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On multiple-class prediction of issuer credit ratings

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SUMMARY

For multiple-class prediction, a frequently used approach is based on ordered probit model. We show that this approach is not optimal in the sense that it is not designed to minimize the error rate of the prediction. Based upon the works by Altman (*J. Finance* 1968; **23**:589–609), Ohlson (*J. Accounting Res.* 1980; **18**:109–131), and Begley *et al.* (*Rev. Accounting Stud.* 1996; **1**:267–284) on two-class prediction, we propose a modified ordered probit model. The modified approach depends on an optimal cutoff value and can be easily applied in applications. An empirical study is used to demonstrate that the prediction accuracy rate of the modified classifier is better than that obtained from usual ordered probit model. In addition, we also show that not only the usual accounting variables are useful for predicting issuer credit ratings, market-driven variables and industry effects are also important determinants.

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KEY WORDS: industry effect; issuer credit rating; market-driven variable; ordered probit model; optimal cutoff value; selection bias

1. INTRODUCTION

Credit ratings are extensively used by practitioners and regulators as a surrogate measure for the creditworthiness of bonds and companies. The ratings represent rating agency's opinions and risk assessments for bonds and companies. There are two basic types of credit ratings, the bond rating and the issuer credit rating. The former attempts to measure the likelihood of the default or delayed payments of a bond issue. The latter is an overall assessment of the creditworthiness of a company.

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Currently, there are many widely recognized rating agencies, such as Moody's Investors Service, Standard and Poor's Ratings Services (S&P's), etc. They routinely provide credit ratings for bonds and companies.

This study focuses on the S&P's long-term issuer credit rating (LTR). According to the definition given by S&P's, the LTR focuses on the obligor's capacity and willingness to meet its long-term financial commitments. To determine the LTR of a company, S&P's examines a profile called the corporate rating analysis and methodology profile. The profile contains two types of information of the company. The first type of information is available publicly, for example, public financial data. The second type of information is collected through a proprietary process from industry characteristics, competitive bargain positions, interviews with management team, etc. Thus, S&P's rating procedure is in part common knowledge. However, the major determinants of S&P's LTR are basically not clear. In this paper, we propose to first identify important predictors of S&P's LTR, selected from publicly available market data, accounting data, and industry classification.

Based on the Compustat North America (COMPUSTAT) database, there were 8039 companies in the year 2005 having stock traded on the New York Stock Exchange, American Stock Exchange, or NASDAQ. However, among those 8039 companies, there were only 20.46% (1645) companies having S&P's LTRs. This means that most of companies do not have their S&P's LTRs. In this paper, our next focus is to forecast ratings of those companies 'without' S&P's LTRs. Pettit *et al.* [1] reported that the new faces in the pool of companies with S&P's LTRs have lower rating category on the average. Blume *et al.* [2] also found similar results that bond ratings have declined, but the decline could be due to the use of more stringent rating standards in assigning ratings. On the other hand, we do not pursue the issue of rating forecast for companies 'with' S&P's LTRs for two reasons. First, if a company is once rated by S&P's, then it will be continuously rated unless a special event happens to the company, for example, bankruptcy. Second, the continuously rated companies have relatively unchanged rating categories in general [1]. Thus, it seems less interesting in discussing the ratings of these companies.

There are several important developments in statistical techniques for constructing classification models. These techniques include multiple regression analysis [3–5], multiple discriminant analysis [6–8], ordered probit model [9–11], ordered and unordered logit models [10], etc. We refer to the monograph by Altman *et al.* [12] for a detailed introduction of statistical classification models.

In this paper, we focus on the study based on ordered probit model.[‡] We point out in Section 2.1 that its prediction rule is basically equivalent to a method using cutoff value $\frac{1}{2}$. In the two-class classification problem, Altman [15], Ohlson [16], and Begley *et al.* [17] suggested that the classification rule based on cutoff value $\frac{1}{2}$ is not optimal. We extend their idea to the multiple-class prediction problem and propose a variation of the usual ordered probit model based on an optimal cutoff value. To be more precise, the usual ordered probit model approach is modified by replacing cutoff value $\frac{1}{2}$ with a data-dependent optimal cutoff value p^* in [0,1]. The optimal cutoff value p^* is determined by minimizing the error rate on the estimation sample. This idea is straightforward and can also be easily extended to the ordered logit model or multiple discriminant analysis.

[‡]Owing to the superiority in explaining and predicting, the ordered probit model has been adopted by a number of studies such as Kaplan and Urwitz [9], Ederington [10], and Gentry *et al.* [11]. Also, the test procedure of sample selection bias is only available for the ordered probit model [13]. On the other hand, it is not suggested using discrete explanatory variables in multiple discriminant analysis [14, p. 641]. In this paper, the industry effects on S&P's LTR were estimated through coefficients of eight indicator variables. Given these industry indicator variables, it is not adequate to use multiple discriminant analysis to predict S&P's LTRs.

