Effects of Input Redundancy on Time Optimal Control

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Abstract Due to the popularity of the systems with input redundancy, this paper focuses on the problems with input redundancy, where we concern about the effects of adding new input redundancy into the controllable systems. Time optimal control problems are discussed, where such effects are evaluated by the optimal time. Based on the assumption of the existence and uniqueness of the optimal control, the paper proves that increasing the number of input redundancy will result in a strict reduction of the optimal time from the same initial state if there exists non-idle channel among the redundant input channels. Moreover, if the problem is normal, then all of the redundant input channels are used to shorten the optimal time. On the other hand, without the assumption of normality, the optimal time will also be smaller for the redundant system as comparing to the original system if at least one of these redundant input channels is completely controllable. Finally, two numerical examples are deployed to demonstrate the main results of this paper.

Key words Input redundancy, idle channel, time optimal control, redundant system

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In a traditional aeroplane, there are three different kinds of control surfaces to generate three different moments which are needed for flying the plane, i.e., horizontal tails generate the moment of pitch, rudders control the moment of yaw, as well as the rolling moment is manipulated by the ailerons. Nowadays, with rapid development of technology, more than twenty control surfaces (actuators) are available for aircrafts, such as leading edge flaps[1]; thrust-vectoring moment generators and vortical lift fans[2]; trailing edge flaps and spoilers on the leading edge extensions[3]; side force generators[4]. From a view of system dynamics, adding these extended/redundant control surfaces is equivalent to add additional columns into the input matrix B of the system if the inputs appear linearly. In 1970′s, Wonham pointed out that a single-input system can replace a multi-input system equivalently by introducing a state feedback and finding an auxiliary vector in the column space of the input matrix[5−6]. Meanwhile, the properties such as stabilizability, controllability, and poles assignment will remain invariant after adding new input redundancy (or input extension) if the original system already possesses them. However, beyond these invariance, this paper will investigate the extra benefits brought by the new input redundancy, where the optimal time is used to evaluate the improvements of the input extensions.

Obviously, adding redundant control surfaces will possibly result in some improvements of dynamical performance for an aeroplane, which are demonstrated in numbers of experiments. For example, in 1970′s, the thrust vectoring was introduced to satisfy the demands of designing short-takeoff and vertical-landing (STOVL) aircrafts, such as Harrier and Yak-36. Soon after that, NASA developed the experimental aircraft X-31 to investigate the advantage of using thrust vectoring to improve the in-flight maneuverability[7]. Costes reported that the killing ratio was significantly increased for a vectored thrust plane against a conventional control fighter[8]. Meanwhile, the aircrafts implemented with thrust vectoring are capable to perform the supermaneuver such as Pugachev-Cobra and Herbst maneuvers[9], which can result in a rapid speed reduction and a sharper turn. These supermaneuver abilities become vital characteristics to takeover the enemy during the dogfights[10]. With respect to time optimal control problems, input extension also reduces the optimal time. Schneider concluded that the reduction of the minimum turning time is related to adding the thrust-vectoring[11].

Different from the experimental results listed above, in this paper, we will discuss the advantages of adding input redundancy through a theoretical perspective. Firstly, the invariance of the controllability and pole placement under input extension will be proven. Then, after defining and examining the existence of idle channels, which represent the unused input channels, the system with input redundancy will own a faster optimal time than the one without input redundancy if there exists non-idle channel among the redundant input channels. According to a well-known input constraint Π, the normality of the optimal control problem will play a key role in the reduction of the optimal time, although which is a quite strong condition since all of the input channels are completely controllable. A weaker condition requires only one of the redundant input channels to be completely controllable.

