

Gauge Invariance and the Goldstone Theorem

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Abstract

This manuscript was originally created for and printed in the “Proceedings of seminar on unified theories of elementary particles” held in Feldafing Germany from July 5 to 16 1965 under the auspices of the Max-Planck-Institute for Physics and Astrophysics in Munich. It details and expands upon the Guralnik, Hagen, and Kibble paper that shows that the Goldstone theorem does not require physical zero mass particles in gauge theories and provides an example through the model which has become the template for the unified electroweak theory and a main component of the Standard Model.

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Introduction

This manuscript was created for and printed in the “Proceedings of seminar on unified theories of elementary particles” held in Feldafing Germany from July 5 to July 16 1965 under the auspices of the Max-Planck-Institute for Physics and Astrophysics in Munich. It is based on the talk that I gave at this conference which was essentially identical to earlier talks at Imperial College (then my home institution) and Edinburgh. It expands and details the arguments given in the Guralnik, Hagen and Kibble paper [13] demonstrating that the Goldstone Theorem does not require a physical massless particle in gauge theories and examines the explicit scalar electrodynamic model which now forms the basis of the unified model of weak and electromagnetic interactions and inspired “Higgs” searches at LEP, the Tevatron and the LHC. This paper shows how the arguments evolved from my earlier PRL paper [10]. As in the Guralnik, Hagen and Kibble paper, this paper keeps close track of the physical degrees of freedom and explicitly shows, in leading order of the model, that a massive vector boson and a scalar boson (whose mass is generated by higher order contributions) describe the entire excitation spectrum. In the radiation gauge solution of this model the Goldstone theorem is not valid and consequently imposes no constraints requiring zero mass. The Lorentz gauge solution, which yields the same physical results, obeys the Goldstone theorem (as it must) by having irrelevant massless gauge excitations. The original work was supported by the U.S. National Science Foundation through a postdoctoral fellowship. Except for this introduction, the acknowledgements and the associated header information, I have attempted to make this article identical in content to the original complete with some potentially confusing notation, typographic errors and (minor) errors in language and physics. I am posting this here because I have been receiving requests for copies since the original proceedings are not readily available.

Original Feldafing Conference Paper

The impressive success of the method of broken symmetries in non-relativistic problems have understandably led to the hope that similar techniques might be the key to at least some of the problems of the relativistic theory. The early calculations [1] [2] demonstrated that the requirement that the vacuum expectation value of a field operator be non-vanishing, could indeed lead to solutions of field theoretic problems not realized by normal perturbative methods while not drastically destroying the normal structure expected of a relativistic field theory. Nevertheless, to date, these methods have not resulted in any particularly gratifying insights into nature except for a possible indication of how the photon can be regarded as a composite particle [3]. The desirable guarantee that the photon has zero mass which naturally comes from this technique, in fact, seems to be reflective of a limitation which prohibits its application to a wide range of problems. Indeed, it was realized at the initial stages that a broken symmetry might always be associated with a zero mass particle. General proofs of this were given by Goldstone, Salam and Weinberg, and Bludman and Klein [4].

To emphasize our point of view we give here a proof which is completely equivalent to the usual proofs, but makes points essential to our discussion more transparent. Assume that we are given some time independent generator Q on fields φ_i such that

$$[Q, \varphi_i(y)] = t_{ij} \varphi_j(y) \quad (1)$$

and

$$Q = \int d^3x j^0(x) . \quad (2)$$

To “break” the symmetry Q the requirement

$$t_{ij} \langle 0 | \varphi_j | 0 \rangle = n_i \neq 0$$

is imposed. That n_i is independent of y combined with energy momentum conservation gives a definite statement of the nature of the excitation spectrum of φ_i . In this case

$$\langle 0 | [j^0(x), \varphi_i(y)] | 0 \rangle = n_i \int \frac{e^{ip(x-y)}}{(2\pi)^3} G(\vec{p}, p^0) d^4p,$$

