Naming Game on Adaptive Weighted Networks

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Abstract

We examine a naming game on an adaptive weighted network. A weight of connection for a given pair of agents depends on their communication success rate and determines the probability with which the agents communicate. When the preference toward successfully communicating agents is not so strong, the model behaves similarly to the naming game on a complete graph. In particular, it quickly reaches a single-language state, albeit some details of the dynamics are different from the complete-graph version. Much different behaviour appears when the preference toward successfully communicating agents is stronger and the model gets trapped in a multi-language regime. In this case gradual coarsening and extinction of languages lead to the emergence of a dominant language, albeit with some other languages still being present. A comparison of distribution of languages in our model and in the human population is discussed.

keywords: adaptive weighted networks, naming game

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1 Introduction

Language is based on a certain set of conventions socially shared by a group of users. Recently, increasing scientific efforts have been made to understand mechanisms that are responsible for the emergence and subsequent evolution of language [16, 9, 23, 5]. Processes that underlie these mechanisms are very complex and to a great extent spontaneous. They result from cultural interactions between users and it seems that the most promising approach to model them is computational simulations [12, 25, 26, 13, 27], especially those based on the so-called multi-agent systems [6, 24] .

An important class of such models originates from the naming game introduced by Steels to describe processes leading to the establishment of a shared vocabulary, i.e., a set of mappings between words and meanings [28]. In the naming game, agents are involved in pairwise interactions, which direct the model to a state of linguistic synchronization, i.e., to the conformance of agents' vocabularies. Let us emphasize that while conversations of agents take place strictly locally (each involves only two agents) and go on without any central control, nevertheless the final result of these processes, i.e., a common vocabulary shared by all agents, emerges globally.

To specify the naming game model, one has to define the topology of interactions between agents. For mathematical and computational simplicity, a complete-graph topology is often assumed, where each agent can interact with any other agent [3]. Another possibility is to place agents at sites of a regular lattice and allow interactions only between the nearest neighbours [2]. However, real networks of social interactions seem to be much more complex than the above mentioned graphs [31, 33, 1]. To take into account the heterogeneous nature of social interactions, the naming game on scale-free networks and on small-world networks were also studied. Although some quantitative differences in the dynamics were observed, the overall behaviour, namely a relatively fast convergence to the monolingual state, was predominantly the same [11, 10, 22].

Such a behaviour is in contrast with the multi-language structure of the human population. Indeed, despite gradual extinction, there are thousands of languages still in use, and at a time scale of tens or hundreds of years, many languages, especially those relatively common ones, seem to be very stable. In the present paper, we show that the naming game defined on an adaptive weighted network allows us to examine such stable multi-language structures. Interactions in our model try to mimic certain features of social interactions.

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In particular, the weights of connections between agents differentiate the probabilities of their mutual communication. Moreover, these weights are adaptive and depend on the success rate of past communication attempts of agents. Let us notice that the formation of multi-language states has already been observed by Dall'Asta *et al.* for complex networks with a community structure [11]. However, their networks are a bit artificial with a community structure imposed "by hand". In our opinion, the multi-language structure of human population is a result of a certain dynamic process, which, we hope, is to some extent captured in our model.

2 Model and numerical method

In the original formulation [28], the naming game describes cultural transmission within a single generation of agents. Agents are involved in pairwise negotiations and try to establish a common vocabulary for a certain number of objects present in their environment. Most of the research, however, is limited to a single-object case (for a recent review see [21]) since it seems to be sufficient to capture the essence of the dynamics also in a more general case. Multi-object [17, 30, 20] as well as evolutionary [19, 18] versions of the naming game were also studied.

The naming game is also related to models used to describe opinion formation, with the voter model being the prime example [7]. Let us stress again that the main emphasis in the naming game model is on the cultural (single-generation) transmission of language. An alternative approach to the language evolution, where inter-generational interactions play an important role, is called Iterated Learning Model and was used in some other contexts [15, 29].

In our model we consider a population of N agents playing the single-object naming game. Each agent has its lexicon, i.e., a list of words (initially empty). Basic steps of the game are as follows:

1. First, a speaker i and a hearer j are selected $(i \neq j)$. Then the speaker selects randomly one of the words from its lexicon (if the lexicon is empty, the agent generates a word randomly) and communicates this word to the hearer.

