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Monetary Policy with Interest on Reserves

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In response to the emerging financial crisis of 2008, the Federal Reserve decided to increase the liquidity of the banking system. For this purpose, the Federal Reserve introduced or expanded a number of programs that made it easier for banks to borrow from it. For example, commercial banks were able to obtain additional loans through the Term Auction Facility, which the banks would then hold in their reserve accounts with the Federal Reserve. As a result of the combined financial market interventions, the balance sheet of the Federal Reserve increased from about \$800 billion in September 2008 to more than \$2 trillion in December 2008. Over the same time period, the reserve accounts of commercial banks with the Federal Reserve increased from about \$100 billion to \$800 billion. In late 2008 the Federal Reserve also announced a program to purchase mortgage-backed securities (MBS) and debt issued by government-sponsored agencies. Since then, outright purchases of agency MBS and agency debt have essentially replaced short-term borrowing by commercial banks on the asset side of the Federal Reserve's balance sheet, and the volume of outstanding reserves increased again to about \$1.1 trillion by the end of 2009. Given the magnitude of outstanding reserves, there is some concern these reserves might limit policy options once the Federal Reserve decides to pursue a more restrictive monetary policy. Yet, another change in the available policy instruments might lessen this concern: Starting in October 2008, the Federal Reserve began to pay interest on the reserve accounts that banks hold with the Federal Reserve System.

How should one think about monetary policy when reserve accounts earn interest? To study this issue, I introduce a stylized banking sector into a simple baseline model of money that is at the core of much research in monetary

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economics. In this framework I address an admittedly rather narrow theoretical question, but this question is fundamental to any theory of monetary policy. Namely, does the payment of interest on reserves affect issues of price level determinacy? An indeterminate price level might be undesirable since it can give rise to price level fluctuations driven by self-fulfilling expectations. In this context it is shown that the amount of outstanding reserves has only limited implications for the conduct of monetary policy.

Price level determinacy is studied in a theoretical framework that specifies not only monetary policy, but also fiscal policy, e.g., Leeper (1991) or Sims (1994). Monetary policy is described as setting a short-term nominal interest rate in response to inflation, and fiscal policy is described as setting the primary surplus in response to outstanding government debt. For the baseline monetary model without a banking sector, one obtains price level determinacy if monetary policy is active, that is, it responds strongly to the inflation rate, and fiscal policy is passive, that is, it responds strongly to government debt.[1] Price level determinacy is also obtained when monetary policy is passive and fiscal policy is active. For the modified model with a banking sector, I find that this characterization of price level determinacy is not materially affected, whether or not interest is paid on reserves. I obtain a determinate price level when monetary policy is sufficiently active and fiscal policy is sufficiently passive, or vice versa. Furthermore, the magnitude of outstanding reserves may not matter at all, and if it does matter the impact of reserves is small.

Earlier theoretical work on paying interest on reserves was concerned that this policy would lead to price level indeterminacy. Sargent and Wallace (1985) argue that, depending on how interest on reserves is financed, an equilibrium might not exist or the price level might be indeterminate.[2] In terms of the above characterization of monetary and fiscal policy, these results obtain because the assumed financing schemes for interest on reserves make monetary and fiscal policy both passive or both active. My results are in line with the recent work of Sims (2009), who studies the monetary and fiscal policy coordination problem when interest is paid on money in a baseline monetary model. The results are also related to Woodford's discussion (2000) of monetary policy as an interest rate policy in environments where the role of money is diminished over time.

The framework of this article is not suited to address the question of whether interest on reserves allows a separation of monetary policy from credit policy as suggested by Goodfriend (2002) and Keister, Martin, and McAndrews (2008). In this article I use a reduced form representation of liquidity preferences by households to model distinct household demand

[1] The terminology follows Leeper (1991).

[2] Smith (1991) raises similar concerns on price level determinacy in an extended version of the environment studied by Sargent and Wallace (1985).

functions for cash, bank demand deposits, and government bonds, but the model of the financial system's attitude toward the liquidity of assets in the financial system is even more rudimentary. First, I do not allow for credit risk; and second, the banks' attitudes toward liquidity risk are captured by one exogenous parameter, the desired ratio of liquid assets to deposits. The fact that the volume of reserves is of only limited relevance for price level determinacy therefore does not say anything about the ability of reserves to enhance the liquidity of the financial system.

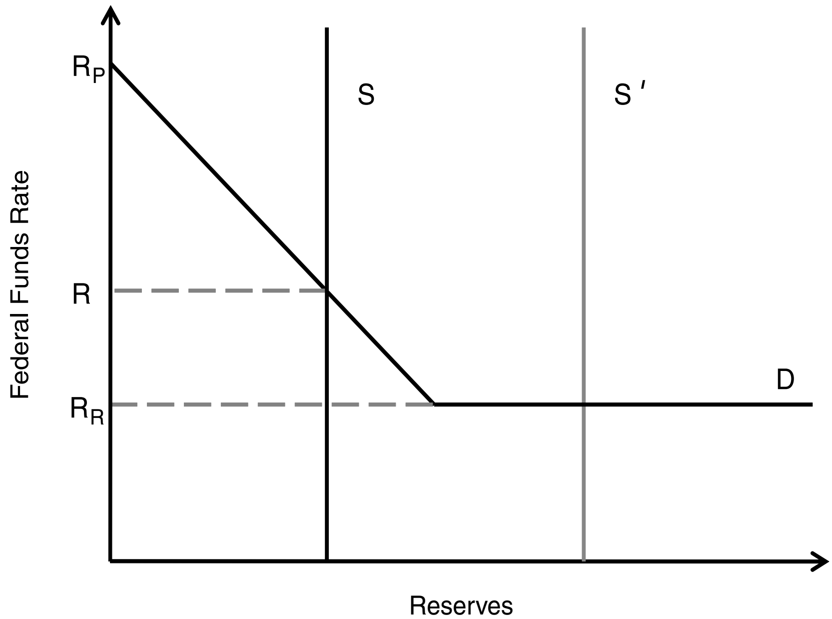
The analysis of the conduct of monetary policy when interest is paid on reserves is based on a stylized model of the economy. Before proceeding with this analysis I will review the mechanics of the Federal Reserve's interest rate policy in the next section. This section also provides an opportunity to describe how the interventions of the Federal Reserve in financial markets in 2008 affected its ability to conduct interest rate policy. Section 2 then reviews Leeper's joint analysis (1991) of monetary and fiscal policy in a baseline monetary model, and Section 3 adds a stylized banking sector to the baseline monetary model. The banking model of this section introduces the payment of interest on reserves into a simplified version of the environment studied by Canzoneri et al. (2008). Section 4 concludes.

1. THE MECHANICS OF INTEREST RATE POLICY

Most central banks implement monetary policy through an interest rate policy. That is, they target a short-term interest rate and adjust their target in response to changes in economic conditions. Federal Reserve monetary policy appears to be well-approximated by a policy rule that increases the targeted interest rate more than one-for-one in response to an increase of the inflation rate and decreases the targeted interest rate in response to declines in economic activity as measured by a declining gross domestic product growth rate or an increasing unemployment rate. This kind of behavior has become known as the Taylor rule. The short-term interest rate that the Federal Reserve targets is the federal funds rate—that is, the interest rate that U.S. banks charge each other for overnight loans. This section provides a short review of the mechanics of how the Federal Reserve influences the federal funds rate, and how paying interest on reserves affects its ability to target this rate. The review takes a very stylized view of the federal funds market, as in Ennis and Weinberg (2007). For a more detailed description of the process see Ennis and Keister (2008).