1 Problem statement

A standard time optimal control problem without input redundancy can be described as[12−15]:

Problem 1. Minimize the final time $T_o(x_0)$ subject to a linear time-invariant (LTI) system

$$\dot{x}(t) = Ax(t) + B_o u_o(t) \tag{1}$$

from the initial state $x(0) = x_0$ to the final state $x(T) = 0$ with the input constraint $u_o(t) \in \Pi$, where $A \in \mathbf{R}^{n\times n}$, $B_o \in \mathbf{R}^{n\times r}$, and Π is defined as

$$\Pi = \Big\{ u(t) : [0, +\infty) \to \mathbf{R}^{d(u)};\quad |u_i(t)| \le 1, \quad i = 1, \cdots, d(u) \Big\} \tag{2}$$

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in which $d(u)$ denotes the dimension of the input vector $u(t) \in \mathbf{R}^{d(u)}$.

On the other hand, the time optimal control problem with input redundancy is:

Problem 2. Minimize the final time $T_e(x_0)$ subject to the LTI system with input redundancy:

$$\dot{x}(t) = Ax(t) + B_e u_e(t) \tag{3}$$

where $B_e = [B_o\ B_1]$ and $B_1 \in \mathbf{R}^{n\times k}$. The corresponding control vector with input redundancy is

$$u_e(t) = \begin{bmatrix} u_o(t) \\ u_1(t) \end{bmatrix}$$

under the input constraint $u_e(t) \in \Pi$, where Π is defined in (2) and $u_1(t) \in \mathbf{R}^k$.

2 Preliminaries

The Hamiltonian functions of Problems 1 and 2 are

$$H_o(x(t), \lambda_o(t), u_o(t), t) = 1 + \lambda_o^{\mathrm{T}}(t)Ax(t) + \lambda_o^{\mathrm{T}}(t)B_o u_o(t)$$

and

$$H_e(x(t), \lambda_e(t), u_e(t), t) = 1 + \lambda_e^{\mathrm{T}}(t)Ax(t) + \lambda_e^{\mathrm{T}}(t)B_e u_e(t)$$

where $\lambda_o(t), \lambda_e(t) : [0, T] \to \mathbf{R}^n$. According to Pontryagin′s maximum principle (PMP)[16], the minimum of the Hamiltonian function of Problem 1 will satisfy:

$$\min_{u_o\in\Pi} H_o \iff \min_{u_o\in\Pi}\{\lambda_o^{\mathrm{T}}(t)B_o u_o(t)\} = \min_{u_o\in\Pi}\{q_{o1}(t)u_{o1}(t) + \cdots + q_{or}(t)u_{or}(t)\}$$

where $q_o(t) \in \mathbf{R}^r$ is the switching vector whose components are defined as:

$$q_{oi}(t) = b_{oi}^{\mathrm{T}}\lambda_o(t), \quad i = 1, \cdots, r$$

b_{oi} is the i-th column of the input matrix B_o in (1). Referring to Problem 2, it can be similarly concluded that

$$\min_{u_e\in\Pi} H_e = \min_{u_e\in\Pi}\{q_{e1}(t)u_{e1}(t) + \cdots + q_{e(r+k)}(t)u_{e(r+k)}(t)\}$$

where $q_e(t) \in \mathbf{R}^{r+k}$ is the switching vector and

$$q_{ej}(t) = b_{ej}^{\mathrm{T}}\lambda_e(t), \quad j = 1, \cdots, r+k$$

b_{ej} is the j-th column of the input matrix B_e in (3).

Then, through the switching vectors $q_o(t)$ and $q_e(t)$, the singularity of Problems 1 and 2 can be defined as follows.

Definition 1. Problem 1 (or Problem 2) is normal, if all of the components $q_{on}(t)$ (or $q_{en}(t)$) of the switching vector $q_o(t)$ (or $q_e(t)$) satisfy

$$q_{on}(t) = 0 \text{ (or } q_{on}(t) = 0)$$

only at some isolated points during the whole optimal time interval, where $n = 1, \cdots, d(u)$.

According to [17], a normal time optimal control problem will lead to a bang-bang control, in which all of the input channels work at their extreme values during any nontrivial time interval (i.e., $t_1 \neq t_2$).

Lemma 1[12]. If all the eigenvalues of the system matrix A in (1) have non-positive real parts, the optimal control for Problem 1 exists.