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To study important predictors of S&P's LTR, we considered 24 variables as potential predictors in our data analysis section. These variables include four market-driven variables, 19 accounting variables, and industry effects. According to the efficient market hypothesis, stock prices reflect all publicly available information, including that contained in the accounting variables [18]. Thus it is reasonable that market-driven variables may reflect the rating of a company. The market-driven variables have been considered in the papers by Shumway [19], Bharath and Shumway [20], and Chava and Jarrow [18] for bankruptcy prediction. Their importance to bond ratings has also been noted by Blume *et al.* [2]. The 19 accounting variables were also considered in Altman [15], Poon [21], and Pettit *et al.* [1] measuring six aspects (size, financial leverage, coverage, cash flow, profitability, and liquidity) of financial health of a company. The industry effects on S&P's LTR were studied by introducing indicator variables in this paper. However, they could also be studied by using a latent variable approach to describe the unobservable heterogeneity (see [22, 23]). We note that it is important to include industry effects in the analysis, because different industries may face different types of business risk and adopt different accounting conventions. In this paper, industries were classified through the levels of standard industry classification (SIC) code. The industry effects on bond rating and bankruptcy prediction also have been considered by Perry *et al.* [24] and Chava and Jarrow [18], respectively.

The studied data were collected from COMPUSTAT and Center for Research in Security Prices (CRSP) databases. Our sample consisted of 736 companies having complete values of the 24 potential predictors for the year 2004 and receiving S&P's LTRs in April 2005. The sample was further divided into the estimation sample and holdout sample. The longevity of the S&P's LTR was adopted as a factor to separate the sampled companies into the holdout and estimation samples.[§] According to S&P's Research Insight North America Data Guide [25, p. 54], S&P's began to use the term LTR on 1 September 1998. Companies receiving S&P's LTRs in consecutive seven years (April 1999–April 2005) were classified into the estimation sample. The rest of the sampled companies were classified into the holdout sample. Based on our division principle, 504 companies were divided into the estimation sample and 232 companies into the holdout sample. In this paper, the estimation sample was used to determine the effective predictors of our model, and the holdout sample was employed to demonstrate the performance of the prediction rule.

To examine whether our divided samples (estimation and holdout samples) induced selection bias, a procedure based on ordered probit model with sample selection was performed using LIMDEP 8.0 to test the null hypothesis of no selection bias caused by the above sample division. The result of the test shows no rejection of the null hypothesis of interest at 5% level of significance.

Before performing the selection bias test, a stepwise selection procedure was used to objectively determine the effective predictors for ordered probit model. The final list of the selected predictors includes two market-driven variables, three accounting variables, and industry effects. The values of estimated coefficients of the selected market-driven and accounting variables all agree with their expected signs. This indicates that the result of the variable selection basically is correct, and market-driven variables and industry effects are also important to the prediction of S&P's LTR.

[§]Given the pool of companies with S&P's LTRs, our estimation companies solely correspond to the rated ones, and our holdout companies the new faces. Their purified composition agrees with our purpose to forecast ratings for companies without S&P's LTRs. On the other hand, one may separate the sampled companies by random allocation. Random allocation has the advantage of eliminating the need to test for selection bias since the resulting estimation and holdout samples have the same composition structure. However, each of the latter samples contains both rated companies and new faces. Such mixed composition does not agree with our prediction purpose.

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We remark that to study the difference between the unsolicited and the solicited ratings, Poon [21] suggested profitability and sovereign credit risk as two major factors in determining S&P's LTR. The sample for her study consists of 265 companies in 15 countries, while ours contains 736 companies selected from the COMPUSTAT database.

In our analysis, we also used the selected predictors and the ordered probit model to predict S&P's LTRs for the 232 holdout companies. The empirical results show that the prediction accuracy rate of the ordered probit model with cutoff value $\frac{1}{2}$ is 72.84%. In contrast, using optimal cutoff value p^*, the prediction accuracy rate becomes 77.16%.

The remainder of this paper is organized as follows. Section 2 introduces our method for predicting S&P's LTR. Section 3 describes the data under study. Section 4 presents the empirical results. Finally, conclusions are given in Section 5.

2. METHODS

In this section, we propose a variation of ordered probit model depending on the optimal cutoff value so that the prediction power for classifying S&P's LTR categories can be improved. The ordered probit model is briefly introduced below, and its detailed introduction can be referred to Kaplan and Urwitz [9].

Suppose that there are m categories among S&P's LTRs, where $m \geqslant 2$. Define R as the ordinal response variable. $R = j$ denotes that the S&P's LTR of a company belongs to the category j, where $j = 1, 2, \ldots, m$. The larger the value of R, the better the S&P's LTR category. We aim to predict the values of R for companies without S&P's LTRs.

To describe the ordered probit model, we first assume that there exists a latent variable R^* relating to the S&P's LTR assessment R. Here R is a random variable representing the creditworthiness of a company. The relation between R and R^* is described by

$$\begin{aligned} R^* &= \alpha + \beta x + \varepsilon \\ R &= j \quad \text{if } \alpha_{j-1} < R^* \leqslant \alpha_j \quad \text{for } j = 1, 2, \ldots, m \end{aligned} \tag{1}$$

Here α is an intercept, β a vector of coefficients, x a vector of predictors, and ε a random error. $\alpha_0 = -\infty, \alpha_1 = 0, \alpha_j$ is the threshold parameter between categories j and $j+1$, for each $j = 2, \ldots, m-1$, and $\alpha_m = \infty$. The values of α_j are of ascending order.