2. The game ends successfully when the hearer has the word in its lexicon; in such a case both agents retain only the communicated word in their

lexicons.

3. The game fails when the hearer does not have the word in its lexicon; in this case the word is added to the hearer's lexicon.

After the game, both the number of successes the pair of agents (i, j) have achieved so far as well as the number of all their communication attempts are updated and the communicative success rate for this pair of agents $s_{ij} = s_{ji}$ is calculated as the ratio of these two numbers (of course, initially $s_{ij} = 0$ for all pairs of agents).

The above definition corresponds to the so-called minimal version of the naming game [3]. To complete the definition of our model, we have to specify how we select the speaker and the hearer. Our intention is to simulate an important, in our opinion, criterion: we talk most preferably with those with whom we already have communicated successfully. This is implemented as follows:

- The speaker i is selected randomly.

- The hearer j is selected using the roulette rule, with the probability

$$p_{ij} = \frac{w_{ij}}{\sum_{k=1}^{N} w_{ik}}, \tag{1}$$

where the weights

$$w_{ij} = \begin{cases} s_{ij} + \epsilon & \text{for } i \neq j \\ 0 & \text{for } i = j \end{cases} \tag{2}$$

for $i, j = 1, 2, \ldots, N$. The (positive and typically small) parameter ϵ ensures that a speaker can sometimes play the naming game also with such agents with which its up-to-now communicative success rate is very small or even equals zero.

From the above rules, it follows that our model constitutes an adaptive weighted network. The weights of links, which depend on the success rates that pairs of agents have achieved so far, change in time and control the intensity of their subsequent interactions. Generally, the bigger the success

rate, the more frequent communication attempts, however, due to the parameter ϵ, there is always a possibility to communicate also with such an agent with which there have not been scored any successes so far.

Obviously, the characteristics of our model change during simulation. To examine their time dependence and compare results for different numbers of agents N, we have defined a unit of time as N communication attempts (which corresponds to a single on average communication attempt per agent). We have monitored a number of time-dependent observables in the system, in particular:

- s – the communicative success rate defined as a fraction of all successes during the last N communication attempts, i.e., within the last unit of time. Let us emphasize that s is calculated for N most recent communication attempts in the entire system of N agents while s_{ij}, i.e., the communicative success rate of a pair of agents i and j, which determines the weight w_{ij} (2) of the connection between them, stores the entire history of their interactions.

- L – the number of different words in lexicons of all agents. At later stages of simulations, when most of agents have only one word in their lexicons, L could be interpreted as the number of languages the agents use at this moment of the evolution of the system.

- N_d – the number of agents that have the most common word in their lexicons, i.e., the word that appears in the largest number of lexicons. In some cases we measured also the number of users of less common words.

To smooth-out statistical fluctuations, the measured quantities s, L, and N_d were averaged over independent runs. Some other observables are described in the following text.

3 Numerical results

Initially, all weights $w_{ij} = \epsilon$ for $i \neq j$ and thus the selected speaker chooses any agent as a hearer with equal probability. As the evolution of the model progresses, some pairs of agents (i, j) might successfully communicate and that increases the weight w_{ij}. Consequently, an agent will communicate with

some agents more often than with others and after some time clusters of such agents will be formed, with communication taking place mainly within these clusters. Hence, linguistic synchronization is quickly reached inside clusters and all agents in a cluster eventually use the same language, i.e., they have the same (and only one) word in their lexicons. One should keep in mind, however, that the structure of the network set by the collection of the weights w_{ij} is dynamic and even strong connections ($w_{ij} \approx 1$) might become weak later on (and also weak connections might get stronger). The details of the dynamics and the final state of the model depend on the parameters N and ϵ, as discussed below.

3.1 Single-language regime

Although communication between agents takes place mainly within clusters, some outside-cluster communication attempts are also made. This is because the weight (2) is positive even for agents with (so far) zero success rate. As a result of such attempts, some agents might change the cluster they belong to; eventually, some clusters might even disintegrate. Such a process resembles the coarsening dynamics and the order/disorder transition found in other versions of the naming game [3].

