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# Forecasting credit migration matrices with business cycle effects – a model comparison

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Migration matrices are considered a major determinant for credit risk management. They are widely used for credit value-at-risk determination, portfolio management or derivative pricing. It is well known that migration matrices show strong variations and cyclical behavior through time. We compare a factor model approach and numerical adjustment methods for estimation and forecasting of conditional migration matrices. Our findings show that the methods may lead to quite different forecasting results. Although the numerical adjustment methods fail to outperform the naive approach of taking previous year's migration matrix as an estimator, the one-factor model provides significantly better in-sample and out-of-sample results. Additionally, on the basis of a chosen risk-sensitive goodness-of-fit criteria, we are able to interpret the results in terms of risk.

**Keywords:** credit risk; transition matrices; forecasting; credit VaR

## 1. Introduction

In credit risk management, transition matrices have become major inputs for risk management, credit value-at-risk (VaR) and derivative pricing. This is a consequence of the increased popularity of rating-based models for credit risk evaluation and management in the last decade.

On the one hand, this popularity is due to the straightforwardness of the approach. The models use the rating of a company as the decisive variable when it comes to evaluate the default risk of a bond or loan. Hence, they avoid various difficulties of the structural models, where it is necessary to determine a company's value and volatility.

Also, the upcoming new capital accord (Basel II) encourages banks to base their capital requirement for credit risk on internal or external rating systems (Basel Committee on Banking Supervision 2001). With Basel II, the Bank of International Settlements aims to strengthen risk management systems of international financial institutions. A vast majority of international operating banks will use the so-called internal-rating-based approach to determine capital requirements for their loan or bond portfolios (Deutsche Bundesbank 2005). Thus, assigned ratings and corresponding default probabilities, but also the probabilities for rating changes, will be determinants of a bank's credit risk management. Thus, to calculate VaR figures especially for internal loan portfolios, a main input will be an adequate transition matrix for the bonds or loans.

Unfortunately, owing to cyclical behavior of the economy, credit spreads and migrations are not constant through time. Helwege and Kleiman (1997) and Alessandrini (1999) have shown that default rates and credit spreads clearly depend on the stage of the business cycle. Nickell,

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Perraudin, and Varotto (2000) provided insight that probability transition matrices of bond ratings also vary with the state of the economy. By separating the economy into two states or regimes, Bangia et al. (2002) show significant differences in the loss distribution of credit portfolios. Further investigating the issue, Trück and Rachev (2005b) show that such changes in migration or default behavior through time lead to substantial effects on risk figures for credit portfolios. Krüger, Stötzel, and Trück (2005) analyze time homogeneity and Markov behavior of an internal rating system and find significant changes in credit migration matrices through time. Thus, to measure and forecast changes in migration behavior as well as determining adequate estimators for transition matrices can be considered as a major issue in rating-based credit risk modeling.

Still, despite the obvious importance of recognizing the impact of business cycles on rating transitions, the literature is rather sparse on this issue. The first model developed to explicitly link business cycles to rating transitions was in 1997 *CreditPortfolioView* (CPV) by Wilson (1997) and McKinsey and Company (1998). Belkin, Forest, and Suchower (1998b) and Kim (1999) use a one-factor model whereby ratings respond to business cycle shifts. The model is extended to a multifactor credit migration model by Wei (2003). Nickell, Perraudin, and Varotto (2000) propose an ordered probit model that permits migration matrices to be conditioned on the industry, country domicile and the business cycle. Finally, Jarrow, Lando, and Turnbull (1997) and Lando (2000) provide numerical adjustment methods in order to obtain risk neutral migration matrices being in line with market credit spreads.

The focus of this article is to compare two of the most common approaches for conditioning credit migration matrices. The one-factor model implemented in the *CreditMetrics* framework will be confronted with numerical adjustment methods for migration matrices initially suggested in the framework by Jarrow, Lando, and Turnbull (1997). We will investigate the in-sample and out-of-sample performance of these models in prediction of conditional credit migration matrices subject to business cycle effects. The results are benchmarked against the standard approach of using historical average transitions or last year's migration matrix for credit VaR calculation.

The article is set up as follows. Section 2 gives a brief overview on the use of transition matrices in credit risk management concentrating on the stability of migration matrices and credit VaR, credit derivatives and internal rating systems. Section 3 reviews methods for adjusting credit migration matrices to business cycle effects. Hereby, the focus is set on the one-factor model approach and numerical adjustment methods that will be used later in the empirical analysis. Section 4 describes the considered data and variables that were used in order to condition migration matrices to business cycle effects. Section 5 provides empirical results on forecasting transition matrices as well as the interpretation of those results, and Section 6 concludes.

## 2. Transition matrices in credit risk

### 2.1 *Reduced form models*

Since the gaining popularity of reduced form models in the 1990s, see e.g. Fons (1994), the approach can be considered as one of the major classes in credit risk modeling. Before that most credit risk models were based on the structural approach developed by Merton (1974), calculating default probabilities and credit spreads using the value of the company as the driver of credit risk. In the model developed by Fons, for the calculation of credit risk, the input variables were the rating of a company as well as the historical default probabilities for the corresponding rating classes. As an extension of this approach, not only the event of default can be considered but also the probabilities of rating changes for a company or an issued bond. Downgrades or upgrades are taken very seriously by market players to price bonds and loans, thus affecting the risk premium

Table 1. Average 1-year transition matrix of Moody's corporate bond ratings for the period 1982–2001.

| | Aaa | Aa | A | Baa | Ba | B | C | D |
|---|---|---|---|---|---|---|---|---|
| Aaa | 0.9276 | 0.0661 | 0.0050 | 0.0009 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| Aa | 0.0064 | 0.9152 | 0.0700 | 0.0062 | 0.0008 | 0.0011 | 0.0002 | 0.0001 |
| A | 0.0007 | 0.0221 | 0.9137 | 0.0546 | 0.0058 | 0.0024 | 0.0003 | 0.0005 |
| Baa | 0.0005 | 0.0029 | 0.0550 | 0.8753 | 0.0506 | 0.0108 | 0.0021 | 0.0029 |
| Ba | 0.0002 | 0.0011 | 0.0052 | 0.0712 | 0.8229 | 0.0741 | 0.0111 | 0.0141 |
| B | 0.0000 | 0.0010 | 0.0035 | 0.0047 | 0.0588 | 0.8323 | 0.0385 | 0.0612 |
| C | 0.0012 | 0.0000 | 0.0029 | 0.0053 | 0.0157 | 0.1121 | 0.6238 | 0.2389 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

and the yield spreads. Hence, those so-called credit events also have an impact on the price of a bond or on the risk associated with a portfolio. Information on historical defaults and rating upgrades or downgrades is provided by the major rating agencies, for example, by *Moody's* or *Standard and Poor's*. They can also be calculated by the banks themselves for their internal loan portfolio based on an internal rating system, as it is suggested in the new Basel Capital Accord (Basel Committee on Banking Supervision 2001). In both cases, the available information on a company or an obligor is summarized by a single validation of its capability to pay back debt obligations that is pronounced in the form of a rating. An exemplary transition matrix is given in Table 1 showing the average 1-year transition probabilities of Moody's corporate bond ratings for the period 1982–2001. Using migration matrices, for simplification, it is generally assumed that only the current rating of a company has to be considered and not its whole history of rating changes. This assumption is generally referred to as the Markov assumption. A further important assumption of these models is that all companies in a certain rating category have the same probability of default (PD) or rating migration. Hence, under these assumptions for calculation of the associated risk with a credit portfolio, it is enough to know the current assigned ratings of the loans, the migration matrix and an estimate for the recovery rates of the loans.