The ordered probit model is defined by assuming that ε is a standard normal random variable. Thus, the cumulative probability of the variable R can be written as

$$\begin{aligned} P(R \leqslant j | \theta, x) &= \Phi(\alpha_j - \alpha - \beta x) \quad \text{for } j = 1, 2, \ldots, m-1 \\ P(R \leqslant m | \theta, x) &= 1 \end{aligned} \tag{2}$$

where $\theta = (\alpha_2, \alpha_3, \ldots, \alpha_{m-1}, \alpha, \beta)$ is a vector of parameters, and $\Phi(\cdot)$ the standard normal distribution function. Given model (2), the parameter θ can be estimated by the maximum likelihood method [26]. Let $\hat{\theta} = (\hat{\alpha}_2, \hat{\alpha}_3, \ldots, \hat{\alpha}_{m-1}, \hat{\alpha}, \hat{\beta})$ be the maximum likelihood estimate of θ based on the estimation sample. The ordered probit model assigns category $\hat{R}$ to the company with predictor value x_0, where $\hat{R}$ is defined by

$$\hat{R} = j \quad \text{if } \hat{\alpha}_{j-1} < \hat{\alpha} + \hat{\beta} x_0 \leqslant \hat{\alpha}_j \quad \text{for some } j \in \{1, 2, \ldots, m\} \tag{3}$$

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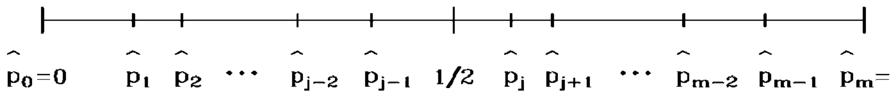

Figure 1. Plot of an artificial example of the distribution of the values of $\hat{p}_j$ and the cutoff value $\frac{1}{2}$.

It is important to point that the prediction rule $\hat{R}$ in (3) is actually equivalent to basing on cutoff value $\frac{1}{2}$ and estimated cumulative probabilities of R:

$$\hat{R}=j \quad \text{if } \hat{p}_{j-1}<1/2\leqslant\hat{p}_j \text{ for some } j\in\{1,2,\ldots,m\} \tag{4}$$

Here $\hat{p}_0=0$, $\hat{p}_j=\Phi(\hat{\alpha}_j-\hat{\alpha}-\hat{\beta}x_0)$, for $j=1,\ldots,m-1$, and $\hat{p}_m=1$. To illustrate inequality (4), Figure 1 shows an artificial example. In Figure 1, interval $[0,1]$ is divided into m subintervals by the $\hat{p}_j$ values, and the jth subinterval has endpoints $\hat{p}_{j-1}$ and $\hat{p}_j$. The value of $\hat{R}$ is determined by the subinterval containing the quantity $\frac{1}{2}$.

In this study, following the idea of Altman [15], Ohlson [16], and Begley *et al.* [17], we propose replacing cutoff value $\frac{1}{2}$ in (4) with some optimal cutoff value p^*. For each value of $p\in[0,1]$, let $\hat{R}(p)$ denote the prediction rule defined in (4) but with $\frac{1}{2}$ replaced by p. We suggest taking p^* as the value p over $[0,1]$ so that the corresponding prediction rule $\hat{R}(p^*)$ has the minimum error rate on the estimation sample. If there are multiple p^* values, then the smallest one is suggested. After calculating the optimal cutoff value p^*, we give rating $\hat{R}^*=\hat{R}(p^*)$ to the company with predictor value x_0. We remark that using p^* has the advantage of reducing the rate of misclassifying a company with worse rating category to better rating category. This seems of importance, since misclassifying worse category to better category might cause severe losses to investors.

3. DATA

The data for this research were collected from COMPUSTAT and CRSP databases. The studied population consists of companies that must (i) adopt calendar fiscal year, (ii) have stock traded on the New York Stock Exchange, American Stock Exchange, or NASDAQ, and (iii) not be a financial services company with the SIC code 6000–6999. The criterion (i) synchronizes the timing of predictors in the sense that all market-driven and accounting variables cover the calendar year 2004. The criterion (ii) makes sure that market-driven variables are available. The criterion (iii) excludes the financial services companies since they are subject to regulations and adopt different accounting conventions.

Based on the COMPUSTAT database, there were 4042 companies in the population, but only 983 companies receiving S&P's LTRs in April 2005. However, among those 983 companies, there were only 736 companies having complete values of the studied 24 predictors for the year 2004. The missing data problem is not unusual in applications, especially when there are many predictors in the model. However, as long as the missingness occurs 'at random' then the sample will not introduce systematic bias in our analyses [27, 28]. We have no reason not to believe that the missingness occurred in COMPUSTAT and CRSP databases is 'missing at random'.

For purpose of later analysis, we decide to divide sample data into two groups. According to S&P's Research Insight North America Data Guide, S&P's began using the term LTR on 1

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September 1998. Thus 504 companies among our 736 sampled companies receiving S&P's LTRs in consecutive seven years (April 1999–April 2005) were classified into the estimation sample. The rest of the sampled companies were classified into the holdout sample.

S&P's LTR ranges from AAA to D. Panel A of Table I presents the frequency distribution of the sampled companies according to their S&P's LTRs. Based on the result from the estimation sample, it seems reasonable to group S&P's LTRs into three categories: {Below BBB} as category 1, {BBB} as category 2, and {AAA, AA, A} as category 3.¶ According to S&P's, firms in the {AAA, AA, A} category mean that they have demonstrated strong capacity to meet their financial obligations. Firms receiving BBB rating mean that they have adequate capacity to meet their financial commitments. However, firms receiving LTR below BBB mean that they were regarded as having speculative characteristics. According to the three S&P's LTR categories, the frequency distribution of the sampled companies is shown in Panel B of Table I.