Lemma 2[12]. Problem 1 is normal if and only if every matrix

$$G_j = [b_j\ Ab_j\ \cdots\ A^{n-1}b_j]$$

is nonsingular, where b_j is the j-th column of matrix B_o and $j = 1, \cdots, r$.

Lemma 3[12]. If Problem 1 is normal and the optimal control exists, then the optimal control is unique.

Obviously, the conclusions in Lemmas 1 ∼ 3 about Problem 1 can be applied to Problem 2.

Lemma 4[18]. The pair (A, B) is controllable if and only if, for every choice of the set Λ, there is a matrix C such that $A + BC$ has Λ for its set of eigenvalues.

The controllability is equivalent to the property that the closed-loop transfer matrix can be assigned to an arbitrary set of poles by a suitable choice of the feedback "gain" matrix C[18].

3 Main results

Lemma 5. The redundant system (3) is controllable if the original system (1) is controllable.

However, it should be pointed out that a change of controllability may happen attributing to the input extension if the original system is uncontrollable. Although the controllability under input extension will be dependent on the property of the original system and the additional columns in B_e, a controllable system will never become uncontrollable due to the extension of the input matrix.

According to Lemmas 4 and 5, the controllability is equivalent to the pole placement and the system with input redundancy will maintain the controllability for a controllable system. Then, the property of pole placement will remain invariant after input extension, which can be expressed as follows:

Lemma 6. The eigenvalues of the closed-loop system for (3) can be assigned arbitrarily if the original system (1) is controllable.

Otherwise, since it is well known that the observability of an LTI system is only associated with the system matrix A and the output matrix C, a change of input matrix B will not change the observability.

As a result, the input extension will retain several properties such as controllability and pole placement for a controllable LTI system, while the observability is immune from this extension in the input matrix. Before presenting the main results of this paper, an important definition of idle channel is introduced as follows:

Definition 2. An input channel is called an idle channel if it is unused during the entire optimal time period, i.e. $u_i(t) \equiv 0$ for all $t \in [0, T^*]$, where $i \in \mathbf{N}$ and T^* is the optimal time.

According to Definitions 1 and 2, it is immediate to conclude that

Lemma 7. There exists non-idle channel in the time optimal control of Problem 1 (or Problem 2) if the problem is normal.

Obviously, the optimal time of Problems 1 and 2 should satisfy:

Proposition 1. For any initial state $x_0 \in \mathbf{R}^n$, it can be concluded that

$$T_e^*(x_0) \leq T_o^*(x_0)$$

where $T_o^*(x_0)$ and $T_e^*(x_0)$ are the optimal time for Problems 1 and 2, respectively.

According to this proposition, the optimal intervals for Problems 1 and 2 should satisfy:

$$[0, T_e^*(x_0)] \subset [0, T_o^*(x_0)]$$

The main contribution of this paper is to find out a condition, which guarantees a strict reduction of the optimal time due to the input redundancy, as follows:

Theorem 1. Assume that the time optimal control of Problems 1 and 2 exist, and the optimal control of Problem 2 is unique, if there exists non-idle channel among the redundant input channels, then the optimal time will satisfy:

$$T_e^*(x_0) < T_o^*(x_0)$$

for any nonzero initial state $x_0 \in \mathbf{R}^n$.

Proof. Denote $u_o^*(t)$ and $u_e^*(t)$ as the time optimal control of Problems 1 and 2, respectively. Without loss of generality, assume that there is only one redundant input channel which has been added into (3), i.e. $k = 1$, then the control with input redundancy can be rewritten as

$$u_e(t) = \begin{bmatrix} u_o(t) \\ u_{r+1}(t) \end{bmatrix} = \begin{bmatrix} u_1(t) \\ \vdots \\ u_r(t) \\ u_{r+1}(t) \end{bmatrix}$$