In some of the most popular industry models such as *CreditMetrics* or CPV, historical transition matrices are used to determine VaR figures for a portfolio as well as adequate bond prices. The most popular rating-based model in the literature is probably the discrete-time Markovian model by Jarrow, Lando, and Turnbull (1997) (JLT). This model also incorporates possible rating upgrade, stable rating and rating downgrade (with "default" as a special event). In their seminal paper, Jarrow, Lando, and Turnbull (1997) model default and transition probabilities by using a discrete time, time-homogeneous Markov chain on a finite state space $S = \{1, \dots, K\}$. The state space $S$ represents the different rating classes. Hereby, the state 1 describes the best credit rating, while $K$ represents the default case. Hence, the $(K \times K)$ one-period transition matrix is

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{12} & \cdots & p_{2K} \\ \cdots & \cdots & \cdots & \cdots \\ p_{K-1,1} & p_{K-1,2} & \cdots & p_{K-1,K} \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad (1)$$

where $p_{ij} \geq 0$ for all $i, j, i \neq j$ and $p_{ii} \equiv 1 - \sum_{\substack{j=1 \\ j \neq i}}^{K} p_{ij}$ for all $i$. Obviously, $p_{ij}$ represents the actual probability of going to state $j$ from state $i$ in one time step. The default state $K$ is an absorbing one. On the basis of the historical default rates and empirical observed credit spreads, Jarrow, Lando, and Turnbull (1997) provide a model that can be used for various aspects in risk management or credit derivative pricing. Hereby, numerical techniques are used to adjust historical migration matrices into risk-neutral ones such that the corresponding default probabilities match observed credit spreads in bond prices.

Overall, for both industry credit VaR models and the pricing of credit derivatives, migration matrices can be considered as one of the major input variables to determine bond prices, credit VaR and credit derivative prices. Hence, a good forecast of credit transition matrices is important for various aspects of risk management and will help to determine adequate risk premiums for bonds, loans and related credit derivatives.

### 2.2 *Stability of migration matrices and credit VaR*

As mentioned earlier, historical transition matrices can be used as an input to estimate portfolio loss distributions and credit VaR figures. Unfortunately, transition matrices cannot be considered to be constant over a longer time period, see Allen and Saunders (2003) for a extensive review on cyclical effects in credit risk measurement. Further, migrations of loans in internal bank portfolios may behave differently than the transition matrices provided by major rating agencies such as *Moody's* or *Standard and Poor's* would suggest (Krüger, Stötzel, and Trück 2005; Weber, Krahnen, and Voßmann 1998). Nickell, Perraudin, and Varotto (2000) show that there is quite a big difference between transition matrices during an expansion of the economy and a recession. The results are confirmed by Bangia et al. (2002), who suggest that for risk management purposes, it might be interesting not only to simulate the term structure of defaults but also to design stress test scenarios by the observed behavior of default and transition matrices through the cycle. Jafry and Schuermann (2004) investigate the mobility in migration behavior using 20 years of Standard and Poor's transition matrices and find large deviations through time. Kadam and Lenk (2008) report significant heterogeneity in default intensity, migration volatility and transition probabilities depending on country and industry effects. Finally, Trück and Rachev (2005b) show that the effect of different migration behavior on exemplary credit portfolios may lead to substantial changes in expected losses, credit VaR or confidence sets for PDs. During a recession period of the economy, the VaR for one and the same credit portfolio can be up to eight times higher than that during an expansion of the economy.

As a consequence, following Bangia et al. (2002), it seems necessary to extend transition matrix application to a conditional perspective using additional information on the economy or even forecast transition matrices using revealed dependencies on macro-economic indices and interest rates. To approach these issues, it is a major concern to be able to judge whether one has an adequate model or forecast for a conditional or unconditional transition matrix. It rises the question what can be considered to be a "good" model in terms of evaluating migration behavior or risk for a credit portfolio. Therefore, to compare different adjustment methods in the literature and to benchmark the conditional forecasts against simple approaches, such as using a historical average or the previous period's transition matrix, will give important insight into the benefit of a conditional perspective.

### 2.3 *Credit derivative pricing*

As mentioned earlier, credit migration matrices also play a substantial role in the modeling and pricing of credit derivatives, in particular, collateralized debt obligations (CDOs). The market for credit derivatives can be considered as one of the fastest growing in the financial industry. The importance of transition matrices for modeling credit derivatives has been pointed out in several studies. Jarrow, Lando, and Turnbull (1997) use historical transition matrices and observed market spreads to determine cumulative default probabilities and credit curves for the pricing of credit derivatives. Bluhm (2003) shows how historical 1-year migration matrices can be used to determine cumulative default probabilities. This so-called calibration of the credit curve can then be used for the rating of cash-flow CDO tranches.

In recent publications, the effect of credit migrations on issues such as credit derivative pricing and rating is examined by several authors, among others by Bielecki et al. (2003), Hrvatin, Neugebauer, and Stoyle (2006), Hurd and Kuznetsov (2005) and Picone (2005). Hrvatin, Neugebauer, and Stoyle (2006) investigate CDO near-term rating stability of different CDO tranches depending on different factors. Next to the granularity of the portfolio, in particular, credit migrations in the underlying reference portfolio are considered to have an impact on the stability of CDO tranche ratings. Pointing out the influence of changes in credit migrations, Picone (2005) develops a time-inhomogeneous intensity model for valuing cash-flow CDOs. The approach used by the author explicitly incorporates the credit rating of the firms in the collateral portfolio by applying a set of transition matrices, calibrated to historical default probabilities. Finally, Hurd and Kuznetsov (2005) show that credit basket derivatives can be modeled in a parsimonious and computationally efficient manner within the affine Markov chain framework for multifirm credit migration, whereas Bielecki et al. (2003) concentrate on dependent migrations and defaults in a Markovian market model and the effects on the valuation of basket credit derivatives. Both approaches heavily rely on the choice of an adequate transition matrix as a starting point.

Overall, the importance of credit transition matrices in modeling credit derivatives cannot be denied. Having adequate estimates or forecasts for migrations in loan or bond portfolios substantially helps the process of calibration, valuation and pricing of these complex products.

### 2.4 *Internal and external rating systems*

Owing to the new Basel Capital Accord, most of the international operating banks may determine their regulatory capital based on internal rating systems (Basel Committee on Banking Supervision 2001). As a consequence, a high fraction of these banks will have ratings and default probabilities for all loans and bonds in their credit portfolio. Hence, more effort will be necessary to evaluate the adequacy and performance of internal rating systems in terms of quantifying the associated risks. Although Weber, Krahnen, and Voßmann (1998) were the first to provide a comparative study on the rating and migration behavior of four major German banks, recently more focus has been set on analyzing rating and transition behavior also in internal rating systems (Euopean Central Bank, 2004; Bank of Japan, 2005). Recent publications include Engelmann, Hayden, and Tasche (2003), Araten et al. (2004), Basel Committee on Banking Supervision (2005), Jacobson, Lindé, and Roszbach (2006) and Krüger, Stötzel, and Trück (2005). Hereby, Engelmann, Hayden, and Tasche (2003) and Basel Committee on Banking Supervision (2005) are more concerned with the validation, respectively, classification of internal rating systems. Araten et al. (2004) discuss issues in evaluating banks' internal ratings of borrowers comparing the ex-post discrimination power of an internal and external rating system. Jacobson, Lindé, and Roszbach (2006) investigate internal rating systems and differences between the implied loss distributions of banks with equal regulatory risk profiles. Finally, Krüger, Stötzel, and Trück (2005) examine time homogeneity and Markov behavior of an internal rating system and find substantial changes in migration matrices through time that significantly affect estimated figures for expected loss or VaR.

As a consequence, further analysis on internal rating systems and migration behavior of loans will have to take into account a conditional forward looking perspective including possible changes in the macro-economic environment. Therefore, to gain additional insight into the performance of different methods for adjusting historical average migration matrices to business cycle effects will help the banks to adequately measure credit VaR of internal loan portfolios.