Obviously, the larger ϵ, the greater the intensity of the outside-cluster communication. The number of agents N is yet another factor that increases the intensity of such processes. This is because for large N an agent belonging to a cluster of a given size has so many more candidates to communicate with (albeit of very small weights $\sim \epsilon$). One might expect that it is the combination of ϵ and N, rather than those parameters taken separately, which is the control parameter of the model. Although we cannot provide a rigorous derivation, some arguments (Figure 1) suggest that in our model, some important processes, which most likely underlie the coarsening, take place at a rate that scales as $N\epsilon^2$. The numerical results presented below are collected in sets corresponding to constant $N\epsilon^2$ and their reasonably good convergence (for increasing N) seems to support the suggestion that $N\epsilon^2$ indeed is the control parameter of the model.

Our simulations show that when $N\epsilon^2$ is sufficiently large, the outside-cluster communication is sufficiently frequent and the model behaves similarly to the naming game on a complete graph, where it is known to reach a state of complete synchronization [3]. Numerical results confirm basically such a behaviour but exhibit some important differences as well. In Figure 2
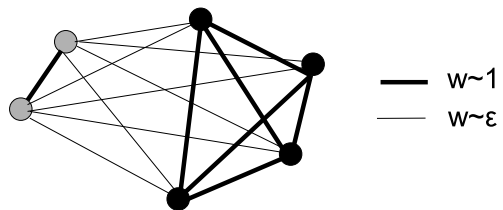
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Figure 1: In the final stage of the coarsening process, almost all agents form a single-language cluster (black circles). To include a given (grey) agent which belongs to a certain small cluster into the main cluster, one needs two steps: (i) a black speaker selects a grey hearer, which results in failure, and the grey agent adds to its lexicon the word used by black agents (the rate of this processes scales as $\epsilon N/N = \epsilon$); (ii) the grey agent as a speaker selects a black hearer and the word it chooses to communicate is the one acquired in step (i) (the rate $\frac{N\epsilon}{N\epsilon+1} \approx N\epsilon$ for small $N\epsilon$). Because the second step is a success, all words but the communicated one are removed from agents' lexicons, and the grey agent becomes black. The combined rate of steps (i) and (ii) should scale as $N\epsilon^2$.
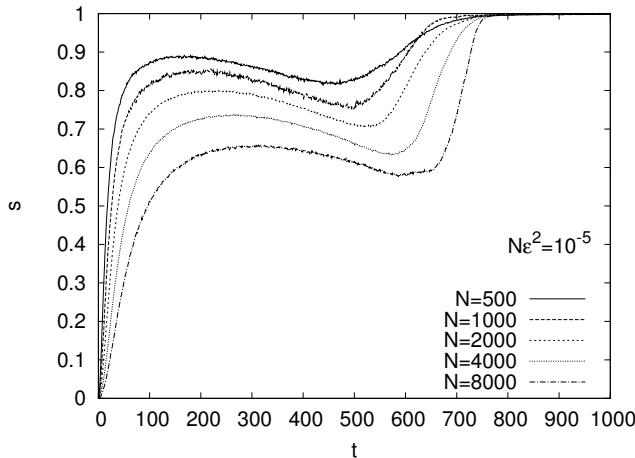
Figure 2: The time dependence of the success rate s calculated for several values of N and for $N\epsilon^2 = 10^{-5}$.

we present the success rate s calculated for $N\epsilon^2 = 10^{-5}$. One can notice that after an initial interval, agents reach the state where they communicate with a large success rate $s \sim 1$. Interestingly, this initial interval is almost the same ($t \sim 750$) for all examined systems even though their sizes N differ substantially.

Since the number of languages L diminishes in time (Figure 3), we conclude that indeed the system evolves toward the single-language state. The inset of Figure 3 shows that at the same time when the success rate s approaches unity ($t \sim 750$, Figure 2), the normalized number of languages L/N drops to 0. Let us also note that the crossing point around $t = 900$ in Figure 3 suggests that in the limit $N \to \infty$ for $t > 900$, there is only one language in the system. Figure 4, where the ratio of users of the most common language N_d/N is plotted, shows that for $t > 750$ almost all agents use the same language.