Note that the companies in our holdout sample are mostly new faces in the pool of companies with S&P's LTRs and have lower S&P's LTRs on the average. Such result agrees with the observation reported in Pettit *et al.* [1]. Blume *et al.* [2] also reported the same observation for bond ratings. The distribution of the companies in the holdout sample shows that there are about 71% companies in the speculative S&P's LTR category. In contrast, there are only about 38% companies in the estimation sample with the speculative S&P's LTR category.

The 24 potential explanatory variables considered in this research for studying the important predictors of S&P's LTR include four market-driven variables [19, 20], 19 accounting variables [1, 15, 21], and industry effects [1, 18]. In order to study industry effects on S&P's LTR, we classified companies according to the first digit of their four-digit SIC codes. The frequency distribution of the sampled companies according to their SIC codes is given in Panel C of Table I. The industry effects were estimated through the coefficients of eight indicator variables in the model. The four market-driven variables were excess return (EXRET), relative size (RSIZE), standard deviation of monthly returns (SIGMA), and the logarithm of the KMV-Merton default probability ($\log_{10}(\pi_{\mathrm{KMV}})$).‖ The 19 accounting variables measure six aspects (size, financial leverage, coverage, cash flow, profitability, and liquidity) of financial health of a company. The definitions of the 24 predictors are given in Table II. Using data in the estimation sample, Table III shows summary statistics and F-tests of equality of the means among the three S&P's LTR categories for the market-driven variables and accounting variables. The p-values in Table III show that testing the null hypothesis of equal means is significant at 0.05 level for each of the four market-driven variables and are significant for all accounting variables, except OM, ROE, CASHR, and QR. This result indicates that most of the variables considered in this paper are effective predictive variables. Table III also shows that, on the average, if a company has larger firm size, smaller

¶The estimation sample was divided into the three categories so that the resulting three cells have approximately equal sizes. On the other hand, if one divides the estimation sample into more categories, then some cells have smaller sizes and the number of threshold parameters increases. Thus the corresponding estimates of parameters in the model may become less precise. The unreported results based on each of the four-seven categories show the same conclusion on studying important predictors of S&P's LTR as, but much worse prediction performance using each of cutoff values p^* and $\frac{1}{2}$ than those based on the three categories.

‖Since the computation of KMV-Merton default probability π_{KMV} requires the market value of equity, it is treated as a market-driven variable. The detailed computation procedure of π_{KMV} can be referred to Bharath and Shumway [20]. For most companies in the estimation sample, their values of π_{KMV} are very close to 0. Thus, $\log_{10}(\pi_{\mathrm{KMV}})$ is considered as an explanatory variable in this study.

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Table I. The frequency distributions of the sampled companies collected from the COMPUSTAT and CRSP databases with complete values of the predictors in the year 2004.

	Estimation companies	Holdout companies
Panel A: S&P's LTR		
AAA	6	0
AA	17	1
A	114	18
BBB	173	48
BB	128	92
B	64	70
CCC	1	2
CC	1	0
C	0	0
D	0	1
Total firms	504	232
Panel B: S&P's LTR category		
{Below BBB}	194	165
{BBB}	173	48
{AAA, AA, A}	137	19
Total firms	504	232
Panel C: SIC code		
100–999	3	2
1000–1999	49	18
2000–2999	116	42
3000–3999	118	60
4000–4999	129	58
5000–5999	29	12
7000–7999	37	28
8000–8999	19	12
9000–9999	4	0
Total firms	504	232

Panels A, B, and C present the frequency distributions of the sampled companies according to different S&P's LTR categories and SIC codes.

financial leverage, larger coverage, larger cash flow, or larger profit, then it has better S&P's LTR category.

4. RESULTS

In this section, we shall apply stepwise selection procedure to objectively choose important predictors of S&P's LTR and apply our suggested method to predict S&P's LTR categories for companies in the holdout sample. We also examine whether our estimation and holdout samples induced bias in sample selection. Section 4.1 gives the results of ordered probit model for testing the selection bias. The results in Table IV show the selected predictors, parameter estimates, and conclusion for testing the null hypothesis of no bias in sample selection. Using the selected predictors, Section 4.2 compares the prediction performance of the ordered probit model with cutoff value $\frac{1}{2}$ and that with optimal cutoff value p^*.

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Table II. The definitions of the studied predictors.