## 3. Conditioning migration matrices to business cycle effects

In this section, we will review some of the approaches on adjusting migration matrices to the business cycle. As it was mentioned earlier, despite the substantial effects of changes in migration behavior on credit risk, only few methods have been suggested in the literature so far. A quite common approach in practice is to use average historical migration matrices instead of conditional ones to determine the VaR for a bond or loan portfolio. However, the literature points out that there are substantial changes in migration behavior that will also affect the risk associated with a credit portfolio (Bangia et al. 2002; Krüger, Stötzel, and Trück 2005; Trück and Rachev 2005b).

The first model to introduce the conditioning of transition matrices was the so-called macro-simulation approach by Wilson (1997) that is implemented in CPV (McKinsey and Company, 1998). Unfortunately, the model has some deficiencies in representing market cumulative default probabilities, especially for longer time horizons (Wehrspohn, 2004), and leaves open some important details of the adjustment procedure of the migration matrix. Nickell, Perraudin, and Varotto (2000) and Hu, Kiesel, and Perraudin (2002) propose the use of Bayesian methods in combination with an ordered probit model for conditioning credit migration matrices. The idea is to combine information from the historical average transition matrix estimate and results from other exogenous variables such as industry, country domicile and the business cycle. Bangia et al. (2002) link business cycle effects and transition matrices by estimating a regime-switching model for quarterly expansion and contraction classifications of the economy. However, simulating rating distributions based on their approach, the authors find no significantly different results for short-term migration and default behavior, compared with using an average migration matrix (Bangia et al. 2002).

The analysis in this article will focus on the probably most popular approaches in practice and theory: adjustments based on factor models, see, for examples, Belkin, Forest, and Suchower (1998b) and Kim (1999) as well as the use of numerical adjustment techniques that were initially proposed by Jarrow, Lando, and Turnbull (1997) and Lando (2000) . In the first class of models, a one-factor model is used to adjust migration matrices to a shift in the business cycle index. The approach is also implemented in one of the most popular industry models, the *CreditMetrics* framework. The second class of models, introduced by Jarrow, Lando, and Turnbull (1997), provides numerical adjustment methods to link historical transition matrices and observed market bond prices to obtain risk neutral migration matrices being in line with market credit spreads. Note that Lando (2000) gives some extensions of the adjustment methods that will also be included in the analysis. This section is dedicated to the description of these two approaches that will also be investigated more thoroughly in our empirical study.

### 3.1 *Adjustments based on factor model representations*

The first class of models for adjusting migration matrices to be considered in this study are approaches based on factor models as they are suggested in Belkin, Forest, and Suchower (1998b), Kim (1999), and Wei (2003). In these approaches, a one-factor model is adopted to incorporate business cycle dynamics into the transition matrix. To implement the technique, in a first step, a credit cycle index is built, which indicates the credit state of the financial market as a whole. The model for the credit cycle index should include relevant macro-economic and financial time series in order to represent the state of the economy adequately. In a second step, the transition matrix is conditioned on the forecasted credit cycle index. Unlike in one-factor default mode models, see, for example, Basel Committee on Banking Supervision (2001) and Finger (2001), the model of conditioning the transition matrix covers also events that lead to upgrades or downgrades.

The so-called credit cycle index $Z_t$ defines the credit state based on macro-economic conditions shared by all obligors during period $t$. The index is designed to be positive in good days and to be negative in bad days. A positive index implies a lower PD and downgrading probability but a higher upgrading probability and vice versa. To calibrate the index, PDs of speculative grade bonds are used, as often PDs of higher rated bonds are rather insensitive to the economic state, see, for example, Belkin, Forest, and Suchower (1998a) and Wilson (1997).

Now let $S_t$ be the speculative grade default probability in period $t$ and $\mu_S$ and $\sigma_S$ denote the historical average and the standard deviation of the inverse normal transformation $\Phi^{-1}(S_t)$ of $S_t$, respectively. Thus, as standardized expression, the credit cycle index

$$Z_t = \frac{\Phi^{-1}(S_t) - \mu_S}{\sigma_S} \tag{2}$$

follows a Gaussian distribution with expectation 0 and standard deviation of 1. As $S_t$ is restricted to lie between 0 and 1, for the regression analysis, a probit model is used. It is assumed that defaults reflect an underlying, continuous credit-change indicator that follows a standard normal distribution. Thus, $S_t$ can be modeled as

$$S_t = \Phi(\beta X_{t-1} + \epsilon_t) \tag{3}$$

where $\Phi$ denotes the standard normal distribution function, $X_{t-1}$ a set of macro-economic variables of the previous period and $\epsilon_t$ a random error term with $E_{t-1}(\epsilon_t) = 0$. The coefficients $\hat{\beta}$ can then be estimated based on the regression model

$$\Phi^{-1}(S_t) = \beta X_{t-1} + \epsilon_t \tag{4}$$

and the forecast for the inverse normal CDF of the speculative grade default probability is $E_{t-1}(\Phi^{-1}(S_t)) = \hat{\beta} X_{t-1}$. Kim (1999) points out that the probit model allows to create an unbiased forecast of the inverse normal CDF of $S_t$, given recent information about the economic state and the estimated coefficients.

The second step is to adjust the transition matrix according to estimated or forecasted values of the credit cycle index. Following the one-factor model suggested by Belkin, Forest, and Suchower (1998b), it is assumed that rating transitions reflect an underlying continuous credit-change indicator $Y_t$ following a standard normal distribution. Further, the credit-change indicator is assumed to be influenced by both a systematic and an unsystematic risk component. Therefore, $Y_t$ has a linear relationship with the systematic credit cycle index $Z_t$ and an idiosyncratic error term $u_t$. The typical one-factor model parameterization (Belkin, Forest, and Suchower 1998a; Finger, 2001) is denoted by.

$$Y_t = w Z_t + \sqrt{1 - w^2} u_t \tag{5}$$

As both $Z_t$ and $u_t$ are scaled to the standard normal distribution, with the weights chosen to be $w$ and $\sqrt{1 - w^2}$, $Y_t$ is also standard normally distributed. Note that $w^2$ can also be interpreted as the correlation between the the systematic credit cycle index $Z_t$ and the credit-change indicator $Y_t$. The probability distribution for the rating change of a company then takes place according to the outcome of the systematic risk index. To apply this scheme to a multi-rating system, it is assumed that conditional on an initial credit rating $i$ at the beginning of a year, one partitions values of the credit-change indicator $Y_t$ into a set of disjoint bins. The bins are defined in a way that the probability of $Y_t$ falling in a given interval equals the corresponding historical average transition rate. This can be performed by simply inverting the cumulative normal distribution function starting from the default column what is illustrated in Figure 1. The corresponding credit score $x_j^i$ for a transition from rating grade $i$ to grade $j$ is calculated according to $x_j^i = \Phi^{-1}(\sum_{k=j}^{K} p_{ij})$. Assume, for example, for a loan in rating class BBB, a default probability of $p_{\mathrm{BBB},D} = 0.0141$ and a migration probability of $p_{\mathrm{BBB},C} = 0.0111$. Then, the first corresponding credit score is $x_D^{\mathrm{BBB}} = \Phi^{-1}(p_{\mathrm{BBB},D}) = \Phi^{-1}(0.0141) = -2.1945$, respectively, the first bin is $(-\infty, -2.1945]$. For the second credit score, we obtain $x_C^{\mathrm{BBB}} = \Phi^{-1}(p_{\mathrm{BBB},D} + p_{\mathrm{BBB},C}) = \Phi^{-1}(0.0141 + 0.0111) = \Phi^{-1}(0.0252) = -1.9566$ and the corresponding bin is $(-2.1945, -1.9566]$. The calculation of the other credit scores and bins is straightforward.