As it is already known, a characteristic time τ to reach a single-language state in the naming game increases with the system size N. For example, it is found that on a complete graph $\tau \sim N^{1.5}$ [3]. To compare this result with the behaviour of our model, we calculated τ for several values of N and the results are presented in Figure 5. They show that τ does not diverge for increasing N but most likely converges to a finite value ≈ 1050, which is approximately consistent with the time scale seen in the behaviour of s, N_d/N, and L.
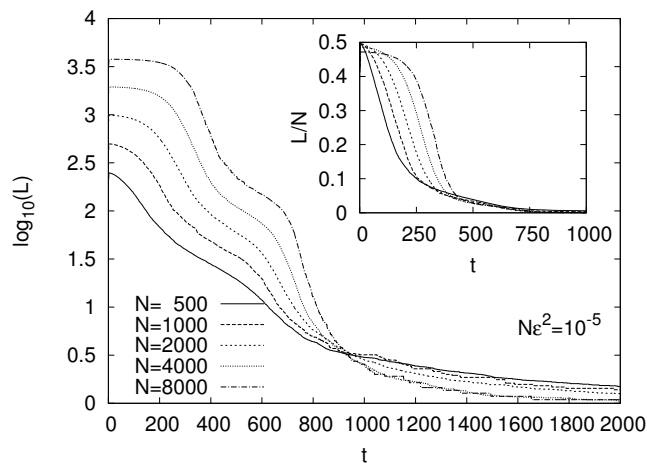
Figure 3: The time dependence of the number of languages L (logarithmic scale) calculated for several values of N and for $N\epsilon^2 = 10^{-5}$. The inset shows the time dependence of the normalized number of languages L/N.
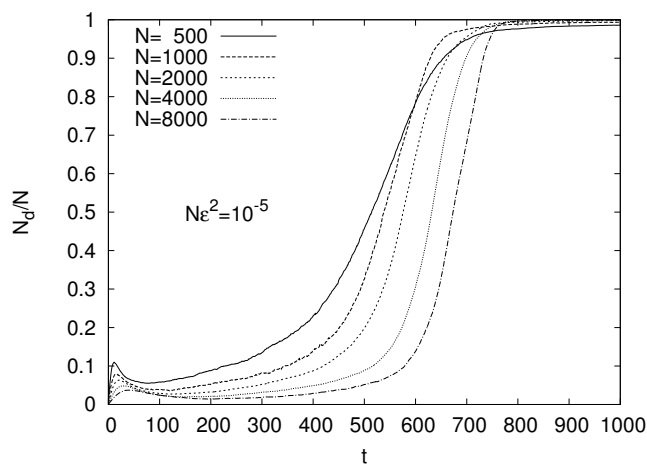

Figure 4: The time dependence of the ratio of agents that use the most common language N_d/N calculated for several values of N and for $N\epsilon^2 = 10^{-5}$.
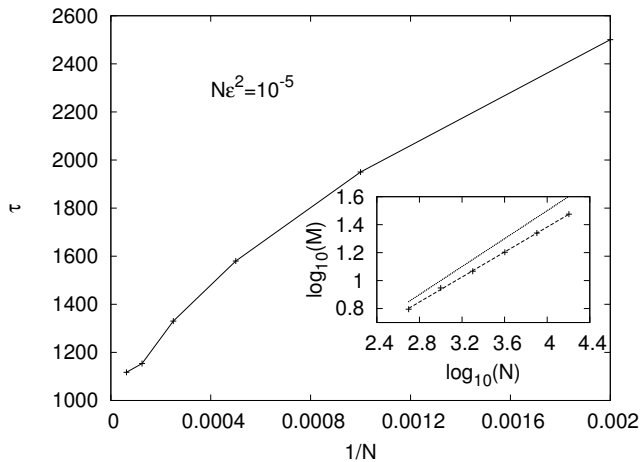
Figure 5: The size dependence of the average time needed to reach a single-language state. Calculations were done for $N = 500, 1000, 2000, 4000, 8000, 16000$ and for $N\epsilon^2 = 10^{-5}$. For each N, usually 100 independent runs were made. The inset shows the size dependence of the averaged maximum number of words M in agents' lexicons (log-log scale). The fitted dashed line has the slope 0.45 and the dotted line corresponds to the complete graph (slope 0.5).