Commercial banks are required to hold particular assets (reserves) against their outstanding liabilities. How many reserves a bank has to hold depends on the types and amounts of its outstanding liabilities. Assets that qualify as reserves are vault cash and accounts with the central bank. Banks hold accounts with the central bank not only to satisfy reserve requirements, but also to facilitate intraday transactions. Private agents engage in transactions and

Figure 1 The Market for Reserves



use their bank accounts to settle payments associated with these transactions. Since not everybody is using the same bank, these payment settlements result in corresponding payment settlements between banks during a business day. Banks use their accounts with the central bank to implement these settlements. A payments transfer from one bank to another can be settled through a debit (credit) to the paying (receiving) bank's account with the central bank. Total inflows and outflows to a bank's account with the central bank during a day need not balance, and at the end of the day a bank's account may have increased or decreased. Furthermore, there is some randomness to settlement transactions and the bank is uncertain as to its end-of-day balance with the central bank.

The uncertainty about payment flows creates a problem for banks since they have to hold a certain balance with the central bank at the end of the day in order to satisfy their reserve requirement. Suppose that at the beginning of the day a bank has some amount of money and has to decide how much to allocate to its reserve account and how much to borrow/lend overnight with other banks at the federal funds rate. If the bank does not allocate enough to its reserve account and at the end of the day its balance falls short of its reserve requirement, it can borrow from the central bank at a penalty rate,

R_P.[3] Alternatively, if at the end of the day the bank's reserve account exceeds its reserve requirement, then the bank foregoes some interest income if the interest rate paid on reserve accounts, R_R, is lower than the federal funds rate.

The optimal response of banks to this settlement uncertainty creates a precautionary demand for reserves, D, that depends on the federal funds rate (Figure 1). The federal funds rate cannot exceed the penalty rate since banks can always borrow at the penalty rate. If the federal funds rate is below the penalty rate but above the interest rate paid on reserves, then the foregone interest income represents an opportunity cost to holding reserve balances. This opportunity cost, however, is declining in the federal funds rate and banks are willing to hold more reserves at lower federal funds rates. Finally, if the federal funds rate is equal to the interest on reserves, then there is no opportunity cost to holding reserves and the demand for reserves becomes infinitely elastic. The equilibrium federal funds rate is bounded by the penalty rate and the interest rate on reserves, and, given the demand for reserves, it is determined by the supply of reserves, S.

In the short run the Federal Reserve controls the federal funds rate through actions that affect the supply of reserves. The particular operating procedure for the Federal Reserve has been that the market desk at the New York Federal Reserve Bank forecasts the daily demand for reserves and then injects or withdraws reserves in order to equalize the predicted federal funds rate with the federal funds rate target set by the FOMC. Except for unusual events, the "effective" federal funds rate during the day—that is, the rate at which intrabank loans occur—is usually very close to the federal funds target rate (Figure 2a).[4] At times, when the Federal Reserve injects large amounts of liquidity for reasons other than interest rate policy, this is no longer true. For example, in response to the events of September 11, 2001, the Federal Reserve wanted to ensure the stability of the financial system and injected large amounts of reserves. This action resulted in an effective federal funds rate that was substantially below the target rate (Figure 2b). At the time, this divergence between perceived liquidity needs and interest rate policy was not considered to be a problem since the liquidity provision was viewed as temporary and to be reversed in a short period of time.

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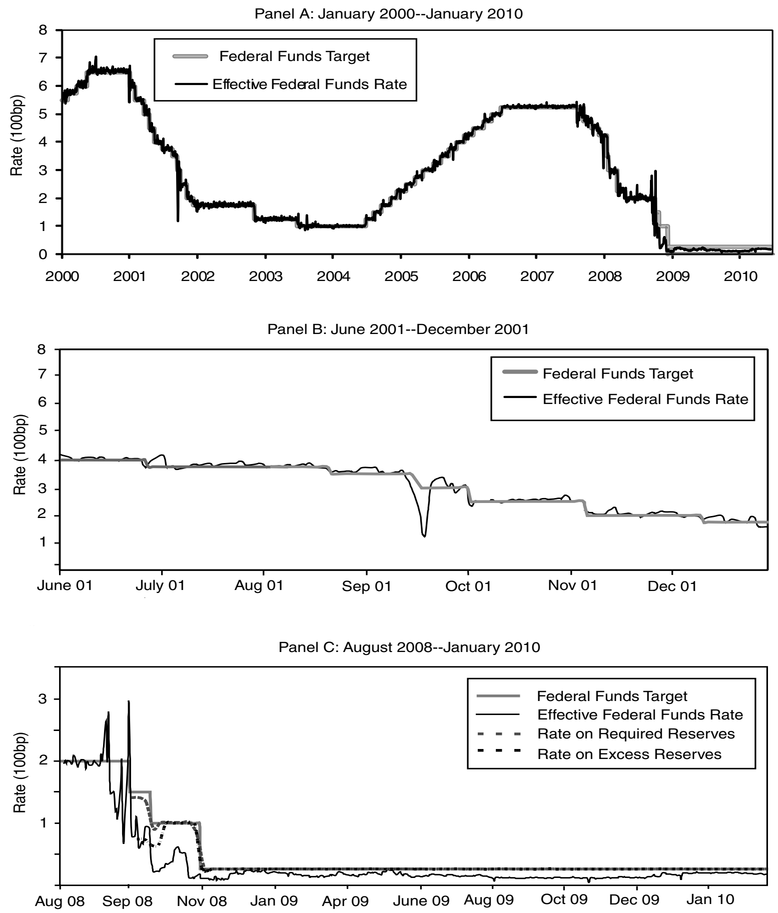
[3] In the United States, banks can borrow from the Federal Reserve against pre-approved collateral at the discount window. The discount rate is set higher than the federal funds target rate, usually 100 basis points (bp). As part of the response to the financial crisis, the Federal Reserve kept the discount rate at 25bp above the target federal funds rate from April 2008 until February 2010. A bank's effective borrowing rate is presumably higher than the discount rate since a bank's borrowing from the discount window is seen as a negative signal on the bank's balance sheet condition.

[4] Interbank lending proceeds through bilateral arrangements and, during the day, the negotiated lending rates can fluctuate substantially. The effective federal funds rate is a value-weighted average of the different loan rates.

Figure 2 Target and Effective Federal Funds Rate



Notes: The data are described in the Appendix.

After the September 2008 bankruptcy of Lehman Brothers, the Federal Reserve increased liquidity substantially in response to the widening financial crises. This was accomplished through the expansion of existing programs, such as the Term Auction Facility and swap lines to foreign central banks, and the creation of new facilities, such as the Commercial Paper Funding Facility. As a result, the Federal Reserve's balance sheet more than doubled over three months and the supply of reserves increased almost tenfold. Even if banks' demand for liquid assets increased at the time, the increase in the supply of reserves was large enough to drive the effective federal funds rate significantly below the stated federal funds target (Figure 2c).

Unlike the events of September 11, 2001, the divergence in this case between effective and target federal funds rates created a problem for the conduct of interest rate policy since the increased liquidity provision was not viewed as a short-lived measure. To deal with this problem, the Federal Reserve in October 2008 started paying interest on reserves.[5] The Federal Reserve Board initially set the interest rate on reserves below the target federal funds rate, but by early November 2008, after several modifications, the interest rate on reserves was essentially the target federal funds rate.[6]

The rationale for this policy is based on the discussion above that suggests that paying interest on reserves puts a floor to the federal funds rate (Figure 1). Thus, even if the Federal Reserve increases the supply of reserves to a point where the demand for reserves becomes infinitely elastic, e.g., S' in Figure 1, the federal funds rate should not fall below the rate paid on reserves. This suggests that with interest on reserves the Federal Reserve can separate the provision of liquidity from its interest rate policy, e.g., Goodfriend (2002). Furthermore, once the Federal Reserve pays interest on reserves, it has the choice between two policy instruments: It can continue to target a market interest rate, such as the federal funds rate, above the interest paid on reserves; or it can increase the supply of reserves sufficiently and bring the federal funds rate down to the interest paid on reserves and then adjust the interest rate it pays, e.g., Lacker (2006). The first approach targets a lending rate for banks that still contains some counterparty risk, while the second approach sets the risk-free lending rate for banks.