so the application of (1) and (2) yields

$$\int d^3p \delta^3(\vec{p}) G(\vec{p}, p^0) = \delta(p^0) \quad (3)$$

If it is assumed that the theory is relativistically invariant and that the broken symmetry requirement in no way interferes with the relativistic structure, equation (3) requires that $G(\vec{p}, p^0) = \delta(p^2) p^0 \epsilon(p^0) +$ possible irrelevant terms. Thus we conclude that φ_i excites a zero mass particle. This remarkably simple result is the Goldstone theorem. It is the only known exact statement on the excitation spectrum of a field operator and consequently has been viewed with suspicion since its inception. There is, however, little question that these results are correct when the large number of implicit existence assumptions which have gone into the proof and which are usually valid in other contexts hold here as well. The fact that the Goldstone bosons of one useful broken symmetry theory, the superconducting electron gas, are massive, has motivated considerable interest in how to get around the assumptions of the theorem without too drastically mutilating the underlying field theoretic structure which made its proof possible [5].

Historically, the first attempts were made with the belief that even in non-relativistic theories the assumptions of the theorem are still valid but that "spurious states" enter to avoid the zero mass particle conclusion. The basic observation [6] is that non-relativistic theories may be imbedded in a relativistic theory by the introduction of a time-like vector $\lambda^u = (1, 0, 0, 0)$. A straightforward analysis demonstrates that Fourier transform

$$\langle 0 | [j_u(x), \varphi(y)] | 0 \rangle = (A + B\epsilon(k^0)) k_u \delta(k^2) + C \lambda_u \delta^3(k) \delta(\lambda \cdot k) + [k_u (\lambda \cdot k) - \lambda_u k^2] D(k^2, \lambda \cdot k) + E \lambda_u \delta(\lambda \cdot k) .$$

The term proportional to C is the one on which attention was initially focussed. It is most unusual in that it represents an isolated state that might be interpreted as a transition

between the various degenerate vacua which are formed as a consequence of breaking the symmetry. In fact, though this term can simulate the behavior of the global generator Q , it cannot consistently be interpreted as contributing to the density commutator since $\langle 0|[j^\circ(x)\varphi(y)]|0\rangle\big|_{x^0=y^0} \propto \delta^3(\vec{x}-\vec{y})$ + possible irrelevant terms while

$$\int d^4k e^{ik(x-y)} C \lambda^0 \delta^3(k) \delta(\lambda \cdot k) \big|_{x^0=y^0} = C \big|_{k^0=0} .$$

Consequently, no escape from the Goldstone theorem can ever occur in this manner. This point has been made in rather different language by Lange [7]. The ‘‘spurious’’ contribution proportional to E cannot be disposed of, nor is its interpretation quite so simple. It is the sort of term that one is not likely to obtain when using usual approximation methods, and to my knowledge no model exists where this term appears when the calculation is properly handled. In principle, however, there is no reason to believe that a simple non-relativistic model demonstrating this transition cannot be formulated. Also, such transitions might be important in apparently relativistic theories where Lorentz invariance is broken. The Bjorken theory in its original current non-conserving approximation is an admittedly poor example of where such terms appear in the check of the Goldstone commutator [8].

The third term is the one that is important for the understanding of how the Goldstone theorem can break down. Indeed such a term appears in broken symmetry theories involving vector gauge fields [9]. As written it seems innocent enough, but it must be appreciated that the particle associated with such a term will always be massive and will necessarily involve the breakdown of the global conservation law and hence a negation of the usual assumptions made to prove the Goldstone theorem. Thus, this term will give results entirely different in nature from those derived from massless particles of ‘‘spurions’’. We shall return to this important point later.

It is our object first to demonstrate how the old arguments on the connection between gauge invariance and zero mass can be expressed in an analytic way through the use of the Goldstone theorem [10]. Then by studying the breakdown of this connection, we will have a guide toward the understanding of how the method of broken symmetries need not lead to massless physical particles.