Using the bins calculated from the average historical transition matrix, the conditional transition probabilities based on the outcome of the credit cycle index $Z_t$ are calculated. On average days, we obtain $Z_t = 0$ for the systematic risk index, and the credit-change indicator $Y_t$ follows a standard normal distribution. A positive outcome of the credit cycle index $Z_t$ shifts the credit-change indicator to the right-hand side, whereas in the case of a bad outcome of the systematic risk index, the distribution moves to the left hand side. Thus, for each year with a positive or negative outcome of the systematic credit cycle index, the conditional transition rates will deviate from the average historical migration matrix. Thus, we have to find a shift such that the probabilities associated with the bins defined above best approximate the given year's observed transition rates. Hence, $w$ is determined so as to minimize the discrepancies between the model transition probabilities and the observed transition probabilities. Recall that $w$ represents the weight or influence of the systematic risk index on the credit-change indicator, whereas $w^2$ measures the correlation between the two variables. The conditional transition probability $p_t(i, j|Z_t)$ for rating state $i$ to another rating state $j$ has the ordered probit model

$$p_t(i, j|\hat{Z}_t) = \Phi\left(\frac{x_{j+1}^i - w\hat{Z}_t}{\sqrt{1-w^2}}\right) - \Phi\left(\frac{x_j^i - w\hat{Z}_t}{\sqrt{1-w^2}}\right) \tag{6}$$

Note that hereby $\hat{Z}_t$ denotes the forecasted outcome of the credit cycle index. The estimation problem then results in finding the "optimal" $w$, such that the difference between the transition

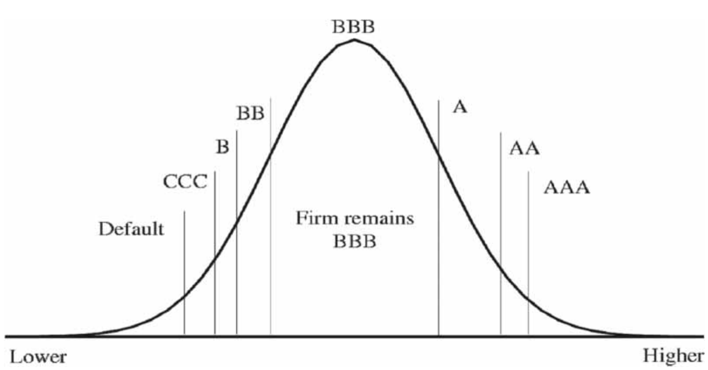


Figure 1. Corresponding credit scores to transition probabilities for a company with BBB rating (compare Belkin, Forest, and Suchower 1998b).

probabilities forecasted by the model and the actual observed ones is minimized according to some distance criterion. Belkin, Forest, and Suchower (1998b) suggest to minimize the following expression

$$\min \sum_j \sum_i \frac{n_{t,j}[p_t(i,j) - p_t(i,j|Z_t)]^2}{p_t(i,j|Z_t)(1 - p_t(i,j|Z_t))} \tag{7}$$

where $n_{t,j}$ denotes the number of transitions from initial grade $i$ to $j$ in the year $t$. Further, the observations are weighted by the inverse of the approximate sample variances $p_t(i, j|Z_t)$. Alternative approaches for weighting the differences between forecasted and observed migration probabilities can be used and will be described in later sections. For the estimation of the ordered probit model, we refer to Maddala (1983).

Wei (2003) extends the one-factor model representation by a multi-factor, Markov chain model for rating migrations and credit spreads. The model allows transition matrices to be time-varying and further driven by rating-specific latent variables. These variables can encompass a variety of economic factors including business cycle effects. The univariate one-factor model such as that of Belkin, Forest, and Suchower (1998b) or Kim (1999) can then be considered as a special case with no rating-specific shift. Similar to one-factor models, the starting point is the assumption that there exists an average transition matrix $\bar{P}$ with fixed entries representing average, per-period transition probabilities across all credit cycles. Further, it is assumed that the entries in a transition matrix for a particular year will deviate from the averages, and the size of the deviations is dependent on the condition of the economy. A further assumption is that the size of the deviations can be different for each rating category. As in our empirical investigation we concentrate on the more common one-factor model approach, for further model description and parameter estimation we refer to Wei (2003).

### 3.2 *Numerical adjustment methods*

In this section, we will briefly review two numerical adjustment methods for migration matrices suggested by Lando (2000). The methods were initially designed to match transition matrices with default probabilities implied in market spreads of bonds (Jarrow, Lando, and Turnbull 1997). However, given estimates for conditional default probabilities based on the macro-economic situation, they can also be used to adjust the transition matrices to business cycle effects. To illustrate the adjustment methods, let us consider the continuous-time case where a time-homogeneous Markov chain is specified via the $(K \times K)$ generator matrix $\Lambda$. A generator matrix can be denoted by

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1K} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2K} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{K-1,1} & \lambda_{K-1,2} & \cdots & \lambda_{K-1,K} \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{8}$$

with negative diagonal elements $\lambda_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{K} \lambda_{ij}$ for $i = 1, \ldots, K$, representing the intensities of jumping from rating state $i$ to state $j$, see, example, Jarrow, Lando, and Turnbull (1997) and Israel, Rosenthal, and Wei (2000). As in the discrete case, the default state $K$ is again an absorbing one.

For a $(K \times K)$ generator matrix $\Lambda$, the series

$$P(t) = \exp(t\Lambda) = \sum_{k=0}^{\infty} \frac{(t\Lambda)^k}{k!} \tag{9}$$

converges to the $K \times K$ $t$-period transition matrix. Lando (2000) describes three different methods to modify the transition matrices such that default probabilities implied in bond prices are matched. Out of those, we will consider two methods, one of them is very similar to the approach suggested in Jarrow, Lando, and Turnbull (1997). The starting point for the procedure is a 1-year average historical transition matrix $P$ and the associated generator matrix $\Lambda$ with $P = \exp(\Lambda)$. Let us further assume that we have estimated conditional 1-year default probabilities $\hat{p}_{iK}$ based on the forecasted outcome of the economy. The aim is then to create a transition matrix $\tilde{P}$ in a way that the estimated conditional default probabilities match the corresponding entries in the last column of $\tilde{P}$.

Thus, a procedure for adjusting the average transition matrix $P$ is required. Jarrow, Lando, and Turnbull (1997) suggest to transform the generator matrix $\Lambda$ into $\widetilde{\Lambda}$ such that the default probabilities in transition matrix $\tilde{P} = \exp(\tilde{\Lambda})$ equal the derived $\hat{p}_{iK}$ for all rating classes $i = 1, \ldots, K$.

The first adjustment method (Num I) modifies the default column of the generator matrix by multiplication with a factor $\pi_i$ $(i = 1, \ldots, K-1)$ and simultaneously modifies the diagonal element of the generator according to

$$\begin{array}{lcl} \widetilde{\lambda}_{1K} = \pi_1 \cdot \lambda_{1K} & \text{and} & \widetilde{\lambda}_{11} = \lambda_{11} - (\pi_1 - 1) \cdot \lambda_{1K} \\ \widetilde{\lambda}_{2K} = \pi_2 \cdot \lambda_{2K} & \text{and} & \widetilde{\lambda}_{22} = \lambda_{22} - (\pi_2 - 1) \cdot \lambda_{2K} \\ \cdots & & \cdots \end{array}$$

and for row $K-1$

$$\widetilde{\lambda}_{K-1,K} = \pi_{K-1} \cdot \lambda_{K-1,K} \text{ and}$$
$$\widetilde{\lambda}_{K-1,K-1} = \lambda_{K-1,K-1} - (\pi_{K-1} - 1) \cdot \lambda_{K-1,K}$$

such that for the new transition matrix $\widetilde{P} = \exp(\tilde{\Lambda})$, the last column equals the estimated conditional PDs $\hat{p} = (\hat{p}_{1K}, \hat{p}_{2K}, \ldots, \hat{p}_{K-1,K}, 1)$.