Variable	Definition
Panel A: *market-driven variables*	
EXRET	Monthly return on the firm minus the value-weighted CRSP NYSE/AMEX/NASDAQ index return cumulated to obtain the yearly return
RSIZE	Logarithm of each firm's market equity value divided by the total NYSE/AMEX/NASDAQ market equity value
SIGMA	Standard deviation of each company monthly stock returns
$\log_{10}(\pi_{\mathrm{KMV}})$	Logarithm of the KMV-Merton default probability
Panel B: *accounting variables*	
Size	
$\log_{10}(\mathrm{TA})$	Logarithm of total assets
Financial leverage	
TDEBITDA	Toal debt/(EBIT+DA), DA: depreciation plus amortization
EM	Total assets/equity
LDC	Long-term debt to capital
TDC	Total debt to capital
SDC	Short-term debt to capital
Coverage	
EBITINT	EBIT/interest expenses
EBITDAINT	(EBIT+DA)/interest expenses
Cash flow	
FFO	Net income from continuing operations, plus DA, deferred income taxes, and other non-cash expense
INT	Interest expenses
CASHEQ	Total cash and equivalent
Profitability	
RETA	Retain earnings/total assets
OM (%)	Operating margin after depreciation
ROC (%)	Return on capital
ROE (%)	Return on equity
ROA (%)	Return on assets
Liquidity	
CASHR	Cash ratio
QR	Quick asset ratio
CR	Current ratio
Panel C: *Indicator variables for industry effects*	
SIC_1	1 if SIC code is within 100–999, and 0 otherwise
SIC_2	1 if SIC code is within 1000–1999, and 0 otherwise
SIC_3	1 if SIC code is within 2000–2999, and 0 otherwise
SIC_4	1 if SIC code is within 3000–3999, and 0 otherwise
SIC_5	1 if SIC code is within 5000–5999, and 0 otherwise
SIC_6	1 if SIC code is within 7000–7999, and 0 otherwise
SIC_7	1 if SIC code is within 8000–8999, and 0 otherwise
SIC_8	1 if SIC code is within 9000–9999, and 0 otherwise

Note: The SIC code 4000–4999 was used as the reference level in studying the industry effects. The financial service companies with the SIC code 6000–6999 were excluded from the study, since they are subject to regulations and adopt different accounting conventions.
Panels A, B, and C present the definitions of market-driven variables, accounting variables, and industry indicator variables, respectively.

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Table III. Summary statistics and F-tests of the estimation sample.

Variable	Mean	Median	Standard deviation	Minimum	Maximum	p-Value
Panel A: {*Below BBB*}						
EXRET	0.188	0.105	0.453	−0.951	1.707	0.001**
RSIZE	−4.106	−4.087	0.517	−5.337	−2.704	0.000**
SIGMA	0.100	0.089	0.043	0.030	0.356	0.000**
$\log_{10}(\pi_{KMV})$	−20.499	−10.200	37.192	−307.653	−0.032	0.000**
$\log_{10}(TA)$	3.345	3.277	0.519	2.307	4.516	0.000**
TDEBITDA	4.966	3.320	12.962	−1.179	178.338	0.001**
EM	6.730	3.060	20.153	1.295	227.106	0.005**
LDC	0.491	0.495	0.210	0.000	0.928	0.000**
TDC	0.530	0.537	0.208	0.000	0.991	0.000**
SDC	0.039	0.014	0.073	0.000	0.512	0.000**
EBITINT	3.631	2.233	7.347	−31.123	48.853	0.000**
EBITDAINT	6.160	3.692	8.819	−26.747	58.987	0.000**
FFO	322.789	138.057	642.040	−451.149	5512.000	0.000**
INT	139.457	50.133	255.348	0.500	1958.000	0.009**
CASHEQ	350.184	113.337	651.787	0.000	4244.000	0.000**
RETA	−0.003	0.047	0.390	−3.730	0.923	0.000**
OM	5.972	8.560	74.724	−1014.605	57.198	0.101
ROC	2.647	3.447	11.385	−50.279	38.266	0.000**
ROE	−2.545	7.512	152.947	−2008.112	345.580	0.091
ROA	1.650	2.220	7.433	−36.379	30.131	0.000**
CASHR	0.592	0.400	1.168	0.000	14.540	0.231
QR	1.172	0.942	1.195	0.154	15.021	0.334
CR	1.790	1.591	1.344	0.332	16.781	0.030*
Panel B: {*BBB*}						
EXRET	0.128	0.046	0.356	−0.417	3.227	
RSIZE	−3.568	−3.565	0.504	−5.225	−2.282	
SIGMA	0.060	0.058	0.021	0.018	0.144	
$\log_{10}(\pi_{KMV})$	−46.287	−35.825	45.548	−307.653	−0.335	
$\log_{10}(TA)$	3.778	3.751	0.501	2.534	5.393	
TDEBITDA	2.306	2.060	1.847	−12.324	7.809	
EM	3.148	2.503	3.084	1.338	34.329	
LDC	0.359	0.371	0.162	0.000	0.918	
TDC	0.404	0.398	0.168	0.000	0.950	
SDC	0.045	0.032	0.050	0.000	0.307	
EBITINT	10.496	5.054	30.032	−3.481	346.237	
EBITDAINT	15.118	7.776	43.383	−0.984	498.772	
FFO	1101.827	512.700	1635.879	−137.000	14973.028	
INT	204.737	86.506	297.835	0.359	1807.000	
CASHEQ	828.016	268.964	1869.804	0.000	15778.539	
RETA	0.206	0.196	0.225	−0.991	0.940	
OM	14.167	12.163	9.623	−1.252	47.997	
ROC	7.191	6.827	7.601	−33.119	45.298	
ROE	13.330	12.108	12.168	−58.236	80.414	
ROA	4.914	4.525	4.288	−22.503	24.544	
CASHR	0.454	0.246	0.594	0.000	3.527	
QR	1.081	0.883	0.723	0.160	4.798	
CR	1.583	1.435	0.858	0.409	5.360	

