}d\hat{e}/dt$, changes its course of acceleration. The loss occurs in and around the surface, $(4/3)\,[4\pi(q^2/m_0c^2)^2]$, of the charged particle and involves the characteristic time constant $\Gamma/m_0 \equiv q^2/m_0c^3$. Noting this feature, it is attempted to work out the possible power loss, $P_\varepsilon \equiv (1/m_0)^2\Gamma^3\dot{v}^2(d\hat{e}/dt)^2$ of a universal oscillator having natural frequency $\omega_0 = (2\Gamma/m_0)^{-1}$, when it is driven by the transverse force, $-\Gamma\dot{v}d\hat{e}/dt$. The driven oscillation in the plane orthogonal to the acceleration can be described in universal representation of two orthogonal modes as $d^2Q_1/d\tau^2 + 2\,\varsigma dQ_1/d\tau + Q_1 = \cos(\varpi\tau)$ and $d^2Q_2/d\tau^2 + 2\,\varsigma dQ_2/d\tau + Q_2 = \sin(\varpi\tau)$, where the non-dimensional quantities are defined as $\tau = \omega_0 t$. Q_1 and Q_2 are the orthogonal components of $\tilde{Q} = \tilde{q}/q_0$, $\tilde{Q}$ representing internal coordinates ($\tilde{q}$). The quantity q_0 expresses the displacement amplitude of the periodic force, $-\Gamma\dot{v}d\hat{e}/dt$ and is given by $q_0 = \Gamma\dot{v}\left|d\hat{e}/dt\right|/(m_0\omega_0^2)$.

The damping ratio ς given by $2\varsigma \equiv \omega'/\omega_0$ is non-dimensional expression of the damping constant per unit mass, and the argument $\varpi\tau$ is identical to ωt, ω being angular frequency of the periodic force. It may be noted that damping constant for an oscillator of mass m_0 in absolute representation is $m_0\omega'$ and damping power loss is $m_0\omega'\omega_0^2 q_0^2$. For the two modes of the universal oscillator, the power input of the external force through its displacements in the internal coordinate are respectively given as $P_1(\tau) = (dQ_1/d\tau)\cos(\varpi\tau)$ and $P_2(\tau) = (dQ_2/d\tau)\sin(\varpi\tau)$. The frictional dissipation in the respective cases are $D_1(\tau) = 2\varsigma(dQ_1/d\tau)^2$ and $D_2(\tau) = 2\varsigma(dQ_2/d\tau)^2$. The power input in each mode should be effectively damped through friction loss so that the vibration excitation remains minimal and electron's internal energy continues to be close to m_0c^2. Thus for all τ, the values of $D_1(\tau)$ and $D_2(\tau)$ approach to those of $P_1(\tau)$ and $P_2(\tau)$ respectively, that is, $D_1(\tau) \approx [\cos^2(\varpi\tau)]/2\varsigma$, and $D_2(\tau) \approx [\sin^2(\varpi\tau)]/2\varsigma$. Accordingly, in the non-dimensional representation the total dissipation rate in the internal modes is $D(\tau) \approx 1/2\varsigma$, and in absolute representation the rate is given by $(1/2\varsigma)m_0\omega_0^3 q_0^2$. As $q_0 = \Gamma\dot{v}\left|d\hat{e}/dt\right|/(m_0\omega_0^2)$, the rate becomes $(1/2\varsigma)\dot{v}^2\left|d\hat{e}/dt\right|^2\Gamma^2/(m_0\omega_0)$. This result shows that, with the choice of the equality, $(2\varsigma\omega_0)^{-1} = \Gamma/m_0$, the power loss due to frictional damping of the internal modes will be identical to the dissipative power of the accelerated charged particle, $P_\varepsilon \equiv (1/m_0)^2\Gamma^3\dot{v}^2(d\hat{e}/dt)^2$, as was obtained from the electrodynamic consideration. The equality corroborates to critical damping ($\varsigma = 1$) of the internal modes with their natural frequency $\omega_0 = (2\Gamma/m_0)^{-1}$. It may be recalled that Γ/m_0 is the characteristic time involved in ALD equation. The overall exercise shows that the power P_ε can manifest as dissipation from critically damped internal modes of the oscillator no soon it encounters with changed course of acceleration by the action of external force. The power input through the transverse force component of the external field, $(q/m_0)\overline{E}_\varepsilon$ gives the measure of dissipative loss, $(1/2\varsigma)\dot{v}^2 |d\hat{e}/dt|^2 \Gamma^2/(m_0\omega_0) \equiv (1/8)\dot{v}^2 |d\hat{e}/dt|^2 m_0/\omega_0^3$.