An additional comparison with a complete-graph version can be obtained from the analysis of the number of words kept in the lexicons of agents. Typically, this number (as a function of time t) first gradually increases and then decreases as the single-language state is being approached. It is known that the averaged maximum number of words (per agent) M on a complete graph increases asymptotically as $N^{0.5}$ [3]. Numerical results for our model, shown in the inset of Figure 5, suggest that M increases asymptotically as $N^{0.45}$. Such a discrepancy is yet another indication that the dynamics of our model, despite qualitative similarities, differs from the complete-graph version.

3.2 Multi-language regime

When $N\epsilon^2$ is sufficiently small, the outside-cluster communication attempts are very rare, which results in a much different behaviour of our model. In the inset of Figure 6, one can notice that the success rate s reaches unity around $t = 10^3$, however, at that time the ratio of agents using the most common language N_d/N is much smaller than unity. Only after $t \approx 3 \cdot 10^4$, this fraction significantly increases, which suggests the emergence of a dominant
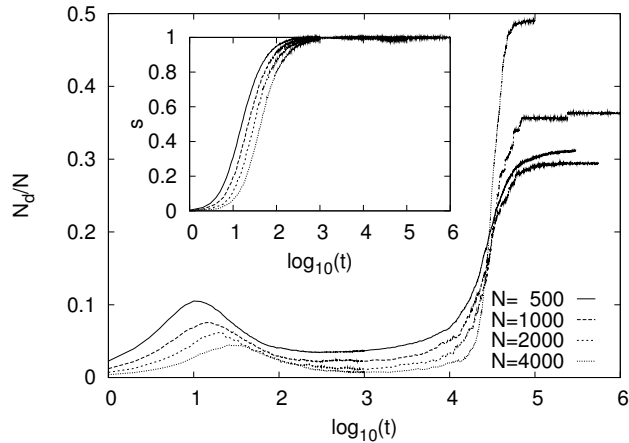
Figure 6: The time dependence of the ratio of agents that use the most common language N_d/N calculated for several values of N and for $N\epsilon^2 = 10^{-9}$. The inset shows the time dependence of the success rate s.

language (for large N the fraction N_d/N for large t most likely exceeds 0.5 and thus there would be only one dominant language). However, the dominant language only partially invades the system, since some other languages also persist, apparently for an arbitrarily long time (Figure 7). Indeed, around $t = 10^5$ the number of languages saturates at $L \sim 10$ and does not seem to diminish even up to $t = 10^6$. Although there are several languages for $t > 10^5$, their number is only a small fraction of the number of agents N (inset in Figure 7).

In the multi-language regime one can thus distinguish three phases. The first one ($t < 10^3$) can be called pre-linguistic, since the success rate remains relatively low. Next ($10^3 < t < 3 \cdot 10^4$) there is a phase with many languages but without a dominant language ($N_d/N \ll 1$). In this phase the number of languages gradually decreases. In the third phase ($t > 3 \cdot 10^4$) several languages exist but one of them emerges as a dominant one and is used by a large fraction of agents. Let us notice that for increasing N, the emergence of a dominant language becomes more and more abrupt.

To get some insight into a possible mechanism that stabilizes the multi-language state, we calculated the numbers of users of the 50 most common languages. We made only a single run (no averaging over independent runs) and Figure 8 shows how these numbers change in time. Initially, one can
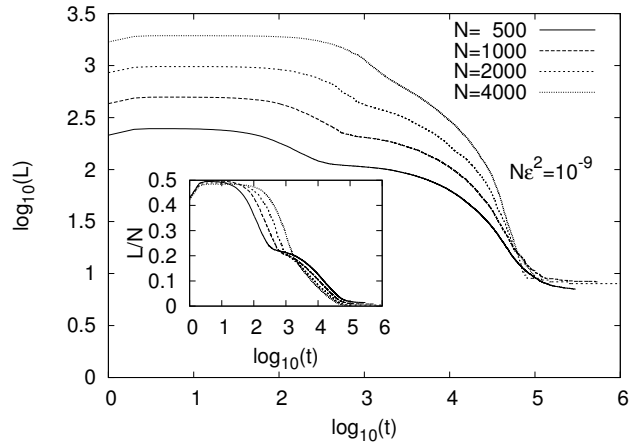
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Figure 7: The time dependence of the number of languages L calculated for several values of N and for $N\epsilon^2 = 10^{-9}$. The inset shows the time dependence of the normalized number of users L/N.