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Table III. *Continued.*

Variable	Mean	Median	Standard deviation	Minimum	Maximum	p-Value
Panel C: {*AAA*, *AA*, *A*}						
EXRET	0.034	0.031	0.211	−0.545	0.842	
RSIZE	−3.164	−3.169	0.609	−4.943	−1.696	
SIGMA	0.052	0.046	0.022	0.017	0.152	
$\log_{10}(\pi_{\text{KMV}})$	−74.181	−60.009	61.667	−307.653	−0.793	
$\log_{10}(\text{TA})$	4.036	4.032	0.542	2.476	5.291	
TDEBITDA	1.933	1.498	1.841	0.010	16.102	
EM	2.729	2.336	1.270	1.248	9.168	
LDC	0.286	0.274	0.150	0.000	0.756	
TDC	0.356	0.344	0.165	0.004	0.759	
SDC	0.070	0.055	0.066	0.000	0.282	
EBITINT	20.805	10.680	31.539	−1.916	205.840	
EBITDAINT	28.143	13.603	43.117	1.309	303.040	
FFO	2920.325	1073.190	5179.117	−3991.000	40551.000	
INT	264.979	77.700	545.486	3.740	4891.279	
CASHEQ	2326.562	559.000	4265.138	0.000	23135.000	
RETA	0.357	0.333	0.255	−0.307	0.914	
OM	16.168	14.581	8.584	−3.508	53.244	
ROC	11.558	10.295	8.786	−33.743	44.282	
ROE	19.449	15.639	17.540	−35.835	101.736	
ROA	7.250	7.056	5.257	−22.146	20.397	
CASHR	0.462	0.277	0.567	0.000	3.344	
QR	1.025	0.902	0.624	0.292	4.184	
CR	1.494	1.349	0.768	0.354	5.294	

Note: The notations ** and * indicate the significance of the F-test at the 1 and 5% levels, respectively. The p-values refer to the F-tests of equality of the means among the three S&P's LTR categories based on the estimation sample. Panels A, B, and C present the results for the three S&P's LTR categories {Below BBB}, {BBB}, and {AAA, AA, A}, respectively.

4.1. *Testing selection bias*

To examine whether our criterion for dividing the overall sample into estimation and holdout samples induced selection bias, the procedure of ordered probit model under our particular sample selection was performed. This procedure was designed by simultaneously applying the ordered probit model with multiple classes to the estimation sample, and applying the two-class probit model to the particular sample in which classes 0 and 1 were assigned to companies in the holdout and estimation samples, respectively (see Model 1 of Table IV). The detail of this approach can be found in Greene [13]. Before performing it, a stepwise selection procedure provided by SAS was first used to objectively determine the predictors for each of the two models. The significance level for including and excluding a predictor in the selection procedure was set as 5%, a default value provided by SAS.

The final list of the selected predictors in the ordered probit model includes industry effects, RSIZE, SIGMA, LDC, FFO, and RETA. Results in Table III, by using F-test for testing equality of three means (corresponding to the three S&P's LTR categories), show that the last five selected predictors are all significant at the 1% level. The two selected market-driven variables, RSIZE and SIGMA, stand for the market capitalization and the risk of a company, respectively. The three selected accounting variables, LDC, FFO, and RETA, measure a company's financial leverage,

Table IV. Maximum likelihood estimates of the parameters in Models 1 and 2.

	Model 1		Model 2	
Variable	Coefficient	*p*-Value	Coefficient	*p*-Value
Panel A: *ordered probit specification*				
α_2	1.6781	0.0000**	1.6962	0.0000**
Intercept α	5.8699	0.0000**	6.0403	0.0000**
SIC_1	−1.4021	0.1162	−1.4238	0.0950
SIC_2	−0.4749	0.0430*	−0.4819	0.0337*
SIC_3	−0.0359	0.8528	−0.0386	0.8348
SIC_4	−0.3311	0.0919	−0.3355	0.0762
SIC_5	−0.7423	0.0092**	−0.7504	0.0092**
SIC_6	−0.6764	0.0096**	−0.6883	0.0084**
SIC_7	−1.3759	0.0075**	−1.3978	0.0004**
SIC_8	−0.3308	0.4597	−0.3117	0.6038
RSIZE	0.7941	0.0235*	0.8679	0.0000**
SIGMA	−21.1330	0.0000**	−21.4097	0.0000**
LDC	−2.4886	0.0000**	−2.4383	0.0000**
FFO	0.000075	0.0335*	0.000078	0.0322*
RETA	1.5633	0.0001**	1.6135	0.0000**
Panel B: *two-class probit sample selection specification*				
Intercept	0.3932	0.6332		
RSIZE	0.3963	0.0011**		
$\log_{10}$(TA)	0.3641	0.0041**		
TDC	0.7031	0.0063**		
RETA	0.4142	0.0003**		
Panel C: *model fit test*				
Chi-squared statistic	0.2674	0.6051	453.5957	0.0000**
d.f.	1		13	
ρ	−0.2166	0.7838		

Note: The notations ** and * indicate the significance of the test at the 1 and 5% levels, respectively. The notations d.f. and ρ stand for the degree of freedom and the correlation coefficient between the error term of the ordered probit model and that of the two-class probit model, respectively.
Model 1 denotes the ordered probit model with sample selection. Model 2 stands for the usual ordered probit model. The variables used in each of the ordered probit approach and the two-class probit approach were selected from using the stepwise selection procedure. The significance level for including and excluding a predictor in the selection procedure was 5%. The *p*-values refer to the Wald chi-squared tests for testing the significance of parameters. Panel A shows the inference results using the ordered probit approach under Models 1 and 2. Panel B gives the inference results using the two-class probit approach under Model 1. Panel C presents the results for model fit test.

cash flow, and profitability, respectively. The results of our analysis indicate that industry effects and market-driven variables are also important to the S&P's LTR.