The idea in the second adjustment procedure (Num II) is quite similar. However, instead of changing just the default and diagonal elements of the rows, the adjustment of the generator matrix is based on multiplying the whole row of the generator matrix by a factor $\pi_i$ $(i = 1, \ldots, K-1)$. Note that the factor $\pi_i$ is often referred to as the so-called risk premium. Thus, in the second numerical adjustment approach, we have to solve

$$\widetilde{\Lambda} = \begin{pmatrix} \pi_1 \cdot \lambda_{11} & \pi_1 \cdot \lambda_{12} & \cdots & \pi_1 \cdot \lambda_{1K} \\ \pi_2 \cdot \lambda_{21} & \pi_2 \cdot \lambda_{22} & \cdots & \pi_2 \cdot \lambda_{2K} \\ \cdots & \cdots & \cdots & \cdots \\ \pi_{K-1} \cdot \lambda_{K-1,1} & \pi_{K-1} \cdot \lambda_{K-1,2} & \cdots & \pi_{K-1} \cdot \lambda_{K-1,K} \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{10}$$

for $\pi_i$, $i = 1, \ldots, k$ such that the last column of $\widetilde{P} = \exp(\tilde{\Lambda})$ equals the vector $\hat{p}$.

Unfortunately, Lando (2000) does not give any information on how exactly this numerical procedure affects the probability mass of the original average historical migration matrix. Also note that after the adjustment, the generator matrix has to be examined if it still fulfills the criteria, namely, non-negative off-diagonal elements and row sums of zero for each row. For further discussion on the existence of a so-called "true" generator, see, for example, Israel, Rosenthal, and Wei (2000) or Trück and Rachev (2005b).

## 4. Data and models

This section will provide considered data and models in our empirical analysis. We will compare the in-sample and out-of-sample performance of adjustment methods for forecasting credit migration matrices. Hereby, we consider Moody's credit migration matrices for the US market from 1984 to 1999. The in-sample period includes a history of 10 years from 1984 to 1993, whereas we use a 6 year period from 1994 to 1999 to evaluate the out-of-sample forecasting ability of our models. The compared approaches include one-factor models and numerical adjustment procedures, as they were described in the previous section. As benchmark results, we will also use the average historical migration matrices and the transition matrix of the previous period as forecasts for next year's migration matrix.

### 4.1 *The credit cycle index*

To determine one-period ahead forecasts of conditional PDs and the credit cycle index, we use a multiple regression model of the form

$$\Phi^{-1}(S_t) = c_0 + \sum_{j=1}^{d} c_j X_{j,t-1} + \epsilon_t, \quad t \in \mathbb{N} \tag{11}$$

The process dynamic is influenced by the vector $X_{t-1}$ of $d$ exogenous macroeconomic variables of the previous period. Using equations (2) and (6), we can then calculate forecasts for the one-period ahead credit cycle index $Z_t$ and the conditional default probabilities $\hat{p}_t(i, D|Z_t)$ for each rating class. Table 2 displays the included variables in the multiple regression model. Both a variety of macro-economic variables as well as credit spreads and differences between long-term and short-term treasury bonds were considered. Having only 10 observations from 1984 to 1993 for both default probabilities and macro-economic variables, in order to avoid overfitting, not more

Table 2. Included variables for the multiple regression model for credit cycle indices.

| Variable | Notation |
|---|---|
| Change in consumer price index | $\mathrm{CPI}_{t-1}$ |
| Change in GDP growth | $\mathrm{GDP}_{t-1}$ |
| Change in annual savings | $\mathrm{SAV}_{t-1}$ |
| Change in manufacturing and sales | $\mathrm{MAN}_{t-1}$ |
| Change in working output per hour | $\mathrm{OUT}_{t-1}$ |
| Change in consumption expenditures | $\mathrm{CON}_{t-1}$ |
| Change in unemployment rate | $\mathrm{UN}_{t-1}$ |
| Treasury yields 10, 5, 3 and 1 year | $\mathrm{TY10}_{t-1}$ etc. |
| Spread between 10-year and 1-year treasury | $\mathrm{STR}_{t-1}$ |
| Spreads on investment grade bonds | $\mathrm{SINV}_{t-1}$ |
| Spreads on speculative grade bonds | $\mathrm{SSPE}_{t-1}$ |

than five exogenous variables were permitted in the regression model. In the following, we will now describe the procedure of model estimation and conditioning of the migration matrices.

### 4.2 *Forecasts using the factor model approach*

Following Kim (1999), the multiple regression model (11) is used for modeling and forecasting the continuous credit cycle index $Z_t$. It is assumed that the index follows a standardized normal distribution. Thus, a probit model will allow us to create unbiased forecasts of the inverse normal CDF of $Z_t$, given the recent information of the last period about the economic state and the estimated coefficients. Note that unlike Kim (1999) who only uses one credit cycle index based on speculative default probabilities, we will consider two credit cycle indices: one for speculative grade and one for investment grade issues. For the investment grade issues, we use cumulative defaults of issuers rated Aaa,Aa,A and Baa, whereas for the speculative grade issues, default probabilities from Ba to C were included. Figure 2 exemplarily reports the observed default frequencies for the non-investment grade rating classes Ba, B and C that were used for the estimation of the speculative grade credit cycle index.

In a second step, the forecasts of the credit cycle indices are used for determining conditional migration probabilities $\hat{p}_t(i, j|Z_t)$. The adjustment is conducted following the procedure described in Section 3.1. However, for finding the optimal weights for the systematic risk indices $w_{Inv}$ and $w_{Spec}$, minimizing the discrepancies between the forecasted conditional and the actually observed transition probabilities, we introduce some model extensions. We allow for a more general weighting of the difference between forecasted and empirical observation for the transition probability in each cell than in equation (7). Hence, the weights for each of the cells are assigned according to some function $f$

$$\min \sum_j \sum_i f(i, j, p_t(i, j), \hat{p}_t(i, j|Z_t)) \tag{12}$$

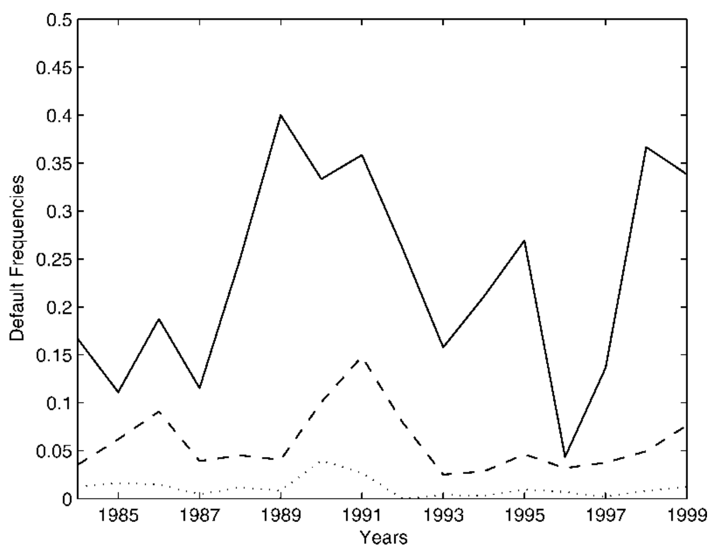


Figure 2. Moody's historical default rates for speculative rating classes Ba (dotted), B (dashed) and C (solid) for the period 1984–99.

where the outcome of $f(i, j, p_t(i, j), \hat{p}_t(i, j|Z_t))$ may be dependent on the row $i$ and column $j$ of the cell as well as on the forecasted and actually observed transition probabilities $\hat{p}_t(i, j|Z_t)$ and $p_t(i, j)$.