observe a gradual increase in the numbers of users of the most common languages at the expense of the less prevalent ones. Around $t = 10^5$ only 12 languages are left and at $t = 3 \cdot 10^5$ the least common one (among these 12) becomes extinct. One can notice that there are virtually no other changes between the data at $t = 10^5$ and $t = 3 \cdot 10^5$. Let us also observe that the remaining languages are relatively prevalent and even the least frequently used one has more than 20 users (2% of the population). For comparison, we present analogous data from the single-language regime (Figure 9). Although transiently a number of relatively common languages are formed in this case, eventually only one of them survives. Thus it is not a sheer number of users that determines whether a language will survive or not but also the value of the control parameter $N\epsilon^2$, which plays an important role.

We are tempted to think that the behaviour of our model in the multi-language regime bears some similarity to the evolution of human languages. If so, some very basic characteristics of human languages could be reproduced within our model. In Figure 10, we present the distribution of users of the 20 most common existing languages [32] compared with analogous distributions obtained for our model. We made a simulation for $N = 10^3$ and $\epsilon = 10^{-6}$, which corresponds to the multi-language regime. The simulation time was chosen in such a way that the percentage of users of the most common lan-
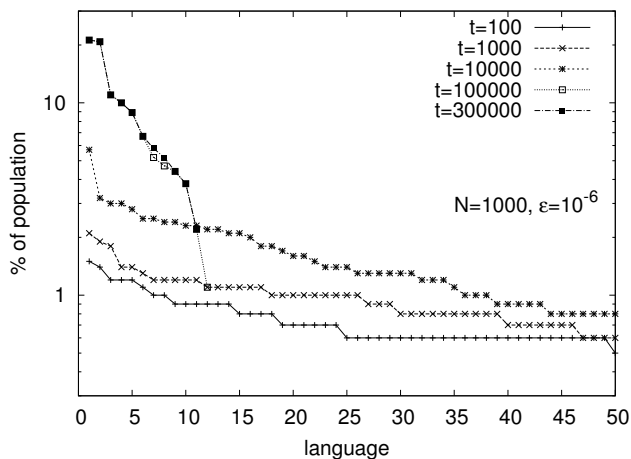
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Figure 8: The percentage of population constituted by the users of the 50 most common languages, calculated for $N = 1000$ and $\epsilon = 10^{-6}$ (i.e., in a multi-language regime).
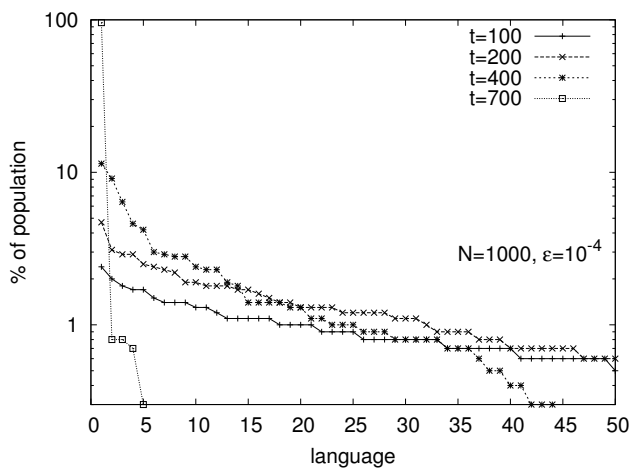