The dissipative principle noted in the oscillator model for the internal modes of an accelerated charge is expected to be applicable to macroscopic system when its elementary components like phonons and electrons, undergo change in their acceleration course by the action of external force. Generally, an oscillating mass (m_0) while undergoing change in its acceleration course by an external force critically dampens the power input at the rate of $(1/8)\dot{v}^2 |d\hat{e}/dt|^2 m_0/\omega_0^3$, where ω_0 is oscillation frequency, $\dot{v}$ and $|d\overline{e}/dt|$ respectively are instant acceleration and magnitude of its angular sweeping rate.

Action of shear force on a macroscopic body in its relative motion with respect to a second body in the frictional contact to result in dissipative energy loss can be modeled as interactive dynamics of a large number of phonon and charge oscillators constituting the interface of the two bodies. The shear force action causes stress induced oscillations of phonons and charge fields of the two surfaces forming the interface, and as the result the oscillators will suffer change in their acceleration course. Phonon modes with their instant accelerations given by $\ddot{\overline{q}}_i = -\varpi_i^2\overline{q}_i$ will experience torques to result in change of the acceleration course by $(2\pi/\varpi_i)|d\hat{e}_i/dt|$. $\overline{q}_i$ and ϖ_i are respectively displacement coordinates and oscillation frequencies and $|d\hat{e}_i/dt|$ is the magnitude of angular sweeping rate of the acceleration $\ddot{\overline{q}}_i$ ($\hat{e}_i$ representing unit vector of $\ddot{\overline{q}}_i$). With the shear velocity $\overline{V}$ of one surface over the other, the angular rate can be generally expressed as $|d\hat{e}_i/dt| = \eta_i V/q_i$ wherein $\eta_i V$ serves as the velocity component that transversely act on the radius vector $\bar{q}_i$. η_i involves not only the projection, $\cos(\bar{V} \wedge \bar{q}_i)$ but also the sheared coupling efficiency, the efficiency being decided by magnitude of coupling force normal to the interface engaging the two bodies. The frictional dissipation in the i[th] mode of oscillation is therefore given by $P_{\varepsilon;i} = (1/8)\dot{v}_i^2 |d\hat{e}_i/dt|^2 \mu_i/\varpi_i^3 \equiv \mu_i \varpi_i \eta_i^2 V^2/8$, where μ_i is the effective mass involved in the oscillation mode. The dissipation rate derived out of stress induced oscillation of phonons at the interface of the two bodies is therefore expressed as $P_\varepsilon \equiv \sum_i g_i \mu_i \varpi_i \eta_i^2 V^2/8$, where g_i is the degeneracy of the mode. The sum will be replaced by the following integral wherein the degeneracy factor is replaced by spectral distribution function, $g(\omega)$ of phonon modes: $P_\varepsilon \equiv (\tilde{\mu}\tilde{\eta}^2 V^2/8) \int_0^{\varpi_m} g(\varpi)\varpi \, d\varpi$, $\tilde{\mu}$ and $\tilde{\eta}$ being the mean values of all μ_i and η_i. For a surface of unit area, $g(\varpi)$ can be expressed as $g(\varpi) = 2\sqrt{2}\pi(\varpi/v_s^2)$, $\int_0^{\varpi_m} g(\varpi)\, d\varpi \approx 3N_s$, v_s being magnitude of stress velocity on the surface and N_s is the number of atoms per unit surface area. The upper limit of the angular frequency works out to be $\varpi_m^2 \approx (3/\sqrt{2}\pi) N_s v_s^2$. Considering the fact that the dissipation is not possible along the application of shear force because the force cannot change acceleration course in that direction, two third of the $3N_s$ modes will be effectively contributing to the dissipation. Accordingly, for the calculation of P_ε one uses $(2/3)g(\varpi)$ function instead of $g(\varpi)$ in the integration argument and writes $P_\varepsilon \equiv (\tilde{\mu}\tilde{\eta}^2 V^2/8)(2/3) \int_0^{\varpi_m} 2\sqrt{2}\pi(\varpi/v_s^2)\varpi \, d\varpi$, that is $P_\varepsilon \equiv [(\sqrt{2}\pi/18)\tilde{\mu}\tilde{\eta}^2 V^2/v_s^2]\varpi_m^3$. On substitution of ϖ_m expression in P_ε followed by simplification of the numerical part gives