The actual experience with interest on reserves does not completely support this argument. Since November 2008, the effective federal funds rate has been consistently below the interest rate paid on reserves. In fact, starting in December 2008, the FOMC decided to announce a target range for the federal funds rate between 0 and 25bps. This continues to be the policy as of the writing of this article. On the positive side, since February 2008, the effective federal funds rate has traded closer to the interest rate paid on

[5] In 2006 Congress authorized the Federal Reserve to pay interest on reserves starting in 2011. At the time, the main motivation for paying interest on reserves was to eliminate the "tax distortion" implied by the absence of interest payments on reserves. For example, banks would engage in activities whose sole purpose was to minimize their holdings of "reservable" accounts.

[6] On October 6, 2008, the Federal Reserve Board announced that it would pay interest on the depositary institutions' reserve account at 10bp (75bp) below the federal funds rate target for required (excess) reserves. On October 22, the Board changed the rate paid on excess reserve balances to the lowest Federal Open Market Committee (FOMC) target rate in effect during the reserve maintenance period less 35bp. Finally, on November 5, 2008, the rate on required reserves was set equal to the average target federal funds rate over the reserve maintenance period, and the rate on excess balances was set equal to the lowest FOMC target rate in effect during the reserve maintenance period.

reserves. Various reasons have been advanced for this divergence between the effective federal funds rate and the interest rate on reserves. For example, in late 2008 participants in the federal funds market may have been preoccupied with events other than trying to exploit all profit opportunities in the market for overnight credit. More recently it has been argued that the low effective federal funds rate originates from particular lenders in the federal funds market—the government-sponsored enterprises (GSEs) Fannie May and Freddy Mac—who do not have interest-bearing reserve accounts with the Federal Reserve (for example, Bernanke [2009] or Bech and Klee [2010]). Arbitrage competition of depository institutions that can borrow from the GSEs and deposit the proceeds in their interest-bearing reserve accounts should eliminate any spreads between the effective rate and the reserve rate. This competition appears, however, to be limited since the GSEs apparently only engage in lending activities with a limited number of banks.

For the analysis of an interest rate policy when reserves are paying interest, I will abstract from the issues just discussed and assume that the interest rate paid on reserves is the market interest rate. First, for monetary policy I am interested in the opportunity cost to banks, which is the rate on reserves. For this analysis it is irrelevant that nonbank institutions drive the effective rate below the rate on reserves; and even if arbitrage by depositary institutions does not completely eliminate the spread between the rate on reserves and the effective rate, it will at least bound that spread. Second, for the types of aggregate models used in monetary policy analysis, there is no meaningful concept of counterparty risk. Thus, there is no risk premium that distinguishes the interbank lending rate from the riskless rate paid by reserves. Third, these models are not specified in terms of overnight interest rates, but interest rates on short-term government debt. Given that the choices for the policy rates tend to be highly persistent over short periods, this seems like a reasonable approximation. Figure 3 displays the effective federal funds rate and several other short-term interest rates from 1980 to present.[7] As is apparent from Figure 3, most of the time the different short-term interest rates track the federal funds rate quite closely.

In what follows I will study an interest rate policy that pays interest on all reserves at the market interest rate. In particular, I will study the implications of interest on reserves for price level determinacy, and to what extent the amount of outstanding reserves matters. Before proceeding to the model with interest on reserves I first outline the framework of analysis for the case without interest on reserves.

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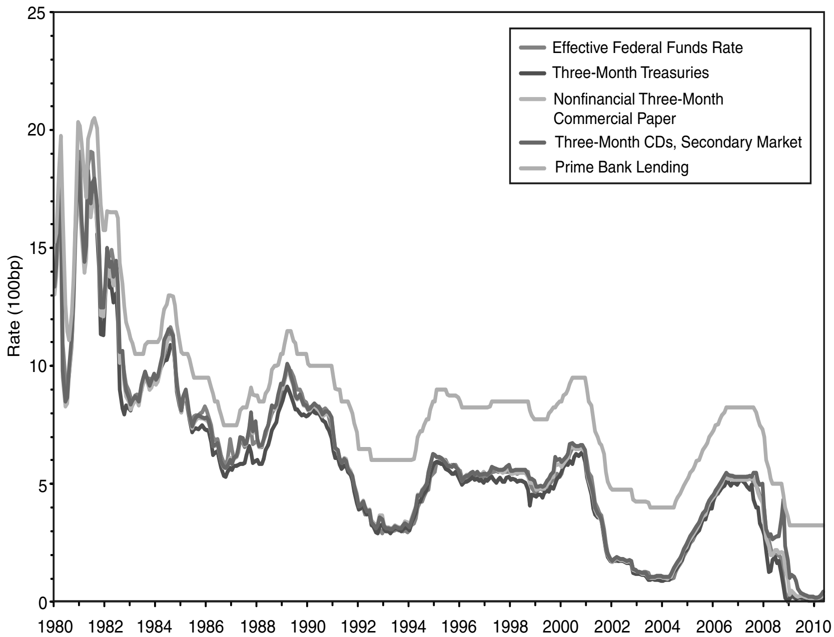
[7] All data are described in detail in the Appendix.

Figure 3 Selected Short-Term Interest Rates



Notes: The data are described in the Appendix.

2. A SIMPLE FRAMEWORK FOR THE ANALYSIS OF MONETARY AND FISCAL POLICY

The following model of an endowment economy has been used extensively in the study of monetary policy. There is one consumption good, c_t, and the consumption good is in exogenous supply. There are two nominal assets issued by the government: fiat money, M_t, and bonds, B_t. The price of the consumption good in terms of fiat money is P_t, and since the consumption good is the only good, P_t is also the price level. Inflation is the price level's rate of change from one period to the next, $\pi_{t+1} = P_{t+1}/P_t$. Bonds pay interest at the gross rate $R_{b,t}$, but fiat money does not. I define real balances and real bonds in units of the consumption good as $m_t = M_t/P_t$ and $b_t = B_t/P_t$.

Households can use both, money and bonds, to save, but holding money also provides some transactions services when households purchase consumption goods. If it was not for the transactions services, households would not want to hold money when bonds pay a positive interest rate since money does not pay any interest. The demand for real balances, equation (1), depends negatively on the opportunity cost of holding money, i.e., the foregone interest income, and positively on the real transactions volume, c_t. The demand for bonds is determined by the Euler equation (2), which equates households' willingness to exchange consumption today for consumption tomorrow with the rate at which households can do that using bonds. The latter is the real rate of return on bonds—that is, how much of the consumption good you obtain tomorrow if you invest one unit of the consumption good today in a nominal bond. Equations (1) and (2) can be derived from simple dynamic representative agent economies that explicitly specify the preferences of households and their budget constraints, e.g., Leeper (1991) or Sims (1994):

$$
\begin{aligned}
m_t &= \mathcal{M}\left(R_{b,t+1}\right) c_t, & (1)\\
1 &= \beta \frac{c_t}{c_{t+1}} \frac{R_{b,t+1}}{\pi_{t+1}}, & (2)\\
v_t &= \frac{R_{b,t} v_{t-1} - \left(R_{b,t} - 1\right) m_{t-1}}{\pi_t} - \tau_t, & (3)\\
v_t &= b_t + m_t. & (4)
\end{aligned}
$$

Equation (3) represents the government's budget constraint. On the left-hand side is the new real debt issued to make interest payments and retire the outstanding debt on the right-hand side. Since debt is nominal, inflation reduces the real amount of debt to be repaid. Furthermore, the government does not pay interest on fiat money. Finally, if the government collects lump sum taxes, τ_t, then less new debt needs to be issued.[8] Equation (4) defines total real government debt as the sum of interest-paying real bonds and non-interest-paying real balances.