To achieve a better interpretation of the results, we will also use risk-sensitive difference indices (Trück and Rachev 2005a) as optimization criteria for the distance between forecasted and actual migration matrix. Recall that based on the estimated model, the parameter $w$ and the shifts on the migrations according to some optimization criteria are determined. In fact, this a crucial point of the model as it comes to forecasting credit migration matrices. Although Belkin, Forest, and Suchower (1998b) suggest to minimize a weighted expression of the form (7), Wei (2003) uses the absolute percentage deviation based on the $L_1$ norm or a pseudo $R^2$ as goodness-of-fit criteria. Trück and Rachev (2005a) show that most of the distance measures suggested in the literature so far do not quantify differences between migration matrices adequately in terms of risk. However, forecasts for transition matrices will be especially used for determining credit VaR, portfolio management and risk management purposes. Therefore, especially risk-sensitive difference indices may be a rewarding approach for measuring the difference between forecasted and observed matrices. Trück and Rachev (2005a) suggest that the difference between a migration matrix $P = (p_{ij})$ and $Q = (q_{ij})$ can be determined in a weighted cell-by-cell calculation. It is based on the distance to the diagonal of the matrix and whether it is on the right- or left-hand side of the diagonal. For example, the use of the following weights may be assigned

$$d(i, j) = (i - j) \cdot (p_{ij} - q_{ij}) \tag{13}$$

Additionally, the authors argue that the credit event is a default, with the most influence on the loss of a credit portfolio. Therefore, changes in the last column of a migration matrix obtain a different weight than deviances in the other cells. The weight is chosen according to the dimension $n$ of the migration matrix. Following Trück and Rachev (2005a), we will include two risk-sensitive directed difference indices in our analysis as optimization criteria

$$D_1(P, Q) = \sum_{i=1}^{n} \sum_{j=1}^{n-1} d(i, j) + \sum_{i=1}^{n} n \cdot d(i, n) \tag{14}$$

$$D_2(P, Q) = \sum_{i=1}^{n} \sum_{j=1}^{n-1} d(i, j) + \sum_{i=1}^{n} n^2 \cdot d(i, n) \tag{15}$$

To compare the results with standard criteria, also the classic $L_1$ and $L_2$ metric

$$D_{L_1}(P, Q) = \sum_{i=1}^{n} \sum_{j=1}^{n} |p_{ij} - q_{ij}| \tag{16}$$

and

$$D_{L_2}(P, Q) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} (p_{ij} - q_{ij})^2} \tag{17}$$

will be considered. Further, the measure of the so-called normalized squared differences $\mathrm{NSD}_{\mathrm{symm}}$

$$D_{\mathrm{NSD}}(P, Q) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{(p_{ij} - q_{ij})^2}{p_{ij}}, \quad \text{for } p_{ij} \neq 0 \tag{18}$$

is included in the analysis. Note that these criteria can also be used to evaluate the distance between forecasted and observed transition matrices for the numerical adjustment methods and the chosen benchmark models.

### 4.3 *Forecasts using numerical adjustment methods*

The second approach involves the numerical adjustment method suggested by Jarrow, Lando, and Turnbull (1997) and Lando (2000). Again, we use a multiple regression model of the form (11). However, as the method requires estimates $\hat{p}_t(i, K)$ for the individual rating classes, for each speculative rating grade $Ba$, $B$, $C$ as well as for the rating class Baa a separate model is estimated. For the investment grade rating classes Aaa, Aa, A, we have to follow a different approach. Considering Moody's historical default frequencies in several years, no default could be observed for the three rating classes. To develop a regression model with only 10 observations, among them several with PD zero should be avoided. Hence, for rating grades Aaa, Aa and A, we decided to use default probabilities from the average historical migration matrix $\bar{P}$ as estimators for the next periods PDs in these rating classes. On the basis of these assumptions for each year, we can estimate the vector for next year's default probabilities in the individual rating classes

$$\hat{p} = (\hat{p}_{\mathrm{Aaa,D}}, \hat{p}_{\mathrm{Aa,D}}, \hat{p}_{\mathrm{A,D}}, \hat{p}_{\mathrm{Baa,D}}, \hat{p}_{\mathrm{Ba,D}}, \hat{p}_{\mathrm{B,D}}, \hat{p}_{\mathrm{C,D}}, 1)'$$

Using these estimates and an average historical migration matrix, we can then apply the numerical adjustment methods suggested in Section 3.2.

## 5. Empirical results

In this section, we will provide in-sample and out-of-sample results for the described models and compare them with benchmark results. Hereby, we will evaluate the performance of the chosen models against the standard approach of using historical average transitions or last year's migration matrix for the calculation of credit VaR.

### 5.1 *Regression models*

In a first step, the regression models for the credit cycle index and default probability scores are estimated. Recall that in order to avoid overfitting, at the most five explanatory variables were permitted in each regression model. The in-sample period comprised the empirical default frequencies and the suggested macro-economic variables for the period from 1984 to 1993. Among the models tested, the best results for the speculative grade credit cycle index were obtained using the macro-economic variables "change in GDP growth" $\mathrm{GDP}_{t-1}$, "change in annual savings" $\mathrm{SAV}_{t-1}$, the "change in consumption expenditures" $\mathrm{CON}_{t-1}$, the "change in unemployment rate" $\mathrm{UN}_{t-1}$ and the "spread between a 10- and 1-year treasury" $\mathrm{STR}_{t-1}$ bond. The model gave a coefficient of determination of $R^2 = 0.98$, an $F$-statistic of 43.38 and a corresponding p-value of 0.001; therefore, it was highly significant. For the investment grade credit cycle index, the best results were obtained with a model including the macro-economic variables change in the consumer price index $\mathrm{CPI}_{t-1}$, change in GDP growth $\mathrm{GDP}_{t-1}$, change in consumption expenditures $\mathrm{CON}_{t-1}$ and the change in unemployment rate $\mathrm{UN}_{t-1}$. The model gave an $R^2$ statistic of 0.82, an $F$-statistic of 5.52 and a corresponding $p$-value of 0.045, and so the model was still significant at 5 % level. Further, all regression coefficients were significant and showed the anticipated sign. Note that the fit for cumulated investment grade defaults was clearly worse, but it is generally

Table 3. Parameter estimates for the multiple regression model (in-sample period from 1984 to 1993).

| Variable | Notation | $Z_{\text{Inv}}$ | $Z_{\text{Spec}}$ |
|---|---|---|---|
| Constant | $\beta_0$ | −0.3973 | −0.6182 |
| Change in consumer price index | $\text{CPI}_{t-1}$ | 0.2187 | – |
| Change in GDP growth | $\text{GDP}_{t-1}$ | −0.9338 | −0.2461 |
| Change in annual savings | $\text{SAV}_{t-1}$ | – | 0.1838 |
| Change in consumption expenditures | $\text{CON}_{t-1}$ | −1.3744 | −0.2509 |
| Change in unemployment rate | $\text{UN}_{t-1}$ | 0.3616 | 0.2351 |
| Spread between 10- and 1-year treasury | $\text{STR}_{t-1}$ | – | −0.0057 |

accepted that investment grade defaults are less dependent on business cycle effects than speculative grade issuers (Belkin, Forest, and Suchower 1998a; Nickell, Perraudin, and Varotto 2000). The regression coefficients for speculative and investment grade credit cycle indices $Z_{\text{Spec}}$ and $Z_{\text{Inv}}$ are displayed in Table 3. Estimation of the models for individual rating classes yields $R^2$ statistics between 0.79 and 0.98. Further information on parameter estimates and statistics are available on request to the author.

### 5.2 *In-sample results*

After estimation of the regression model for the credit cycle index, in a next step, we will determine conditional forecasts for migration matrices based on the outcome of the credit cycle index. We first consider the results for the estimated weights of the systematic credit cycle indices $Z_{\text{Spec}}$ and $Z_{\text{Inv}}$. Recall that in the chosen one-factor model approach, $w$ is determined numerically in order to minimize the difference between the conditional forecast $\hat{p}_t(i, j|Z_t)$ and empirically observed migrations $p_t(i, j)$ for all considered transition matrices in the in-sample period. The shift in the credit change indicator and hence, the shift in transition and default probabilities, is then an outcome of the forecasted credit cycle index of the next period and the estimated weight $w$ for the systematic risk factor $Z$.