Consider the simplest possible field theory, that of a massless free spinless particle described by the Lagrangian

$$L = \varphi^u \partial_u \varphi + \frac{1}{2} \varphi^u \varphi_u .$$

Note that since there is no mass term, this L possesses the gauge invariance $\varphi \rightarrow \varphi + n, \varphi^u \rightarrow \varphi^u$. It is our intention to display an operator realization of this transformation. This is easily done since from the field equation $\partial_u \varphi^u = 0$, it follows that

$$L_n \equiv \int d^3x n \varphi^0(x)$$

with n constant is independent of time. In fact L_n does not exist unless n is adjusted to fall off rapidly to zero for large spatial coordinates. We shall, however, proceed formally with n constant, as ignoring this subtlety does not get us into any difficulties. Those who object to this procedure may find comfort either by noting that all our results are derived by

considering well defined commutators of L_n or better still be referring to the paper of Streater which treats these problems rigorously [11]. Since the canonical commutation relations have the form

$$i [\varphi^0(x), \varphi(y)]_{x^0=y^0} = \delta^3(\vec{x} - \vec{y}),$$

it follows that

$$i [L_n, \varphi(y)] = n .$$

In particular, introducing the usual set of states $|a'\rangle$ for which $\langle 0|\varphi|0\rangle = n$, it follows that

$$i \langle 0|[L_n, \varphi(y)]|0\rangle = n ,$$

and consequently the Goldstone theorem states that $\varphi(y)$ excites a massless particle. This of course, is a completely trivial and circular observation for this model. Now these conclusions may be applied to make this example look like a broken symmetry theory. To do this, we introduce a new set of states defined by the relation

$$|na'\rangle \equiv e^{-iL_n}|a'\rangle.$$

Although, formally it appears that the states $|na'\rangle$ are unitarily connected with the states $|a'\rangle$ this is not the case, as L_n is properly defined by a limiting process where $n(\vec{x})$ is taken to be constant over an increasingly large spatial volume. As this volume becomes infinite, $|a'\rangle$ and $|na'\rangle$ become members of different inequivalent representations. In this limit all state vectors of one set become orthogonal to all the state vectors in the other set. It is emphasized that no finite product of operators $\varphi(y)$ can induce a transition between members of different sets of inequivalent representations, and consequently in this special type of theory no ‘‘spurions’’ occur as an expression of such transitions.

It follows at once that

$$\begin{aligned} \langle b'n|\varphi|na'\rangle &= \langle b'|\varphi|a'\rangle + i\langle b'|[L_n, \varphi]|a'\rangle . \\ &= \langle b'|\varphi|a'\rangle + n\langle b'|a'\rangle . \end{aligned}$$

In particular $\langle 0n|\varphi|0n\rangle = n$, so a non-vanishing expectation value of a field operator has been realized through this technique explicitly because of the natural presence of zero mass particles. The realization has occurred through the possibility of constructing states differing from the original states through the addition of an infinite number of zero energy, zero mass particles to these states.

To make full contact with the usual formulation of broken symmetry theories, assume that the field φ has two degrees of freedom

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix},$$

then there is a conserved bilinear local current $j^u = i\varphi^u q \varphi$ with the corresponding global charge $Q = \int d^3x j^0(x)$. Since

$$[Q, \varphi(x)] = q \varphi(x),$$

it follows that

$$\langle 0n|[Q, \varphi(x)]|n0\rangle = q n ,$$

and consequently there is a Goldstone theorem in the usual sense involving a bilinear generator. In this simple case, as always, the theorem is nothing more than an expression of the possibility of making constant gauge transformation on a massless field. However, since here the canonical group is the exact carrier of the broken symmetry this statement has a particularly nice realization through the decomposition

$$\langle 0n|[Q, \varphi(x)]|n0\rangle = i \int d^3x \langle 0|[\varphi^0 q n, \varphi]|0\rangle = i \langle 0|[L_{qn}, \varphi]|0\rangle .$$