To close the model I assume that there is an exogenous endowment of the consumption good such that one can take the time path for consumption as given. I also assume that monetary policy chooses the nominal interest rate in response to the inflation rate, and fiscal policy chooses taxes in response to outstanding real bonds,

$$R_{b,t+1} = f\left(\pi_t\right) \text{ and } \tau_t = g\left(b_t\right). \qquad (5)$$

I characterize the equilibrium time paths for inflation, the interest rate, real balances, real bonds, real debt, and lump sum taxes, $x_t = \left(\pi_t, R_{b,t}, m_t, b_t, v_t, \tau_t\right)$. An equilibrium is then a bounded time path for the variables $\{x_t\}$ that solves the dynamic system defined by equations (1)–(5).[9]

[8] A negative lump sum tax represents a transfer payment to the household. We can interpret lump sum taxes as the government's primary surplus, that is, lump sum tax revenues minus spending net of interest payments.

[9] The equilibrium time paths for real balances and debt have to remain bounded, since they represent solutions to a dynamic optimization problem. Technically, real balances and debt have to satisfy transversality conditions, which state that the limiting value of the discounted future

Monetary policy is said to be active (passive) if the nominal interest rate responds strongly (weakly) to an increase of the inflation rate. Fiscal policy is said to be active (passive) if lump sum taxes respond weakly (strongly) to an increase of real bonds. For a local approximation of the difference equation system, Leeper (1991) shows that for positive interest rates there exists a unique equilibrium if monetary policy is active and fiscal policy is passive, or conversely if monetary policy is passive and fiscal policy is active.[10] The existence of a unique equilibrium in terms of the inflation rate and real balances implies price level determinacy. If both policies are passive then the equilibrium is indeterminate, and if both policies are active an equilibrium will not exist.[11] Sims (1994) shows that these results hold globally in Leeper's model (1991), and not only for local approximations.

The point of this analysis is that price level determinacy is jointly determined by monetary and fiscal policy. To illustrate this point, Figure 4, Panel A1 displays the different regions that characterize equilibrium in terms of the responsiveness of monetary and fiscal policy to the inflation rate and real debt for a standard parameterization of the model.[12] The horizontal axis displays the elasticity of lump sum taxes with respect to real debt, γ, and the vertical axis displays the elasticity of the nominal interest rate with respect to the inflation rate, α. The northeast and southwest regions represent parameter combinations for which there exist unique equilibria. The southeast region represents parameter values in which a continuum of equilibria exists, and the northwest region represents parameter values in which no equilibrium exists.

The intuition for this decomposition of the policy parameter space is fairly straightforward. Substituting the interest rate policy rule (5) into the Euler equation (2) shows that the difference equation describing the dynamics of inflation is independent of fiscal policy. If monetary policy is active, i.e., it responds strongly to past inflation, then this difference equation defines a unique bounded solution for inflation. Furthermore, if fiscal policy is passive,

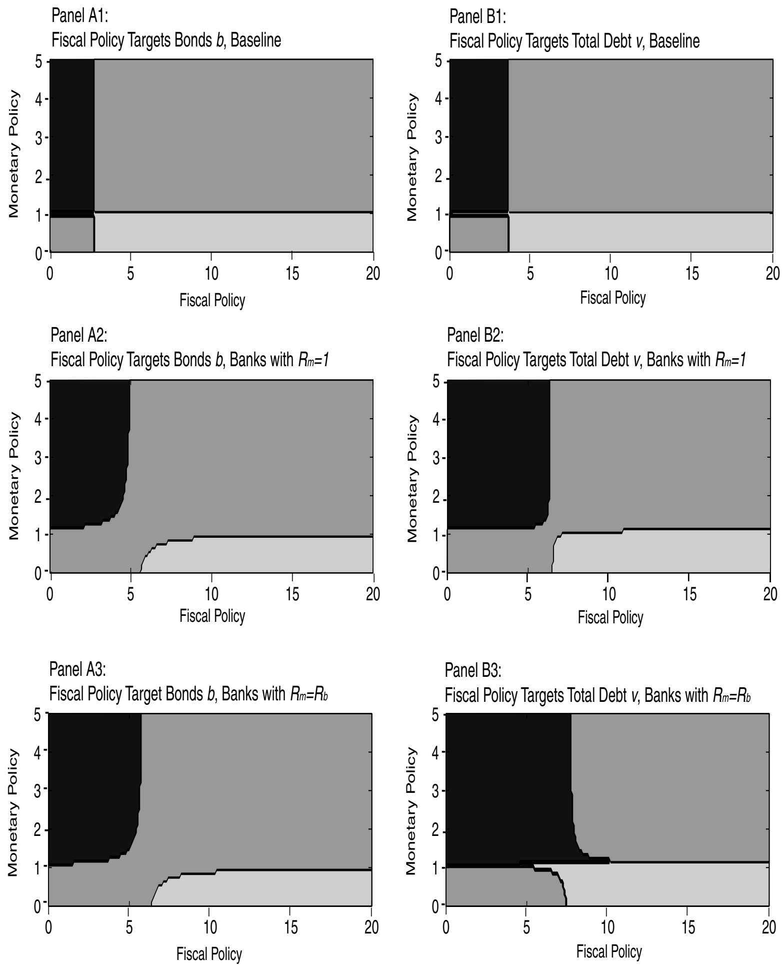
marginal utility of real balances and debt has to be zero. Thus, real balances and debt cannot grow too fast relative to the time discount factor.

[10] For a constant consumption path, $c_t = c$, and given policy targets for inflation and the debt-consumption ratio, equations (1)–(5) define a unique time-invariant solution for the endogenous variables, $x_t = x$, the steady state. I define a local approximation to the equilibrium in terms of small deviations from the steady state, which transforms the dynamic system of equations into a linear difference equation system. For a description of conditions for the existence and uniqueness of a bounded solution to linear difference equation system see, e.g., Sims (2000).

[11] Indeterminacy or nonexistence of an equilibrium raises an issue as to how useful the proposed theory is for the analysis of monetary policy. After all, we are trying to explain a particular outcome for the economy. Indeterminacy can be resolved by refining the equilibrium concept. For example, we might assume that decisions are coordinated on an extraneous random variable that has no relevance for the feasibility of outcomes, a sunspot. This gives rise to fluctuations as a result of self-fullfilling expectations. If no equilibrium exists for certain combinations of monetary and fiscal policy then we might conclude that some policy rules are simply not feasible in the long run (Sargent and Wallace [1981]).

[12] Figure 4 is based on a parameterization of the economy described in Section 3.