Note that none of the accounting variables, which measures the short-term liquidity aspect of a firm, appeared in the final list of selected predictors. This is perhaps not surprising, because the S&P's LTR reflects an obligator's capacity to meet its long-term financial commitments. Furthermore, the S&P's withdraws issuer credit ratings if the companies default on payments or file bankruptcy. The absence of pending bankruptcy or extreme financial distress in the studied population explains why the stepwise selection procedure failed to select the fourth market-driven variable (KMV-Merton default probability).

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The final list of the selected predictors for the two-class probit model using both the estimation and the holdout samples includes RSIZE, $\log_{10}$(TA), TDC, and RETA. Table IV shows the results obtained from performing the procedure of ordered probit model, under our particular sample selection, based on the application of LIMDEP 8.0. Panel C of Model 1 in Table IV shows that the null hypothesis of $\rho=0$ was not rejected at 5% level of significance. Here the null hypothesis of $\rho=0$ stands for no sample selection bias caused by our criterion of dividing the overall sample. On the other hand, Model 2 in Table IV shows the final results of the usual ordered probit model, including p-value of the chi-squared test for model fit. It is also interesting to note that by comparing the parameter estimates of the two ordered probit models (in Models 1 and 2 of Table IV), we find out that their values are approximately equal. Thus since there is no sample selection bias, the results of Model 2 are adopted for predicting S&P's LTR categories for companies in the holdout sample.

From the results of Model 2, the industry indicator variable SIC_1 is non-significant at 5% level. This means that the industry effect with SIC code 100–999 is of no significant difference, comparing with that of the reference level with SIC code 4000–4999. The same remark also applies to the industry effects with SIC code 2000–2999, 3000–3999, and 9000–9999.

It is important to note that the two selected predictors SIGMA and LDC measure various aspects of risk and financial leverage of an issuer, respectively. The larger the values of these two predictors (SIGMA and LDC), the smaller the value of R. That is, these two selected predictors should be negatively correlated with rating R, and the signs of their coefficients should be negative. On the other hand, the rest of the selected predictors RSIZE, FFO, and RETA stand for the market capitalization, cash flow, and profitability of an issuer, respectively. Thus, the larger the values of these three predictors, the better the creditworthiness of an issuer will result. This implies that RSIZE, FFO, and RETA should be positively correlated with rating R, and the signs of their coefficients should be positive. From Table IV, the signs of the values of estimated coefficients for selected predictors RSIZE, SIGMA, LDC, FFO, and RETA all agree with our expectation.

4.2. *Prediction results*

In this subsection, the S&P's LTR categories of the 232 holdout companies were predicted based on the application of Model 2 with cutoff value $\frac{1}{2}$ and the optimal cutoff value p^*. Recall that the optimal cutoff value p^* was taken as the minimum among those p arriving at the minimum error rate of the prediction rule $\hat{R}(p)$ over $[0,1]$, based on applying Model 2 to the estimation sample. It was selected on the 10 001 equally spaced grids of p in $[0,1]$. We plot cutoff value p against the corresponding error rate of $\hat{R}(p)$ in Figure 2. The figure shows that the optimal cutoff value should be $p^*=0.3706$.

The classification results for the 504 estimation companies are given in Table V. Table V shows that Model 2 with the optimal cutoff value $p^*=0.3706$ has the same accuracy rate 71.03% as that with cutoff value $\frac{1}{2}$. As expected from the definition of optimal cutoff value p^* given in Section 2.2, one interesting point is that Model 2 with optimal cutoff value has better ability in classifying the speculative grade {Below BBB}.

The prediction results for the 232 holdout companies are given in Table VI. Table VI shows that Model 2 with the optimal cutoff value $p^*=0.3706$ has better prediction performance than that with cutoff value $\frac{1}{2}$, since the accuracy rates of their predictions are, respectively, equal to 77.16 and 72.84%. Another interesting point is that Model 2 with optimal cutoff value also has better

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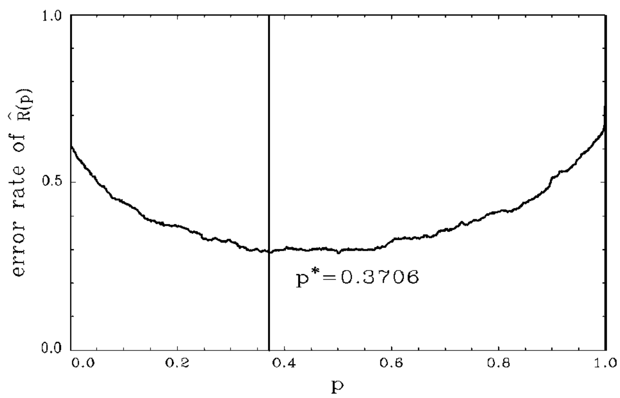

Figure 2. Plot of the error rate among the estimation sample of the prediction rule $\hat{R}(p)$ versus the cutoff value p in [0, 1]. The location of the vertical line in the plot stands for the optimal cutoff value p^*.

Table V. Classification results obtained from the 504 estimation companies.