One point should again be emphasized. The fact that $\langle 0|[L_{qn}, \varphi]|0\rangle$ is non-vanishing is an expression of a fundamental degeneracy of any system with zero mass particles. It is a demonstration that in the system under consideration there are states carrying unit charge relative to the vacuum but of the same energy. On the other hand, though the non-vanishing of the commutator $\langle 0n|[Q, \varphi(x)]|n0\rangle$ is directly due to the realization of such a degeneracy, the states $\langle a'n|$ should not be looked at as being any of these states. It is clear from their definition that $\langle a'n|$ are not eigenstates of the operator Q , but are in fact a superposition of states of all possible charges. They are orthogonal to the states $|a'\rangle$, but form the basis for a physically equivalent description in which the observable current is measured relative to the vacuum and is $j^{u'} = i[\varphi^u q \varphi - \varphi^u q n]$. It is consequently clear that this type of model does not really “break” the symmetry.

The considerations here have been of the most elementary sort, nevertheless they serve as a basis towards the understanding of the Goldstone theorem. The results become much less trivial if we add to the free Lagrangian any interaction whatsoever depending on φ^u and any other fields except φ . In this case all the above statements are still valid.

Now we proceed to the only slightly more complicated problem of free electrodynamics. In the radiation gauge the field equations are

$$F^{uv} = \partial^u A^v - \partial^v A^u$$

and

$$\partial_v F^{uv} = 0$$

with the equal time commutation relations

$$[F^{0k}(x), A^l(y)]_{x^0=y^0} = i[\delta_{kl} - \frac{\partial_k \partial_l}{\nabla^2}] \delta^3(\vec{x} - \vec{y}).$$

From the field equations it follows that

$$L_n = \int d^3x F^{0k} n_k$$

is independent of time. The commutation relations establish that $\frac{1}{i}[L_n, A^k] = \frac{2}{3}n^k$, and consequently just as in the scalar model there is a Goldstone theorem and the possibility of manufacturing a set of states $|na'\rangle \equiv e^{iL_n}|a'\rangle$ for which $\langle a'n|A^k|nb'\rangle = \langle a'|A^k|b'\rangle + \frac{2}{3}n^k$.

In the Lorentz gauge the field equation is

$$\partial_u(\partial^u A^v) = 0 ,$$

while usually the canonical commutation relation is taken as

$$[\partial_0 A^v(x), A^u(y)]_{x^0=y^0} = -ig^{uv}\delta^3(\vec{x} - \vec{y}) . \quad (4)$$

From this time independent generator $L_n = \int d^3x \partial_0 A^u(x) n_u$ is formed which satisfies the relation

$$i[L_n, A^u(y)] = n^u .$$

The states $|na'\rangle$ are formed in the usual manner. The bilinear symmetry which is broken here as in any theory in which the field whose expectation is non-vanishing carries intrinsic spin is Lorentz invariance. In more complicated theories that breaking of this symmetry is extremely dangerous, because of the possibility of the realization of a non-covariant excitation spectrum.

Now consider interacting electrodynamics in the radiation gauge. All is the same as before except that the interaction is inserted by replacing the free equation for F^{uv} by the equation

$$\partial_u F^{uv} = -e_0 j^v .$$

Since this may be rewritten as $\partial_u[F^{uv} + e^0 x^v j^u] = 0$, application of the same reasoning as in the preceding discussion would lead us to claim that

$$L_n = \int d^3x n_k [F^{0k}(x) + e_0 x^k j^0(x)]$$

is independent of time. Since

$$[j^0(x), A^k(y)]_{x^0=y^0} = 0,$$

one would then conclude that

$$i \langle 0|[L_n, A_k(y)]|0\rangle = \frac{2}{3} n_k,$$

independent of y^0 . However, use of the spectral form for $\langle 0|[A^u(x), A^v(y)]|0\rangle$ quickly shows that the above relation cannot be true except when $e_0 = 0$, and in fact the correctly computed right hand side is $\frac{2}{3} n_k$ for $x^0 = y^0$ so that L_n generates equal time constant gauge transformations, but for $x^0 \neq y^0$ it depends on all values of mass in the excitation spectrum of $A_k(x)$. That there is no time independent generator of constant numerical gauge transformations and hence no Goldstone theorem is a simple restatement of the well established fact that there is no dynamical reason why the physical photon should have zero mass [12].

