Comment on "Nuclear structure corrections in muonic deuterium"

A recent letter [1] investigated the effects of deuteron polarizability in the μD atom. The importance of understanding this atom has been heightened because of the possible light it could shed on the proton radius puzzle [2]. Here we confirm, clarify, and emphasize the significance of the result [1] that the effects of the third Zemach moment are canceled by part of the deuteron polarizability effects.

We focus on Coulomb effects and take the Hamiltonian to be the sum of muon kinetic energy T_μ , evaluated using the muon-deuteron reduced mass m_r, the muon-proton Coulomb interaction V_C and the nuclear Hamiltonian H_N with $H = T_\mu + V_C + H_N$. We set up perturbation theory such that excitation of the deuteron is not included in first order. Thus define the one-body central interaction $U(r)$

$$U(r) \equiv \langle D|V_C|D\rangle = \int d^3R|\Phi_D(\mathbf{R})|^2 \frac{\alpha}{|\mathbf{r}-\mathbf{R}|}, \quad (1)$$

where $\mathbf{r}(\mathbf{R})$ is the muon (proton) coordinate.

Friar [3] derived a well-organized perturbation theory to treat the difference between the potential U and the Coulomb interaction between point particles. This gives the well-known term dependent on the proton mean-square charge radius and also the effects of the third Zemach moment:

$$\delta E_Z = -\frac{\pi}{3}\alpha^2\phi^2(0)R_2^3 m_r, \quad (2)$$

$$R_2^3 \equiv \int d^3Rd^3R'|\phi_D(\vec{R})|^2|\phi_D(\vec{R}')|^2|\vec{R}-\vec{R}'|^3. \quad (3)$$

We shall show that this is the term that is canceled by part of the effects of the deuteron polarizability.

To do this, split the Hamiltonian into a dominant term H_0 and a perturbative term H_1, with $H_0 \equiv T_\mu + U(r) + H_N, H_1 \equiv V_C - U, H = H_0 + H_1$. The lowest-energy eigenstates of H_0 are product states of $T_\mu + U$, (ϕ_n) and the deuteron. The label n represents both discrete and continuum states. The ground state is defined to have $n = 0$, with energy $-E_0 \equiv \epsilon_0 + E_D > 0$.

The energy shift δE caused by H_1 is given by

$$\delta E = -\langle\phi_0, D|H_1 \frac{\Lambda}{E_0 + H_0} H_1|\phi_0, D\rangle \quad (4)$$

where Λ is the projection operator excludes the ground state of H_0. Furthermore, the matrix elements of H_1 vanish unless the deuteron is excited: $\langle\phi_0, D|H_1|\phi_n, D\rangle = 0$, so the deuteron can **not** appear in the intermediate states.

To isolate the term that cancels δE_Z, neglect the operator U in the energy denominator of Eq. (4), and use the plane wave representation for the intermediate muon states to obtain

$$\delta E = \int_0^\infty dE \int d^3Rd^3R'\phi_D^*\langle\vec{R}|E\rangle\langle E|\vec{R}'\rangle P_E(\vec{R},\vec{R}')\phi_D(\vec{R}'), (5)$$

where the states $|E\rangle$ are the neutron-proton continuum states and [1]

$$P_E(\vec{R},\vec{R}') \equiv -\alpha^2\phi^2(0)\int \frac{d^3q}{(2\pi)^3}\left(\frac{4\pi}{q^2}\right)^2\left(E+E_0+\frac{q^2}{2m_r}\right)^{-1} \times[e^{i\vec{q}\cdot(\vec{R}-\vec{R}')} - 1 - q^2\frac{(\vec{R}-\vec{R}')^2}{6}]. \quad (6)$$

The integral over q can be done analytically, and the result expanded in the small parameter $\beta \equiv \sqrt{2m(E+E_0)}|\vec{R}-\vec{R}'|$. Keeping **only** the lowest-order term which is independent of energy, we find [1]

$$P(\vec{R},\vec{R}') \approx \frac{-\pi}{3} m_r|\vec{R}-\vec{R}'|^3\alpha^2\phi^2(0). \quad (7)$$

The influence of this energy independent (EI) term is obtained by using Eq. (7) in Eq. (5). The evaluation is simplified by using the completeness relation in the form $\int_0^\infty dE|E\rangle\langle E| = I - |D\rangle\langle D|$. Using I in Eq. (5) forces $\vec{R} = \vec{R}'$ and the resulting term vanishes. Thus the integral over continuum states is replaced by $-|D\rangle\langle D|$ yielding

$$\delta E_{\rm EI} = \frac{\pi}{3} m_r\alpha^2\phi^2(0)R_2^3 = -\delta E_Z. \quad (8)$$

The influence of the third Zemach moment is canceled by a term arising from the deuteron polarizability! All remaining terms in the deuteron polarizability lead to reducing the energy of the 2S state by -1.60 meV [1].

Using the ANL V18 potential gives $\delta E_Z \approx -0.37$ meV, so the total deuteron polarizability correction in [1] is actually -1.23 meV. The difference between -1.23 meV and -1.60 meV would have substantial impact on μD experiments that seek to measure the deuteron radius and possibly resolve the proton radius puzzle.

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