$P_\varepsilon = 0.1369(\tilde{\mu}\tilde{\eta}^2V^2)v_s N_s^{3/2}$. For metal like niobium (bulk density ρ, 8.57 g/cc, and $\tilde{\mu} \approx$ 92.9064 a.m.u.) N_s is 1.4759x10^{15} cm^{-2}, and the sound speed in the metal (v_s) is 348000 cm s^{-1}. Thus for $\tilde{\eta} \approx 1$ and V=1 cm s^{-1}, P_ε value for the unit shearing area works out to be 4.17x10^5 ergs s^{-1}, and this leads to the value of frictional damping constant as 0.425 kg s^{-1}. For a small shearing surface like 50 nm x 5 nm, it is 1.06x10^{-12} kg s^{-1}. The phonon modes of silicon probe imparting the shear action on Nb surface add further dissipation. For silicon ($\tilde{\mu} \approx$ 28.0855 a.m.u.), N_s and v_s being 1.35x10^{15} cm^{-2} and 843310 cm s^{-1}, the contribution to dissipation works out to be 2.69x10^5 erg s^{-1} for 1 cm^2 surface of silicon. Thus for the shear action over about 250 nm^2 surfaces of Nb and Si, the observable value of the frictional coefficient due to phonons is 1.75x10^{-12} kg s^{-1}.

In metallic lattice, there are additional components of dissipative losses from shear action on the oscillations of (i) positively charged cores in the electric field of conduction electrons, and (ii) electron plasma itself. Besides vibrations in the normal modes, the charged cores undergo oscillations described as $\ddot{\bar{q}}_i' = (4\pi n_e zq^2 / \mu_i)\bar{q}' \equiv (m_0 / \mu_i)z\omega_p^2\, \bar{q}'$, where n_e is the density of conduction electrons, q is the electronic charge, zq is charge of the ith core of effective mass μ_i, m_0 is electronic mass, and $\omega_p = \sqrt{(4\pi n_e q^2 / m_0)}$ is the frequency of electron plasma in the metal. Noting that the core is undergoing harmonic oscillation with the frequency of $\omega_i' = \omega_p\sqrt{(m_0 / \mu_i)z}$, the action of a shear force in orthogonal direction to the linear oscillation will lead to the frictional dissipation, $P_{\varepsilon;i}' = \mu_i\omega_i'\eta_i'^2V^2 / 8$. As mentioned before, $\eta_i'V$ serves as component of the shear velocity $\bar{V}$. Thus the integral dissipation of all the charged cores per unit surface area of the lattice is given by $P_\varepsilon' = (\mu\omega'\eta'^2V^2 / 8)(2/3)N_s$, wherein the expression is written for the cores each with uniformly unit charge and effective mass (μ). When one considers that each of the N_s phonon modes can associate with an electron of either of the spin states ($\pm 1/2$), then the total number of vibration modes is double of what has been counted so far, leading to $P'_\varepsilon = (\mu\omega'\eta'^2 V^2/8)(4/3)N_s$. For Nb metal with its one electron per atom in the conduction band, n_e is $5.55\text{x}10^{22}$ cm^{-3}, and so ω_p is $1.34\text{x}10^{16}$ s^{-1}. Thus, using $\eta' \approx 1$ and V=1 cm s^{-1}, P'_ε is given by $12.34\text{x}10^5$ ergs s^{-1}. The dissipative loss due to shear action on electron plasma will be smaller as compared to P'_ε because of the lighter mass of electron and it (case (ii)) is not considered. The overall exercise shows that the dissipation losses involving conduction electrons and phonons for Nb-Si interfacial shearing action are in the ratio of 1.8: 1. The dissipation manifesting out of the conduction electrons' influence on core dynamics will be realized only above superconducting transition of the metal. In the superconducting region, however, this additional frictional loss over the phonons contribution will be absent because of coherent dynamics of the phonon modes coupled to cooper pairs formed out of the conduction electrons. The phonons contribution remains to be present in the superconducting region. The stated results are quite in agreement with recently reported findings [4,5].

References:

1. P.A.M. Dirac, Proc.Roy.Soc. London A167 (1938), 148.
2. S.W. Hawking, Nature 248, 30-31 (1974).
3. W.G. Unruh, Phys. Rev. D 14, (1976) 870.
4. A. Dayo, W. Alnasrallah, and J. Krim, Phy. Rev. Lett., 80 (1998) 1691.
5. Marcin Kisiel, Enrico Gnecco, Urs Gysin, Laurent Marot, Simon Rast and Ernst Meyer, Nature Materials j Advance online publication j www.nature.com/naturematerials