Our argument has failed because in a theory which is not manifestly covariant one is not able to demand that when

$$\frac{\partial}{\partial x^u} \langle 0|[J^u(x), \Phi(y)]|0\rangle = 0$$

then

$$\int d^3x \langle 0|[J^0(x), \Phi(y)]|0\rangle$$

is independent of $(x^0 - y^0)$. This is because causality cannot be invoked to demonstrate that the other surface integrals that arise in the application of Gauss's theorem to the above equation vanish. This statement should not be construed to mean that radiation gauge electrodynamics is acausal. It must be remembered that $A^u(y)$ is an unphysical field so there is no reason to be concerned if acausality appears in the study of some of its commutators.

Interacting Lorentz gauge electrodynamics has the field equations $-\partial^2 A^u = e j^u$ and $\partial_u j^u = 0$ together with the commutation relations given by equation (4). It is easily found that

$$i \langle 0|[A^u(x), A^v(y)]|0\rangle = (g^{uv} - \frac{\partial^u \partial^v}{\partial^2}) [Z_3 \Delta(x-y; 0) + \int_{>0}^{\infty} dk^2 B(k^2) \Delta(x-y, k^2) + \frac{\partial^u \partial^v}{\partial^2} \Delta(x-y; 0)] \quad (5)$$

Since we are dealing with a manifestly covariant theory the quantity

$$L_n = \int d^3x n_u [\partial_0 A^u(x) - e_0 x^u j^0(x)]$$

is independent of time as is indeed verified by direct computation which yields the expected result

$$i \langle 0|[L_n, A^\lambda(y)]|0\rangle = n^\lambda. \quad (6)$$

Consequently there is a Goldstone theorem for interacting Lorentz gauge electrodynamics. There also is a strong point to be made from this result. Note that (6) is true even if $Z_3 = 0$ and there is no physical zero mass particle. If this should be the case the last term of (5) would be entirely responsible for the consistency of (6). This term has occurred only because we have insisted on the somewhat peculiar commutation relations (4) and being purely gauge it does not contribute to any physical amplitudes. Thus as this simple example illustrates, it is possible to have a Goldstone particle which is of no interest whatsoever. In electrodynamics we are fortunate because the high degree of gauge invariance allows us to quantize in the Coulomb gauge which has all unphysical modes removed. In this gauge it was easily found that no Goldstone theorem followed as an immediate consequence of the structure of the field equations. It is perhaps of interest to note that after we are told there is a zero mass physical photon as a direct result of solving the detailed dynamics for small values of the coupling constant we can, of course, construct a Goldstone theorem by applying the spatially integrated "in" or "out" field operator to the states in the usual way. This is an example of the inverse Goldstone theorem and is entirely after the fact. Because of these observations about the radiation gauge it is no surprise that the zero mass Goldstone modes of Lorentz gauge electrodynamics are purely unphysical. Most theories of interest are not so

highly gauge invariant, and admit quantization only in a fully relativistic manner.

Consequently it is necessary to examine the zero mass excitations very carefully to ascertain whether they correspond to true particles which appear in physical amplitudes.

So far we have used the method of broken symmetries in theories whose Lagrangians are invariant under constant additions to some field. Consequently we have been able to construct the “broken symmetry” states in a straight forward manner from the usual states and to understand exactly what is meant by “breaking” a symmetry. In short, no results obtained are not more or less the direct consequences of the normal field equations, and the methods we use while perhaps amusing are an entirely unnecessary sophistication as far as solving the problem at hand. Indeed what we have done is something of a fraud because the parameter of the “broken symmetry” appears in any formulae in an entirely inert manner just because of the gauge invariance. In so far as any physical interpretation of results is concerned the symmetry under consideration has not really broken at all. Now we wish to study theories which are less gauge invariant and for which the parameter of the breaking appears in the Green’s functions in a physically significant manner. Despite the non-trivial nature of these more complicated theories we will be able to use the preceding results as a guide to construct symmetry breaking theories where there is no physical zero mass particle as a result of a Goldstone theorem.