Figure 4 Price Level Determinacy



i.e., lump sum taxes respond strongly to government debt, then iteration on the transition equation for government debt defined by the government budget constraint (3) defines a unique bounded path for government debt. Conversely, if fiscal policy is active, i.e., lump sum taxes respond weakly to debt, then the unique bounded solution for debt from the government budget constraint defines debt as the discounted present value of future lump sum taxes and seigniorage revenue from money creation. This in turn defines a time path for the price level and thus the inflation rate. The implied time path for inflation need not be the same as the unique time path for inflation implied by an active monetary policy. Thus, active monetary and fiscal policies are inconsistent with the existence of an equilibrium. But if monetary policy is passive, then the difference equation describing the dynamics of inflation is consistent with a continuum of bounded solutions for inflation, in particular the inflation rate implied by the government budget constraint. This case is therefore also known as the fiscal theory of the price level. Finally, if monetary and fiscal policy are both passive, then there exists a continuum of bounded solutions to the system of difference equations, that is, the equilibrium is indeterminate.

Since for positive interest rates there is a uniquely defined demand for real balances, one can think of the interest rate as being supported by open market operations that supply the amount of money that is demanded at the given interest rate, equation (1). If the demand for real balances is characterized by a "liquidity trap"—that is, the demand is flat at a zero interest rate—then open market operations do not affect the equilibrium outcome.

3. INTEREST ON RESERVES AND THE CONDUCT OF MONETARY POLICY

I now describe a simple endowment economy with a banking sector that generalizes the baseline model described in the previous section. In this model banks are required to hold reserves, and one can study if and how the conduct of monetary policy needs to be changed once market interest rates are paid on reserves. I will limit attention to the question of how the payment of interest on reserves affects price level determinacy, that is, existence and uniqueness of an equilibrium.

An Economy with a Banking Sector

Consider a representative agent with preferences over a cash good, c, a credit good, k, real balances, m_h, real demand deposits, d, and real government bonds, b_h. Including these financial assets in preferences introduces a wedge into the asset pricing equations because the assets pay a liquidity premium. There is also a generic asset, a, that does not provide any liquidity services. The demand deposits are offered by a competitive banking sector that uses reserves and government bonds to service the demand deposits. The banking sector also makes loans, l, to the representative agent that are used to finance purchases of the credit good. Fiscal policy affects the evolution of government debt. The environment is a simplified version of Canzoneri et al. (2008).

Household Demand for Assets

The representative agent's preferences are

$$\sum_{t=0} \beta^t \left\{\ln c_t + \gamma_k \ln k_t + \gamma_m \ln m_{h,t} + \gamma_d \ln d_t + \gamma_b \ln b_{h,t}\right\} \qquad (6)$$

and the budget constraint is

$$\begin{aligned} & c_t + k_t + m_{ht} + d_t + b_{ht} + a_t - l_t + \tau_t \\ \leq \quad & y_t + \left[m_{h,t-1} + d_{h,t-1}R_{dt} + b_{h,t-1}R_{bt} + a_{t-1}R_t - l_{t-1}R_{l,t}\right]/\pi_t, \quad (7) \end{aligned}$$

where the nominal interest rate for asset $j = m, d, b$, and l is R_j, the nominal interest rate on the generic asset is R, exogenous income is y, and lump sum taxes are τ. Real balances, demand deposits, and government bonds are assets that provide liquidity services in addition to being a store of value. The liquidity services are represented as direct contributions to a household's utility. The generic asset does not provide any liquidity services and is not included in the household's utility function. By assumption the household has to take out a loan to purchase the credit good

$$k_t \leq l_t. \quad (8)$$

The optimal choices of the household imply the following asset demand equations:

$$\begin{aligned} m_{ht} &= \gamma_m \frac{R_{t+1}}{R_{t+1} - 1} c_t, & (9) \\ d_t &= \gamma_d \frac{R_{t+1}}{R_{t+1} - R_{d,t+1}} c_t, & (10) \\ b_{ht} &= \gamma_b \frac{R_{t+1}}{R_{t+1} - R_{b,t+1}} c_t, & (11) \\ l_t &= \gamma_k \frac{R_{t+1}}{R_{t+1} - R_{l,t+1}} c_t. & (12) \end{aligned}$$

Note that the household's demand for real balances is well-defined even at a zero nominal bond rate. The household's demand for real balances depends on the interest rate of the generic asset and not the bond rate. Furthermore, since bonds provide liquidity services, the bond rate will always be below the generic asset rate. Thus, even if the bond rate is zero the household demand for real balances is uniquely defined. There is no liquidity trap for household demand of real balances.

Intertemporal optimization with respect to the generic financial asset implies the Euler equation

$$1 = \left[\beta \frac{c_t}{c_{t+1}}\right] \frac{R_t}{\pi_{t+1}}, \quad (13)$$

where the term in square brackets is the marginal rate of substitution between consumption today and tomorrow. In the endowment economy equilibrium consumption of the cash and credit good is exogenous. With exogenous consumption, this Euler equation determines inflation conditional on the nominal interest rate for the generic asset.

Two remarks are in order. First, I deviate from the standard asset pricing setup to get potentially well-specified demand functions for real balances and demand deposits. Putting the assets into the utility function is one way to get well-defined demand functions. Alternatively, I could have assumed that these assets lower transactions costs and introduced the relevant cost terms in the budget constraint as in Goodfriend and McCallum (2007). Second, I want to have a simple model of bank lending, so just assume that the "credit" good has to be purchased through a one-period loan taken out from the bank.

Bank Demand and Supply of Assets

A bank takes in demand deposits that provide transactions services for the household and represent a liability to the bank. The bank's assets consist of loans made to the household, and bond and reserve holdings, b_b and m_b. The balance sheet of a bank is

$$l_t + b_{bt} + m_{bt} = d_t. \tag{14}$$

Banks need to hold reserves and bonds to service demand deposits:

$$b_{bt} + m_{bt} = \varphi d_t. \tag{15}$$

This equation represents an assumption on the bank's technology, namely what and how many assets the bank needs in order to generate the demand deposit services for the household. I assume that the bank uses liquid assets, i.e., bonds and reserves, in order to service demand deposits, but it need not hold 100 percent liquid assets, $\varphi < 1$. Furthermore, bonds and reserves are perfect substitutes in the production of demand deposit services.

Banks may also be forced to satisfy a reserve requirement that is imposed by a government regulator:

$$m_{bt} \geq \rho d_t. \tag{16}$$

Alternatively, the reserve ratio can reflect special precautionary preferences of banks for reserves. I assume that $\rho < \varphi$, otherwise banks would not hold other liquid assets besides reserves.[13]

I can assume that there is a representative bank that behaves competitively since the banking technology described above is characterized by constant returns to scale. Whereas banks receive interest on their bond holdings, the payment of interest on reserves (IOR), $R_m \geq 1$, is a policy choice. If bonds pay interest at a higher rate than do reserves, $R_b > R_m \geq 1$, then banks would prefer to hold bonds only against their demand deposits, but they are forced to hold at least a fraction, ρ, of their demand deposits in the form of reserves. If IOR is paid, I assume that interest is paid at the bond rate such that banks are

[13] Canzoneri et al. (2008) provide a more elaborate model of a banking sector that uses resources and not just assets to service demand deposits, and they allow for imperfect substitution between reserves and government bonds.

indifferent between reserves and bond holdings, $R_m = R_b$.[14] To summarize, the bank demand for reserves and bonds is determined by interest rates and reserve requirements as follows

$$m_{bt} = \rho d_t \text{ if } R_{b,t+1} > R_{m,t+1} \geq 1, \quad (17)$$

$$m_{bt} \in [\rho d_t, \varphi d_t] \text{ if } R_{b,t+1} = R_{m,t+1} \text{ or } R_{b,t+1} = 1, \quad (18)$$

$$b_{bt} = \varphi d_t - m_{bt}. \quad (19)$$

In any case, the zero profit condition for making loans and demand deposits determines the deposit rate

$$R_{d,t+1} = (1 - \varphi) R_{l,t+1} + (\varphi - \rho) R_{b,t+1} + \rho R_{m,t+1}. \quad (20)$$

This model for banks' reserve demand exhibits features of a "liquidity trap." First, at a zero bond rate the demand for reserves is indeterminate. Note, however, that the range of indeterminacy is bounded by the required reserve ratio and the desired liquid asset ratio. Second, once IOR is paid at the bond rate, the demand for reserves becomes indeterminate even at positive bond rates. Even though the banks' demand for reserves may be indeterminate within a range, the banks' joint demand for reserves and bonds is always uniquely determined.