Table 4 provides the weights $w_{\text{Inv}}$ and $w_{\text{Spec}}$ for investment grade and speculative grade ratings giving the minimal distance between forecasts and observed migration matrices for the in-sample period 1984–93. Note that dependent on the chosen distance measures, we obtain different outcomes for the weights. For the speculative grade model, we find significantly higher weights of the systematic credit cycle index than for the investment grade model for all optimization criteria. We observe the lowest estimate for the weight $w_{\text{NSD,Spec}} = 0.1698$ for the NSD distance criterion, whereas for the other distance measures, the weight for the speculative credit cycle index is

Table 4. Estimated weights $w$ for the credit cycle index $Z$, representing the influence of $Z$ on the change indicators $Y$.

| Optimization criteria | $L_1$ | $L_2$ | NSD | $D_1$ | $D_2$ |
|---|---|---|---|---|---|
| $w_{\text{Inv}}$ | 0.0504 | 0.1143 | 0.0318 | 0.1089 | 0.1762 |
| $w_{\text{Spec}}$ | 0.2176 | 0.2544 | 0.1698 | 0.2115 | 0.2288 |

Results refer to the in-sample period 1984–93 and are estimated based on the one-factor approach for investment grade (Inv) and speculative grade ratings (Spec).

estimated to be between 0.2115 and 0.2544. For investment grade issues, the estimated weights range from 0.0318 to 0.1762. As mentioned earlier, this is in line with previous results in the literature (Belkin, Forest, and Suchower 1998a; Wilson, 1997). Especially, when the shift is conducted to minimize the distance according to the $L_1$ and NSD distance measure, the influence of the systematic risk factor becomes very small, $w_{L_1,\text{Inv}} = 0.0318$ and $w_{\text{NSD,Inv}} = 0.0504$, respectively. This means that for these criteria, the systematic risk index gives very little explanation for changes in rating behavior. The highest estimate for the weight $w_{D_2,\text{Inv}} = 0.1762$ is obtained when the distance is minimized subject to the risk-sensitive $D_2$ difference index criteria. It seems as if according to $D_2$, also changes in investment grade migration behavior could be explained by the systematic credit cycle index to a certain degree.

We will now investigate the in-sample one-period ahead forecasts results for the different approaches. Table 5 provides in-sample results for mean absolute forecast errors according to the applied difference measures. As mentioned above next to a factor-model approach (Factor) and the numerical adjustment methods (Num I and Num II), two standard benchmark methods were included in the results: using the average migration matrix of the in-sample periods (naive I) or the transition matrix of the previous period (naive II) as forecast for next period's migration matrix. Best results for each distance measure are highlighted in bold. Note that the mean error or standard deviation of the errors for different indices within the columns cannot be compared because of a different scale. However, the results in the rows can be compared and provide the forecasting performance in comparison to other approaches. For each of the considered distance criteria, the one-factor model outperforms all other approaches including the numerical adjustment procedures. In contrast to these results, the numerical adjustment methods fail to provide better results than the naive approach for the criteria $L_1$, $L_2$ and NSD. Especially, Num II that was applied in the seminal work by Jarrow, Lando, and Turnbull (1997) gives rather bad 1-year ahead forecasts based on the estimated default probabilities with the credit cycle index. Considering these results and the relevance of the approach in the literature, we recommend a more thorough investigation on how migration probabilities are changed by these methods in the future.

It is not surprising that the best in-sample results are obtained for the one-factor model approach. On the basis of optimization procedure in (4.2), which chooses the weight for the systematic risk

Table 5. In-sample results for mean forecast errors according to applied difference measures and adjustment techniques.

| | | Distance statistics $D(\hat{P}, P_{\text{obs}})$ | | | | |
|---|---|---|---|---|---|---|
| Distance | Method | Factor | Num I | Num II | Naive I | Naive II |
| $L_1$ | MAE | **0.8058** | 1.3461 | 1.7092 | 0.8809 | 1.0245 |
| | Std | (0.2665) | (0.1415) | (0.2966) | (0.2414) | (0.3056) |
| $L_2$ | MAE | **0.0580** | 0.1802 | 0.2947 | 0.0725 | 0.1022 |
| | Std | (0.0483) | (0.0503) | (0.0885) | (0.0510) | (0.0948) |
| NSD | MAE | **0.2956** | 0.4773 | 0.8774 | 0.3483 | 0.5911 |
| | Std | (0.1363) | (0.1356) | (0.2544) | (0.1800) | (0.4556) |
| $D_1$ | MAE | **0.3218** | 0.7592 | 1.1505 | 1.2417 | 1.4969 |
| | Std | (0.2189) | (0.3765) | (0.9526) | (0.8903) | (0.7522) |
| $D_2$ | MAE | **1.9926** | 6.2919 | 9.4428 | 9.6931 | 10.8898 |
| | Std | (1.3993) | (3.2154) | (4.8858) | (7.1228) | (5.5701) |

The estimation period included 10 years from 1984 to 1993. Best results for each distance measure are highlighted in bold.

factor in order to minimize the distance between the forecasted and empirical transition probabilities, these results could be expected. However, it is interesting to investigate how much the results improved, subject to the considered optimality criteria. For the $L_1$, $L_2$ metric and the NSD difference index, we observe a reduction in the mean absolute error (MAE) by a fraction between 10 and 50%, compared with the naive approaches. The reduction for the risk-adjusted difference indices $D_1$ and $D_2$ is clearly higher. Comparing MAEs between conditional and unconditional estimates for the $D_1$ and $D_2$ criteria, we find that according to the chosen criteria, the improvement is highly significant. Forecasting errors for the naive approaches are approximately four to five times higher, e.g. using naive approaches, the MAEs for the risk-sensitive $D_2$ criterion are approximately $D_{2,\text{NaiveI}} = 9.6931$ and $D_{2,\text{NaiveII}} = 10.89$, whereas for the one-factor model, we obtain an MAE of $D_{2,\text{Factor}} = 1.99$. For these criteria, also the numerical adjustment methods Num I and Num II give better results. As more weight is allocated in the default column, the additional information of PD forecasts for the next period improves the results. Overall, in comparison with the one-factor model, for the numerical adjustment techniques, the forecast errors are still significantly higher.

We also investigated whether the improvement of the forecasting results of the one-factor model was mainly due to the speculative or investment grade rating classes of the migration matrix. Table 6 provides the results of the one-factor model and the naive approaches separately for initial speculative and investment grade ratings. We find that especially for the risk-sensitive evaluation criteria, the improvement using a credit cycle index comes from better forecasts for the speculative grade default probabilities and rating changes. For the rating classes Ba–C, the forecast error is reduced up to 80% when the risk-sensitive measures $D_1$ or $D_2$ are applied. Further, as it is indicated by Table 6, the large deviations from actual observed migration matrices take place in the speculative grade area of the matrix, where more variation can be observed.