Before proceeding to the more pertinent model it is beneficial as an illustration of the above remarks to study the example given by Goldstone [1] with the Lagrangian

$$L = \varphi^u \partial_u \varphi + \frac{1}{2} \varphi^u \varphi_u + \frac{\mu_0^2}{2} \varphi^2 - \frac{\lambda}{4} \varphi^2 \varphi^2 .$$

For the moment the number of components of the field φ is left unspecified. We wish to solve this theory in the factorizable approximation subject to the condition $\langle 0 | \varphi(x) | 0 \rangle = n$. It is clear from the Lagrangian that the transformation $\varphi \rightarrow \varphi + n$ is in no sense an invariance of the theory. In order for the broken symmetry condition to be consistent with the field equations one finds the spectrum distorting condition $\mu_0^2 = \lambda n^2$. Introducing the Green’s function

$$g(x) = i \langle 0 | (\varphi(x) \varphi(0)) | 0 \rangle - i n n$$

it is easily found that

$$g(p) = \frac{1}{p^2} \left[1 - \frac{n n}{n^2} \right] + \frac{n n}{n^2} \frac{1}{p^2 + 2 \lambda n^2} .$$

Note that in this theory the parameter n appears in an entirely non-trivial manner. If φ has only one component there is no zero mass particle. But if φ has two (or more) components a zero mass particle appears as the expression of the conserved current $j^u = i \varphi^u q \varphi$ which yields the generator $Q = \int d^3 x j^0$ and the Goldstone consistency condition $\langle 0 | [Q, \varphi(y)] | 0 \rangle = q n$.

Now we return to our problem, that of constructing a true broken symmetry without a zero mass Goldstone boson. The connection we have established between gauge-invariance and the Goldstone theorem suggests that we should try to find a failure of the gauge invariance yields zero mass argument, and from this construct a broken symmetry theory. This is how the problem was initially solved, but we now take the more direct route of starting with the

Lagrangian

$$L = -\frac{1}{2}F^{uv}(\partial_u A_v - \partial_v A_u) + \frac{1}{4}F^{uv}F_{uv} + \phi^u \partial_u \varphi + \frac{1}{2}\varphi^u \varphi_u + ie_0(\varphi^u q \varphi)A_u.$$

Note that the current $j^u = ie_0 \varphi^u q \varphi$ satisfies the differential conservation law $\partial_u j^u = 0$. Any approximation made on this theory will be required to respect this conservation. We now impose the broken symmetry condition $ie_0 q \langle 0 | \varphi | 0 \rangle = n \equiv \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ and ask for the solution of the above Lagrangian in the lowest factorizable approximation [13]. In this approximation the complete self consistent Green's function calculation is fully simulated by replacing the interaction term by $(\varphi^u n)A_u$ and treating φ as though it had vanishing vacuum expectation. We will thus for this presentation avoid any complications by use of this device. The resulting field equations are

$$F^{uv} = \partial^v A^u - \partial^u A^v$$

$$\partial_v F^{uv} = \varphi^u n$$

$$\varphi^u = -\partial^u \varphi - n A^u$$

$$\partial_u \varphi^u = 0.$$

Note that the current $\varphi^u n$ is conserved. If φ had only one component these would essentially be the equation of one of the models [14] demonstrating that gauge invariance does not always require zero mass. Solving these equations in the radiation gauge and taking (with no loss in generality) $n_2 = 0$ it follows that

$$(-\partial^2 + n_1^2)\varphi_1 = 0$$

$$-\partial^2 \varphi_2 = 0$$

$$(-\partial^2 + n_1^2)A_k^T = 0$$

$$\text{with } \nabla \cdot A_k^T = 0.$$

We thus see that the dimensional broken symmetry parameter n plays the role of a mass, serving to combine the two components of A_k^T and the one component of φ_1 , into the three components of a massive vector meson. The field φ_2 has been inert under this process and while massless, is completely decoupled from the massive excitations. It is found rather directly that

$$\langle 0 | [j^u(x), \varphi(y)] | 0 \rangle \equiv n_1 \langle 0 | [\varphi^u(x), \varphi(y)] | 0 \rangle \quad (7)$$

so that

$$\langle 0|[j^u(x), \varphi_2]|0\rangle = 0 \quad (8)$$

and

$$\langle 0|[j^u(x), \varphi_1(y)]|0\rangle = n_1[-\partial^u + \frac{n^2[\lambda^u(\lambda \cdot \partial) + \partial^u]}{\partial^2 + (\lambda \cdot \partial)^2} \Delta(x - y; n^2)]. \quad (9)$$