Does the proposed "banking" technology make sense? For commercial banks the ratio of cash (including reserves with the Federal Reserve System) plus Treasury holdings relative to deposits has been remarkably stable from 1973 to the end of the 1980s (Figure 5). There was a sharp increase in the early 1990s and then a downward trend that has been reversed since last fall. At the same time, there was a steady decline of the ratio of cash relative to total deposits. Since excess reserves were small relative to required reserves before 2008, this must reflect a steady decline in the required reserve ratio.

Simultaneously with the introduction of IOR in the fall of 2008 and associated with various credit and liquidity programs, the amount of reserves banks hold with the Federal Reserve System has increased dramatically. These higher reserve holdings have not been accompanied by a corresponding decline of other liquid assets, such as treasuries or MBS, or by an increase of demand deposits (Figure 5). In terms of the proposed simple model this would have to be interpreted as a substantial increase in the desired ratio of liquid assets to deposits, φ.

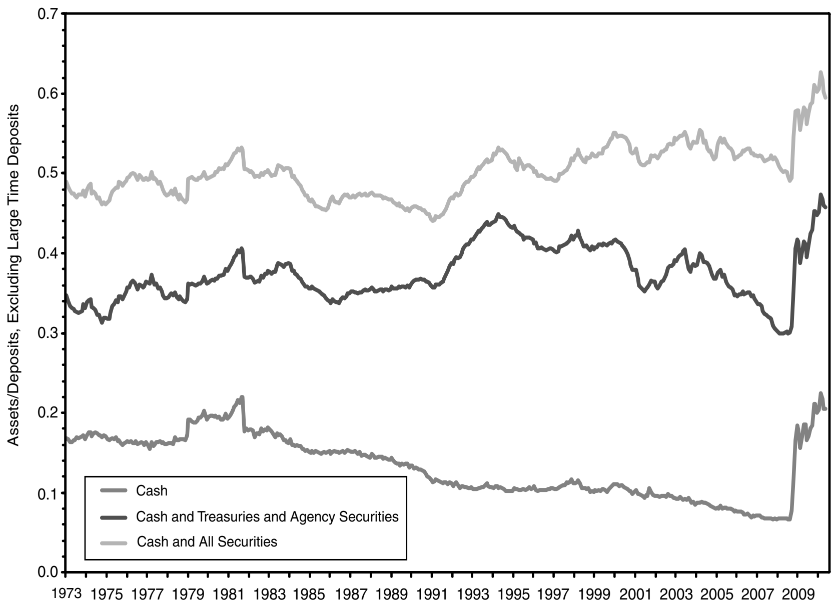
Government Supply of Assets

The government budget constraint is

$$b_t + m_t = \left[R_{b,t} b_{t-1} + m_{h,t-1} + R_{m,t} m_{b,t-1}\right] / \pi_t - \tau_t, \quad (21)$$

[14] In principle the policymaker could decide to make IOR greater than the bond rate, $R_m > R_b$, and reserves would dominate bonds as an asset for banks. I do not consider this case.

Figure 5 Liquid Asset Shares



Notes: The data are described in the Appendix.

where $b = b_h + b_b$ is the total amount of government bonds issued and $m = m_h + m_b$ is the monetary base. In an equilibrium the total amount of government debt has to equal the sum of bank and household bond holdings, and the monetary base has to equal the sum of bank reserves and household cash holdings.

Simplifying the Model

It is possible to simplify the exposition of the model considerably.[15] First, given the exogenous endowment of the cash and credit good, I can use the household demand for loans, (12), and the zero profit condition for banks, (20), to get an expression for the deposit rate:

$$R_{d,t+1} = \mathcal{R}_d \left(R_{t+1}, R_{b,t+1}, R_{m,t+1} \right). \tag{22}$$

[15] For the detailed derivation, see Hornstein (2010).

I can use this function in the household's demand for deposits, (10), and obtain the banks' demand for reserves:

$$m_{bt} = \rho\mathcal{D}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t \text{ if } R_{b,t+1} > R_{m,t+1} = 1, \quad (23)$$
$$m_{bt} \in [\rho, \varphi]\,\mathcal{D}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t \text{ if } R_{b,t+1} = R_{m,t+1} \text{ or } R_{b,t+1} = 1.$$

Aggregate demand for real balances, the monetary base, is then the sum of household demand (9) for cash and bank demand for reserves (23):

$$m_t = \mathcal{M}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t. \quad (24)$$

The demand for monetary base inherits a "flat" indeterminacy range from the banks' reserve demand if the bond rate is zero or interest is paid on reserves.

Analogously to the total demand for real balances, I can define a total demand for government bonds by households and banks:

$$b_t = \mathcal{B}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t. \quad (25)$$

Corresponding to the aggregate demand for real balances, the aggregate demand for bonds also inherits a "flat" indeterminacy range from the banks' demand for bonds. Aggregate demand for total government debt is the sum of the demand for real balances and bonds, equations (24) and (25),

$$v_t = \mathcal{V}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t. \quad (26)$$

As pointed out above, banks' demand for reserves and bonds together is always determinate and the same then applies to the demand for total government debt (money and bonds).

The reduced form of the economy can now be represented by the following set of equations:

$$m_t = \mathcal{M}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t, \quad (27)$$
$$1 = \beta\frac{c_t}{c_{t+1}}\frac{R_{t+1}}{\pi_{t+1}}, \quad (28)$$
$$v_t = \frac{R_{b,t}v_{t-1} - \left(R_{b,t} - 1\right)\tilde{m}_{t-1}}{\pi_t} - \tau_t, \quad (29)$$
$$\tilde{m}_t = \tilde{\mathcal{M}}\left(R_{t+1}, R_{b,t+1}\right)c_t, \quad (30)$$
$$v_t = \mathcal{V}\left(R_{t+1}, R_{b,t+1}, R_{m,t+1}\right)c_t, \quad (31)$$
$$v_t = b_t + m_t. \quad (32)$$

Equation (27) is the aggregate demand for real balances. Equation (28) is the household Euler equation for the generic asset, (13). Equation (29) is the government budget constraint in terms of total debt outstanding v, and $\tilde{m}$ denotes non-interest-bearing government debt. Without interest on reserves, non-interest-bearing debt is aggregate real balances, $\tilde{m} = m$; and with interest on reserves, non-interest-bearing debt is cash holdings by households, $\tilde{m} = m_h$. Equation (31) is the aggregate demand for government debt. Equation

(32) defines total government debt as the sum of real balances and bonds. The baseline model, (1)–(4), is obtained from Section 2 if one assumes that bonds and demand deposits do not provide any liquidity services, $\gamma_b = \gamma_d = 0$; eliminates the credit good, $\gamma_k = k = 0$; and eliminates the banking sector.

Price Level Determinacy with Interest on Reserves

I now show that the simple baseline model from Section 2 and the just described model with a banking sector have very similar implications for how monetary and fiscal policy affect price level determinacy. Whether or not interest is paid on reserves, the model with banking does not materially affect this result. In particular, it appears that the volume of bank reserves does not matter.