	Classified category		
True category	{Below BBB}	{BBB}	{AAA, AA, A}
Panel A: *cutoff value* $\frac{1}{2}$			
{Below BBB}	151	42	1
{BBB}	20	120	33
{AAA, AA, A}	4	46	87
		Accuracy rate $=(151+120+87)/504=71.03\%$	
Panel B: *optimal cutoff value* $p^*=0.3706$			
{Below BBB}	171	23	0
{BBB}	37	120	16
{AAA, AA, A}	10	60	67
		Accuracy rate $=(171+120+67)/504=71.03\%$	

Panel A shows the results using cutoff value $\frac{1}{2}$. Panel B presents the results using the optimal cutoff value $p^*=0.3706$.

ability in predicting the speculative grade {Below BBB}. This is important, since misclassifying speculative grade to investment grade ({BBB} or {AAA, AA, A}) might cause severe losses to investors.

In this analysis, a hypothesis testing was also performed to confirm that Model 2 with the optimal cutoff value $p^*=0.3706$ has better prediction performance than that with cutoff value $\frac{1}{2}$. The one-sided McNemar's binomial test [29] was used to test the null hypothesis of no difference in prediction performance. The results are given in Table VII. The p-value of the one-sided McNemar test is 0.006 indicating the data strongly support that Model 2 with the optimal cutoff value $p^*=0.3706$ should be used instead of that with cutoff value $\frac{1}{2}$.

DOI: 10.1002/asmb

Table VI. Prediction results obtained from the 232 holdout companies.

	Predicted category		
True category	{Below BBB}	{BBB}	{AAA, AA, A}
Panel A: *cutoff value* $\frac{1}{2}$			
{Below BBB}	130	35	0
{BBB}	6	33	9
{AAA, AA, A}	4	9	6
		Accuracy rate $=(130+33+6)/232=72.84\%$	
Panel B: *optimal cutoff value* $p^*=0.3706$			
{Below BBB}	140	25	0
{BBB}	10	34	4
{AAA, AA, A}	4	10	5
		Accuracy rate $=(140+34+5)/232=77.16\%$	

Panel A shows the results using cutoff value $\frac{1}{2}$. Panel B presents the results using the optimal cutoff value $p^*=0.3706$.

Table VII. McNemar test for comparing the ordered probit model with cutoff value $\frac{1}{2}$ and that with optimal cutoff value $p^*=0.3706$ on their prediction performance obtained from the holdout sample.

	Cutoff value $\frac{1}{2}$			
Optimal cutoff value $p^*=0.3706$	Correct	Incorrect	Total	p-Value
Correct	164	15	179	0.006**
Incorrect	5	48	53	
Total	169	63	232	

Note: ** indicates significance at the 1% level.

5. CONCLUSIONS

In this paper, we have shown that the prediction rule based on the usual ordered probit model is equivalent to using classification rule in (4) with cutoff value $\frac{1}{2}$, and hence this approach may not be optimal. We thus have proposed a modified method based on the ordered probit model but using an optimal cutoff value instead. Also, we have performed an empirical analysis to find important predictors of S&P's LTR from the publicly available market and accounting data as well as industry classification. Using the important predictors of S&P's LTR, we have developed a prediction rule combining ordered probit model and optimal cutoff value p^*. Our empirical results demonstrate that the proposed prediction rule has better accuracy rate of prediction than that based on cutoff value $\frac{1}{2}$.

To find the important predictors of S&P's LTR, we have considered 24 potential explanatory variables used in previous studies. They include industry effects, four market-driven variables, and 19 accounting variables. A data set containing 736 companies (504 estimation companies and 232 holdout companies) having complete values of the 24 explanatory variables was collected

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from COMPUSTAT and CRSP databases. The results from the stepwise selection procedure show that the final list of the selected predictors in the ordered probit model contains industry effects, two market-driven variables, and three accounting variables. They were considered as important predictors of S&P's LTR. The values of estimated coefficients of the selected market-driven and accounting variables all agree with their expected signs. Our analysis indicates that industry effects and market-driven variables are also important to the prediction of S&P's LTR.

The longevity of the S&P's LTR was adopted as a factor for separating the sampled companies into the holdout and estimation samples. To examine whether our criterion of dividing the overall sample into estimation and holdout samples induced bias in 'selection', a procedure using LIMDEP 8.0 was performed. Our analysis shows that there is no selection bias in dividing the overall sample into the estimation and holdout samples at 5% level of significance.

In this paper, we have applied a variation of the ordered probit model, depending on optimal cutoff value p^*, to improve the power of prediction. The optimal cutoff value p^* is determined by minimizing the error rate on the estimation sample. Our empirical results show that the ordered probit model with optimal cutoff value p^* has better prediction power than that with cutoff value $\frac{1}{2}$, since their prediction accuracy rates are 77.16 and 72.84%, respectively. Also, the model with optimal cutoff value p^* also has better ability in predicting the speculative grade {Below BBB}.

Finally, there are two possible extensions of the methods considered in this paper. First, Blume *et al.* [2] applied panel data with independence assumption to the ordered probit model to study bond ratings. In this paper, we only used cross-sectional data to study LTRs. In the future, to account for the correlations among panel data, we shall use dynamic ordered probit model with autocorrelation structure [30, 31] to study LTRs. Second, in the future research, the methods considered in this paper will be used for bond rating prediction. The same problem has been done by Pinches and Mingo [6, 7], Blume *et al.* [2], and others.

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