Here we have introduced $\lambda^u = (1, 0, 0, 0)$.

Note these results are consistent with differential current conservation. However, defining $Q(x^0) = \int d^3x j^0(x)$, we find that

$$\langle 0|[Q(x^0), \varphi_1(y)]|0\rangle = -in_1 \cos(x^0 - y^0) n_1 .$$

Consequently, because of the acausal nature of the unphysical commutator there is no globally conserved charge and thus there is no Goldstone theorem to invoke to argue that breaking the symmetries requires that φ_1 excites massless particles. It should be appreciated that these results not only do not contradict, but are actually required by the field equations which show that $(\partial_0^2 + n_1^2)Q = 0$. The fact that we are allowed the possibility of quantizing this theory in the radiation gauge which is not manifestly covariant allows us to strip the problem of unphysical degrees of freedom and to see quickly to the heart of the matter. However, since in most theories one is not allowed this possibility it is good to point out what happens when these equations are examined in the Lorentz gauge. In this manifestly causal situation the global charge Q is of necessity independent of time. We find that just as in the case of interacting Lorentz gauge electrodynamics a pure gauge zero mass part is added to the photon propagator and also becomes associated with the operator φ . It is this purely unphysical mode which guarantees the consistency of the Goldstone theorem.

We have thus found a “true” broken symmetry theory which in its relativistic form satisfies the Goldstone theorem through the existence of unphysical modes, and in its equivalent radiation gauge treatment avoids the restrictions of the Goldstone theorem because no globally conserved symmetry operator exists.

The analysis of the behavior of this model in the radiation gauge reveals why so many broken symmetry non-relativistic models fail to have massless Goldstone bosons. It is simply because when these models have long range forces in their Hamiltonian (such as $\frac{1}{r}$ Coulomb potentials) one must carefully check to see whether the appropriate global generator is time independent. In most cases as an expression of the existence of these long range forces some of the “charge” may oscillate in and out of the boundaries of any box no matter how large and thereby negate any zero mass Goldstone arguments. The previously troublesome problem of the superconducting electron gas at zero temperature with Coulomb interaction is explained by this mechanism. [7, 13] In this case the plasma oscillations have the property that their energy remains non-vanishing as wavelength goes to infinity. Consequently the Goldstone theorem is not applicable to this problem. For details see the paper by Lange. There is a particularly nice point which our relativistic model illustrates quite well. In the superconducting model as soon as the Coulomb interaction is inserted in the Hamiltonian is screened ever so slightly the massless Goldstone bosons appears. This corresponds to the

fact that the surface integrals at infinity converge and consequently that the global charge exists. Note, that this phenomenon would be particularly hard to explain if terms of the form $n^u \delta^4(k)$ carried the explanation of why zero mass particles did not initially appear in this model. Now the analogue of this transition occurs in the model considered above when the smallest amount of bare mass is associated with the field A^u . In that case a physical zero mass particle which serves to satisfy the Goldstone criteria appears.

We conclude with a pessimistic note by pointing out that though the above remarks lead to a much better understanding of the significance of the Goldstone theorem, that it seems to me that we have a long way to go before we find a relativistic model based on the method of broken symmetries which has any real application to physical problems. If we do succeed in finding such a model it is my feeling that it probably will not have physical massless Goldstone bosons since the “getting something for nothing” aspect of the Goldstone theorem when it applies to the physical mass spectrum very seriously limits the way the broken symmetry can “interact” with the “normal” dynamics.

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