At this point, we should also emphasize the advantage of the directed difference indices $D_1$ and $D_2$ as measure for the goodness of fit. It concerns the question of interpretation of the results. Obviously, an MAE of 0.8058 for the $L_1$ norm cannot be interpreted in terms of risk. Using the risk-sensitive difference indices, we are able to give an interpretation of the results from a risk perspective. (Trück and Rachev (2005a)) show that for credit portfolios, differences between migration matrices are highly correlated with the estimated credit VaR. Using Moody's historical migration matrices, for an exemplary credit portfolio, a relationship between credit VaR and the deviation of a transition matrix from Moody's average historical migration matrix is derived. Setting the recovery rates to a constant, the relationship between VaR for the exemplary loan

Table 6. In-sample results (mean absolute errors) separately for speculative grade (ratings Ba, B and C) and investment grade (Aaa, Aa, A and Baa) ratings.

| | Speculative grade | | | Investment grade | | |
|---|---|---|---|---|---|---|
| Distance | Factor | Naive I | Naive II | Factor | Naive I | Naive II |
| $L_1$ | 0.5060 | 0.5756 | 0.6558 | 0.2997 | 0.3052 | 0.3687 |
| $L_2$ | 0.0423 | 0.0557 | 0.080 | 0.0157 | 0.0168 | 0.0222 |
| NSD | 0.1913 | 0.2425 | 0.3548 | 0.1043 | 0.1058 | 0.2363 |
| $D_1$ | 0.2770 | 1.1620 | 1.3375 | 0.0449 | 0.1197 | 0.1628 |
| $D_2$ | 1.6544 | 8.8322 | 8.8505 | 0.3382 | 0.8609 | 1.0393 |

portfolio and $D_2$ is then approximately expressed by (in Mill. Euro)

$$\text{VaR}_{95\%,t} = 138.7675 + 4.7110 \cdot D_{2,t} + \epsilon_t \tag{19}$$

The estimated regression model yields an $R^2 > 0.9$ (Trück and Rachev, 2005a). Hence, owing to the very high correlations between the directed difference index and credit VaR, for exemplary loan portfolios, we would be able to measure our errors on migration matrix forecasts in terms of risk. This means that the mean error of 9.6931 from Table 5, using the average migration matrix $\bar{P}$ as an estimator, could be interpreted for the exemplary portfolio as an average misspecification of VaR of approximately $4.7110 \cdot 9.6931 \approx 45$ Mill. Euro per year. For using the migration matrix of the previous year, we obtain an approximate error of 51 Mill. Euro. Using the one-factor model in order to condition migration matrices to business cycle effects, the MAE is reduced to 1.9926, yielding an average error on 1-year VaR forecasts of 9.38 Mill. Euro for the exemplary portfolio. It is important to point out that these are just approximate numbers for an exemplary portfolio, ignoring variations in Loss Given Default (LGD) figures and other components. However, as a general result, we argue that using the risk-sensitive difference indices as goodness-of-fit measure, the forecast error may also be quantified in terms of risk. We point out that further research on this issue will be required, especially on the sensitivity of the difference indices. Overall, the advantage of an index giving a strong interpretation in terms of risk is obvious.

### 5.3 *Out-of-sample forecasts*

Finally, we used the developed models for out-of-sample forecasting of rating migration behavior. The considered period were the subsequent years from 1994 to 1999. Using a yearly re-estimation of the regression model, the weights of the systematic credit cycle index and conditional PD estimates, forecasts for the migration matrix of the following year were calculated. Hereby, the in-sample estimation period was increased each year from 1984–93 to 1984–94, 1984–95,…,1984–98. Results for the yearly re-estimated weights $w_{\text{Inv}}$ and $w_{\text{Spec}}$ of the systematic credit cycle index using $D_2$ are described in Table 7. Results for the other distance measures are available on request from the author. As it could be expected, the weights change through time and vary between 0.234 and 0.191 for the speculative grade issuers and between 0.168 and 0.149 for the investment grade credit cycle index. Generally, the weight of the systematic credit cycle index decreases for both investment categories through time.

Table 8 finally investigates the out-of-sample performance of the considered models. We find that, especially for the conditional approaches (Factor, Num I and Num II) that use the credit cycle index, the results are not as good as for the in-sample period. This can be explained by the decreasing influence of the systematic credit cycle index for the out-of-sample period that was

Table 7. Re-estimated weights of the credit cycle index for investment and speculative grade ratings in the one-factor model approach using the distance index $D_2$.

| Year | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 |
|---|---|---|---|---|---|---|
| | | | Speculative grade | | | |
| $D_2$ | 0.2340 | 0.2058 | 0.2005 | 0.1914 | 0.1961 | 0.1964 |
| | | | Investment grade | | | |
| $D_2$ | 0.1663 | 0.1681 | 0.1579 | 0.1493 | 0.1529 | 0.1489 |

Table 8. Out-of-sample results for mean absolute forecast errors according to applied difference measures $D_2$ and adjustment techniques.

| Year | Distance statistics $D_2(\hat{P}, P_{obs})$ | | | | |
|---|---|---|---|---|---|
| Method | Factor | Num I | Num II | Naive I | Naive II |
| 1994 | **3.2563** | 6.9856 | 9.2745 | 8.7058 | 4.4462 |
| 1995 | 1.9834 | 3.5110 | 4.2809 | **1.0289** | 7.6769 |
| 1996 | **8.8284** | 14.3459 | 21.7397 | 18.9697 | 17.9408 |
| 1997 | **5.2049** | 10.8375 | 14.5619 | 12.4049 | 6.5649 |
| 1998 | **2.5774** | 8.9729 | 8.0307 | 5.5436 | 17.9484 |
| 1999 | 3.1936 | 7.2204 | 9.7712 | 8.5408 | **2.9973** |
| Average | **4.1740** | 8.6456 | 11.2765 | 9.1989 | 9.5958 |

The out-of-sample period included 6 years from 1994 to 1999. Best results for each year are in bold.

reported in Table 7. Still, the factor model significantly outperforms the numerical adjustment methods Num I and Num II and the benchmark models Naive I and Naive II. Except for the years 1996 and 1999, it gives the best forecasts of next year's migration matrix in each year. Further, the lowest average forecast error is obtained using the factor model approach. However, compared with the in-sample estimation, the average forecasting error is higher and we obtain an MAE of 4.1740. For the naive models, the results for the in-sample and out-of-sample periods are similar, whereas for the numerical adjustment methods, the error also increased. Note that Num II provides the worst results of all models and is outperformed even by the naive approaches.

We point out that the results could have been improved by changing the variables of the macro-economic forecasting model for the credit cycle index. However, in order to guarantee a genuine out-of-sample test of the model, the macro-economic variables were chosen to be the same for in-sample and out-of-sample model evaluation. Still, we conclude that a regular re-estimation of the model for the credit cycle indices may be recommendable.

## 6. Conclusion

In this article, we compared different methods for forecasting transition matrices with business cycle effects. Transition matrices can be considered as a major determinant for the management of portfolio credit risk and VaR determination in rating-based models. Migration matrices due to business cycle effects show strong variations through time that may be captured by conditional estimation procedures. The focus was set on a comparison of two of the most common methods for adjustment of credit migration matrices: a one-factor model initially suggested by Belkin, Forest, and Suchower (1998a) and Kim (1999) and implemented in the *CreditMetrics* framework and numerical adjustment methods by Lando (2000) applied, for example, in the seminal work by Jarrow, Lando, and Turnbull (1997). We used business cycle indicators and Moody's historical migration matrices for the US market to conduct an empirical analysis.

Our findings show that the numerical adjustment methods fail to outperform the naive approach of using an average historical transition matrix or the one of the previous periods as forecasts. In contrast, hand an approach using a one-factor model for the systematic risk factor provided significantly better in-sample and out-of-sample results than the naive approach and numerical methods. Facing the poor results of the numerical adjustment methods, a more thorough investigation on how migration probabilities are changed by these methods should be conducted in the future. Besides, the significantly better forecasting results, the one-factor model also provided weights that could be interpreted in terms of influence of the systematic risk factor on the credit-change indicator. Confirming the results of previous studies (Belkin, Forest, and Suchower 1998a; Nickell, Perraudin, and Varotto 2000), we find that the correlation between systematic risk and rating changes for speculative grade issues is significantly higher than for investment grade issuers.

Finally, we illustrated the advantage of using risk-sensitive difference indices (Trück and Rachev 2005a) for measuring the forecast error of our models. The mean forecast error may also be interpreted in terms of risk for a credit portfolio, if information on credit VaR and correlation between VaR and changes in the performance criteria are provided. It is important to note that most of the evaluation criteria, usually suggested in the literature, are not capable of measuring differences in migration matrices in terms of risk.

Overall, the superior results of the conditional approach suggest further research on the issue in terms of measuring business cycle effects on credit migrations and evaluating the forecast errors.

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