Figure 9: The percentage of population constituted by the users of the 50 most common languages, calculated for $N = 1000$ and $\epsilon = 10^{-4}$ (i.e., in a single-language regime).
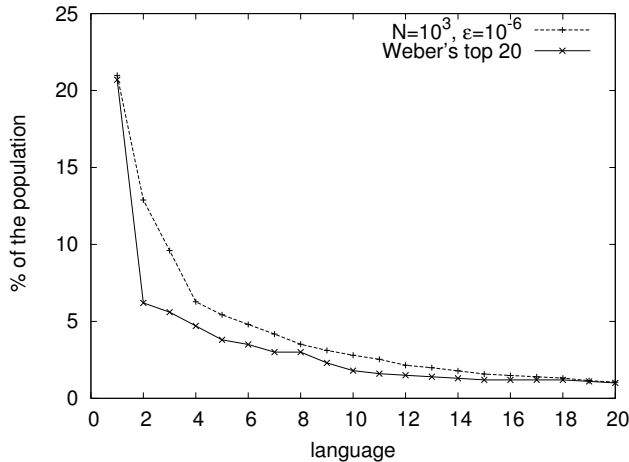
Figure 10: The fraction of the population constituted by the users of the 20 most common languages, calculated for $N = 1000$ and compared with Weber's statistical data [32]. The first three languages according to Weber are Chinese (20.7% of the population, i.e., $1.1 \cdot 10^9$ speakers), English (6.2%, $3.2 \cdot 10^8$), and Spanish (5.6%, $3.0 \cdot 10^8$).

guage was equal to 20.7%, i.e., the percentage of speakers of Chinese. Let us notice that there is a reasonably good agreement even for less common languages and perhaps by modifying the value of $N\epsilon^2$ (within a multi-language regime), a still better fit could be achieved. However, taking into account the complexity of the evolution of human language, such an agreement is to some extent accidental and should be considered with great care.

4   Remarks and Conclusions

In the present paper we examined the naming game on an adaptive weighted network. When the preference due to successful communication in the past is not so strong, the model behaves similarly to the naming game on a complete graph. In particular, it quickly reaches the state of complete synchronization where all agents use the same language. However, the average time needed to reach such a state does not diverge as the number of agents N increases — contrary to the naming game on complete graphs. Moreover, the average maximum number of words in the lexicons of agents increases as $N^{0.45}$, which

is close but different than the $N^{1/2}$ increase that is expected for the naming game on a complete graph.

Quite different and perhaps more interesting behaviour of our model appears when the preference due to successful communication in the past is stronger. In this case our model does not reach complete synchronization but remains trapped in a multi-language state. Three phases can be distinguished in the evolution of our model. In the first one, the average success rate of communicating agents is rather low and most likely this phase corresponds to a prelinguistic phase. In the next phase, although the average success rate is close to unity, many languages exist in the system and none of them dominates. At a certain moment, however, the model enters the third phase where many languages still exist but one of them becomes dominant. Since languages are dynamically equivalent, the emergence of the dominant language might be considered as some kind of spontaneous symmetry breaking. Interesting issues of broken ergodicity or stability of the resulting structures are however omitted in the present paper.

One can speculate about the possibility that human languages evolved similarly to the multi-language scenario. Such an idea gets some support from the comparison of Weber's distribution of the 20 most common languages and the analogous distribution in our model. An interesting, in our opinion, question is whether the rapidly progressing extinction of less widespread ethnic languages will lead to the emergence of a dominant language and if so, when such a situation occurs. With this respect, our work is, of course, inconclusive. Let us notice, however, that the users of the most common language currently constitute about 20% of the human population and according to Figure 6, we might be very close to the transition where the dominant language will emerge.

For models on adaptive networks, there is a subtle point concerning the time scales that govern the evolution of the network itself and of the model (in our case, the naming game). In principle, these processes might have two independent time scales, which in some cases might even be well separated [14, 4]. In our model, however, evolution of weights is strongly coupled with agreeing dynamics of the naming game. Since the success rate stores the entire history of communication between agents, the weights (2) typically lag behind the agreeing dynamics. In particular, these weights will evolve even when the naming game has already reached the single-language state (in such a case, the weights will gradually evolve toward $w_{ij} = 1 + \epsilon$ for every $i \neq j$). It would be desirable to analyse the interplay of these two time scales

in more detail, for example, in the case when the success rate stores only a limited number of last communication attempts. It would also be interesting to have a better understanding of the strong stability and long lifetimes of languages in the multi-language regime. One can consider multi-language states as metastable states of the dynamics and perhaps there is some analogy with metastable states in a certain voter model on complex networks with a strong community structure [8]. Further studies on this subject as well as a more detailed analysis of the structure of the adaptive network is, however, left for the future.

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