The reduced form representation of the economy with a banking sector, equations (27)–(32), appears to be slightly more complicated than the simple baseline model, equations (1)–(4), but the structure of the two economies is very similar. In order to close the model with banking, I again assume that there are fixed endowments of the consumption good, cash and credit; and specify monetary and fiscal policy as responding to inflation and government debt, equation (5). I again study the local properties of the linearized dynamic system defined by equations (27)–(32) and the policy rules (5). In the baseline model, fiscal policy responds to the stock of outstanding real bonds, b, that is, interest-bearing government debt. For reasons that will immediately become apparent, I also consider a fiscal policy that responds to the total stock of government debt, v. I can also do that for the simple baseline model and, comparing Panels A1 and B1 of Figure 4, it is clear that this has no substantial impact on the issue of equilibrium existence and uniqueness.

In order to characterize the implications of monetary and fiscal policy for price level determinacy I need to parameterize the model with banking. Relative to the baseline model, I need to make assumptions on households' steady-state asset holdings (real balances, m , bonds, b, and deposits, d); banks' required reserve ratio, ρ, and desired liquidity, φ; and on steady-state rates of return on the generic asset, R, bonds, R_b, and money, π. I follow Canzoneri et al.'s (2008) calibration of the 1990–2005 U.S. economy. The time period is assumed to be a quarter. The household steady-state ratios of real balances, bonds, and demand deposits to consumption are $m_h/c = 0.3$, $b_h/c = 0.9$, and $d/c = 2.45$. Steady-state nominal interest rates on reserves, bonds, and the generic asset are $R_m = 1$, $R_b = 1.011$, and $R_a = 1.015$. Steady-state inflation is $\pi = 1.007$. The reserve ratio is $\rho = 0.05$ and reflects the ratio of vault cash and bank deposits with the Federal Reserve. The desired liquidity ratio is $\varphi = 0.30$ and reflects the ratio of bank holdings of treasury debt, agency debt, agency MBS, and total reserves to total deposits.

Fiscal Policy Targets Total Debt

Suppose first that fiscal policy targets total debt, v, and that no interest is paid on reserves. Comparing Panels B1 and B2 of Figure 4 it is apparent that the parameter regions that characterize equilibrium existence and uniqueness are qualitatively similar to the baseline model. Price level determinacy is obtained in the northeast region (active monetary policy and passive fiscal policy) and the southwest region (passive monetary policy and active fiscal policy) of the parameter space.

Now suppose that fiscal policy continues to target total debt, but interest is paid on reserves. Because of interest on reserves, total demand for real balances is indeterminate for a range that depends on the reserve ratio and the desired liquidity ratio of banks. Even if the total supply of real balances falls into that range, this does not create a problem for the conduct of monetary policy.

Consider equations (28)–(31) of the reduced form together with the monetary and fiscal policy rules. These equations are sufficient to determine an equilibrium in terms of the inflation rate, interest rates, and total debt, $\left\{\pi_t, R_{t+1}, R_{b,t+1}, v_t\right\}$, if the equilibrium exists. The allocation of total government debt between interest-bearing reserves and interest-bearing debt is irrelevant. In particular, the magnitude of reserves at banks does not matter, as long as the reserves remain within the range of indeterminacy.

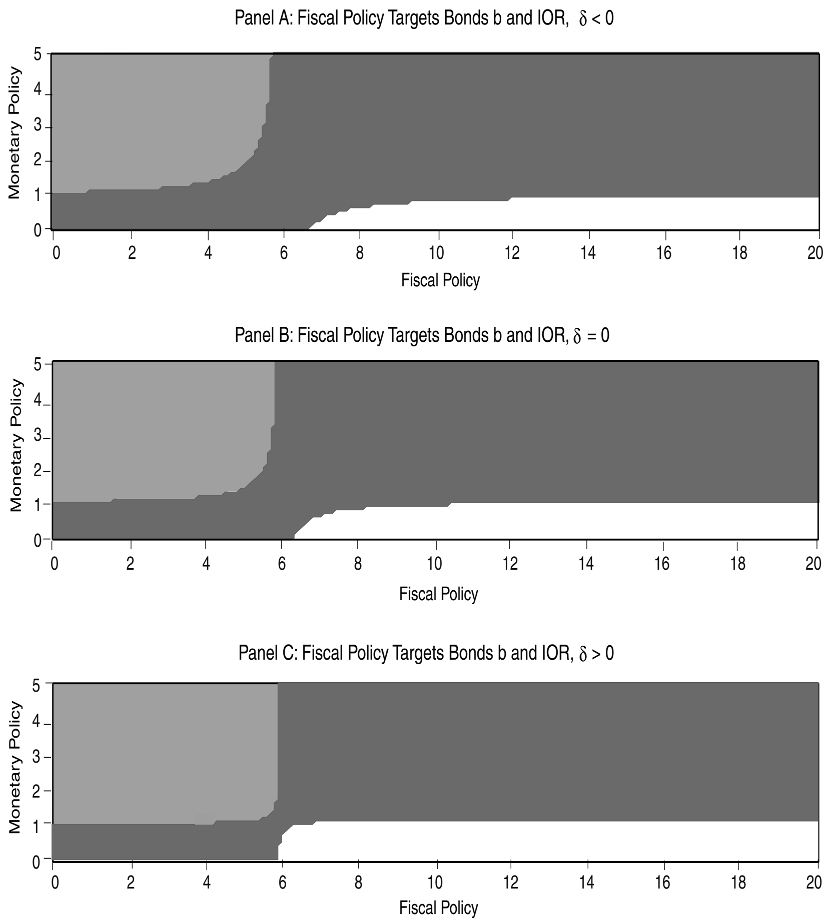
Comparing Panels B2 and B3 of Figure 4 shows that paying interest on reserves has some impact on the issue of price level determinacy. If there is price level determinacy in the northeast region of the parameter space without IOR, then for a given active monetary policy, fiscal policy with IOR has to be somewhat more passive in order for the equilibrium to remain unique.[16] Conversely, in the southwest region of the parameter space, for a given monetary policy, fiscal policy with IOR needs to be more active to obtain price level determinacy.

Fiscal Policy Targets Real Bonds

Now suppose that fiscal policy targets the stock of real bonds, b, rather than total debt, v, but no interest is paid on reserves. Comparing Panels A2 and B2 of Figure 4 shows that for any given monetary policy, fiscal policy can be somewhat more active before losing price level determinacy, either because of nonexistence or nonuniqueness. But now it appears that there is a problem if interest is paid on reserves, since the demand for government bonds—

[16] Recent projections of rapidly expanding fiscal deficits might suggest that fiscal policy has shifted toward a more active stance, that is, taxes are responding less strongly to outstanding debt. If monetary policy were to remain active, fiscal and monetary policy could become inconsistent, that is, an equilibrium would not exist. Thus, the payment of interest on reserves might require a further adjustment of either monetary or fiscal policy to maintain the existence of an equilibrium.

Figure 6 Monetary Policy Targets Reserves



and relatedly the demand for real balances—becomes indeterminate for some range. A well-defined demand for government bonds is, however, needed, since fiscal policy is supposed to respond to the stock of outstanding bonds.

I can resolve the indeterminacy of the demand for bonds through the introduction of an additional policy rule that determines their equilibrium values. For example, the central bank might conduct open market operations (OMO) that adjust real balances in response to the inflation rate:

$$m_t = h(\pi_t), \tag{33}$$

with an elasticity of δ. In other words, because the money demand equation, (27), no longer determines real balances, monetary policy can choose real balances.[17]

Figure 4, Panel A3 graphs the parameter regions for price level determinacy when monetary policy does not adjust real balances in response to the inflation rate, $\delta = 0$. The impact of paying interest on reserves relative to not paying interest on reserves, Figure 4, Panel A2, is similar to the case when fiscal policy targets total debt and not bonds only.

How much paying IOR matters now also depends on the new OMO parameter, δ. Figure 6 displays the parameter regions for price level determinacy when fiscal policy targets real bonds and the OMO parameters are $\delta = 100$ (Panel A), $\delta = 0$ (Panel B), and $\delta = -100$ (Panel C). Given that the OMO response to real balances is essentially a response to bank reserves, one might think that with IOR, monetary policy would have to target both inflation and bank reserves. This interpretation has to be qualified for two reasons. First, bank reserves matter only because I have assumed that fiscal policy targets bonds and not total debt. Second, the graphs in Figure 6 are based on very extreme values for the OMO policy parameter. For δ values that are of similar magnitude as the monetary and fiscal parameters, α and γ, the parameter regions for price level determinacy are essentially the same.

4. CONCLUSION

This article addresses the question of whether paying interest on the reserve accounts that banks hold with a central bank affects the conduct of monetary policy. For this purpose I introduce a stylized model of banks that hold reserves into a standard baseline model of money. This model suggests that paying interest on reserves does not drastically change the implications of monetary policy, implemented as an interest rate policy, for price level determinacy. Furthermore, the amount of outstanding reserves does not appear to be critical for issues of price level determinacy.

The scope of the article is rather narrow. For example, I do not study how the payment of IOR affects the dynamic response of the economy to shocks for given monetary and fiscal policy rules. The model can be used to address this issue if features are added that make money non-neutral, for example, a New Keynesian Phillips curve based on sticky prices. Preliminary results for such an augmented model suggest that for the same monetary and fiscal policy rules the dynamic response of inflation and output to shocks does

[17] We usually think of OMO as determining nominal quantities. I have chosen a policy rule that chooses real balances to keep the exposition simple. One could interpret the proposed policy rule as responding to inflation and to the price level. Alternatively, one could simply start with a policy rule that sets the nominal money stock and study the more complicated system.

depend on whether or not interest is paid on reserves, but the differences are not substantial.

The effects of financial market interventions by central banks, however, cannot be studied in this framework. Since the model's concept of liquidity for the financial sector is rather narrow, the model has nothing to say about central bank provision of liquidity to banks through an increase of the banks' reserve accounts. For example, the model does not provide any rationale for the Federal Reserve's program to purchase agency MBS as opposed to other government debt. Indeed, the simple banking model assumes that agency MBS and treasuries provide the same liquidity services to banks.[18] For a critical review of the Federal Reserve interventions in specific financial markets that gave rise to the expansion of the Federal Reserve's balance sheet, in particular, the volume of reserve liabilities, see Hamilton (2009).

APPENDIX

Figure 2 displays daily data for the federal funds target set by the FOMC and the effective federal funds rate from January 2000–February 2010. In addition, Panel C of Figure 2 also displays the interest rate that was paid on required reserves and on excess reserves from September 2008 on. Figure 3 displays monthly averages from January 1980–February 2010 for the following short-term interest rates: the effective federal funds rate, the three-month constant maturity Treasury rate, the three-month nonfinancial commercial paper rate, the rate for three-month certificates of deposit in the secondary market, and the prime bank lending rate. Figure 5 displays monthly liquid asset ratios of all commercial banks, domestically chartered and foreign related institutions, from January 1973–January 2010 based on the Federal Reserve Board's H.8 table. Securities in bank credit include Treasury and agency securities and other securities. A large part of agency securities consists of MBS issued by GSEs such as the Government Mortgage Association (Ginnie Mae, GNMA), the Federal National Mortgage Association (Fannie Mae, FNMA), or the Federal Home Loan Mortgage Corporation (Freddie Mac, FHLMC). Other securities include private label MBS, among others. Cash includes vault cash and reserves with the Federal Reserve. The liquid asset ratio is calculated relative to bank deposits excluding large time deposits. All series are from Haver.

[18] Given that the GSEs Fannie Mae and Freddie Mac have become wards of the federal government, this does not appear to be such an unreasonable assumption. Indeed, the only reason to distinguish between Treasury debt on the one hand and agency-issued debt and MBS on the other hand appears to be political: GSE-issued debt does not count toward the congressionally mandated federal debt limit.

REFERENCES

Bech, Morten L., and Elizabeth Klee. 2010. "The Art of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market." Finance and Economics Discussion Series 2010-07. Washington, D.C.: Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board.

Bernanke, Ben. 2009. "The Fed's Exit Strategy." *Wall Street Journal*, July 21.

Canzoneri, Matthew B., Robert Cumby, Behzad Diba, and David López-Salido. 2008. "Monetary Aggregates and Liquidity in a Neo-Wicksellian Framework." *Journal of Money, Credit and Banking* 40 (December): 1,667–98.

Ennis, Huberto M., and John A. Weinberg. 2007. "Interest on Reserves and Daylight Credit." Federal Reserve Bank of Richmond *Economic Quarterly* 93 (Spring): 111–42.

Ennis, Huberto M., and Todd Keister. 2008. "Understanding Monetary Policy Implementation." Federal Reserve Bank of Richmond *Economic Quarterly* 94 (Summer): 235–63.

Goodfriend, Marvin. 2002. "Interest on Reserves and Monetary Policy." Federal Reserve Bank of New York *Economic Policy Review* 8 (May): 77–84.

Goodfriend, Marvin, and Bennett T. McCallum. 2007. "Banking and Interest Rates in Monetary Policy Analysis: A Quantitative Exploration." *Journal of Monetary Economics* 54 (July): 1,480–507.

Hamilton, James D. 2009. "Concerns about the Fed's New Balance Sheet." http://dss.ucsd.edu/~jhamilto/fed_concerns.pdf (April 18).

Hornstein, Andreas. 2010. "Technical Appendix for 'Monetary Policy with Interest on Reserves.'" Mimeo, Federal Reserve Bank of Richmond.

Keister, Todd, Antoine Martin, and James McAndrews. 2008. "Divorcing Money from Monetary Policy." Federal Reserve Bank of New York *Economic Policy Review* 14 (September): 41–56.

Lacker, Jeffrey. 2006. "Central Bank Credit in the Theory of Money and Payments." Speech presented at The Economics of Payments II Conference, New York. http://www.richmondfed.org/press_room/speeches/president_jeff_lacker/2006/lacker_speech_20060329.cfm (March 29).

Leeper, Eric M. 1991. "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27 (February): 129–47.

Sargent, Thomas, and Neil Wallace. 1981. "Some Unpleasant Monetarist Arithmetic." Federal Reserve Bank of Minneapolis *Quarterly Review* 5 (Fall): 1–17.

Sargent, Thomas, and Neil Wallace. 1985. "Interest on Reserves." *Journal of Monetary Economics* 15 (May): 279–90.

Sims, Christopher. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 4 (3): 381–99.

Sims, Christopher. 2000. "Solving Linear Rational Expectations Models." http://sims.princeton.edu/yftp/gensys/.

Sims, Christopher. 2009. "Fiscal/Monetary Coordination when the Anchor Cable has Snapped." Presentation at the Princeton Conference on Monetary and Fiscal Policy, Princeton University, May 22.

Smith, Bruce D. 1991. "Interest on Reserves and Sunspot Equilibria: Friedman's Proposal Reconsidered." *Review of Economic Studies* 58 (January): 93–105.

Woodford, Michael. 2000. "Monetary Policy in a World without Money." *International Finance* 3 (July): 229–60.