If $u_{r+1}(t)$ is not an idle channel in the optimal control, there exists a nontrivial time interval $[t_1, t_2]$ and $\xi \in [t_1, t_2] \subset [0, T_e^*(x_0)] \subset [0, T_o^*(x_0)]$, in which it results in

$$u_{r+1}^*(\xi) \neq 0$$

Then, even if the following equation

$$u_e(t) = \begin{bmatrix} u_o(t) \\ 0 \end{bmatrix}$$

holds during several time segments, however, the equality will not be valid during the entire $t \in [0, T_e^*(x_0)]$. Thus, $u_e^*(t)$ and $[u_o^{*\mathrm{T}}(t)\ 0]^{\mathrm{T}}$ are two different controls. Because of the uniqueness of $u_e^*(t)$, if $T_e^*(x_0) = T_o^*(x_0)$, then

$$u_e^*(t) = \begin{bmatrix} u_o^*(t) \\ 0 \end{bmatrix}$$

for all the $t \in [0, T_e^*(x_0)]$, which contradicts to the preceding discussion that $u_e^*(t)$ and $[u_o^{*\mathrm{T}}(t)\ 0]^{\mathrm{T}}$ are two different controls, thus, $T_e^*(x_0) \neq T_o^*(x_0)$. According to Proposition 1, we can conclude that

$$T_e^*(x_0) < T_o^*(x_0) \tag{4}$$

More generally, as the number of redundant input channels goes higher, i.e. $k > 1$ and $k \in \mathbf{N}$, the inequality (4) will be hold as well. □

Based on the proof of Theorem 1, if the existence and uniqueness of the optimal control for both two problems are guaranteed, a sufficient and necessary condition for $T_e^*(x_0) = T_o^*(x_0)$ is:

Corollary 1. Suppose that the time optimal control for Problems 1 and 2 exist, and the optimal time for Problem 2 is unique, then the optimal time for Problems 1 and 2 will be identical from any nonzero initial state $x_0 \in \mathbf{R}^n$, i.e.

$$T_e^*(x_0) = T_o^*(x_0)$$

if and only if all of the redundant input channels are idle channels in the optimal control of Problem 2.

The input constraint Π is illustrated in Fig. 1, where the dimension of input vector $u(t)$ is $d(u) = 2$. It is well known that if Problem 1 (or Problem 2) is normal, the optimal control will be a bang-bang control which only stands at one of the four vertices of a, b, c, d described in the figure. Otherwise, if there exists any idle channel, the corresponding optimal control will be either on the axes or at the origin point. Thus, Problem 2 is normal will imply that none of the redundant input channels is an idle channel among the optimal control. Then, it is immediate to conclude that:

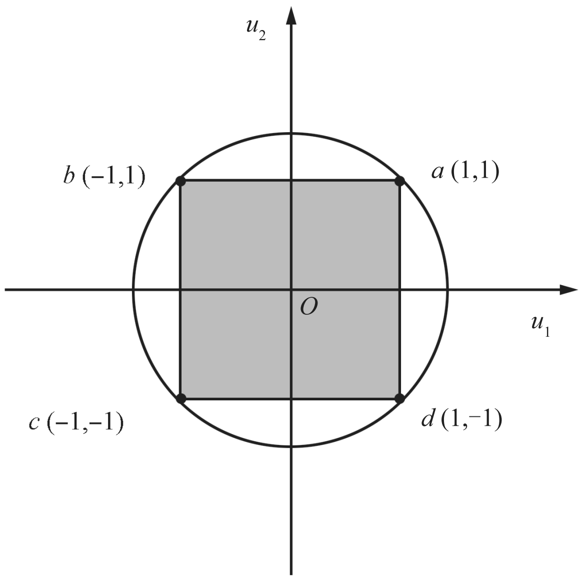

Fig. 1 The input constraint of Π as $u(t) \in \mathbf{R}^2$

Theorem 2. If all the eigenvalues of A have non-positive real parts and Problem 2 is normal, then

$$T_e^*(x_0) < T_o^*(x_0)$$

for any nonzero initial state $x_0 \in \mathbf{R}^n$.

Proof. According to Lemma 1, if all of the eigenvalues of the system matrix A have non-positive real parts, the optimal control $u_o^*(t)$ and $u_e^*(t)$ exist for Problems 1 and 2 from any initial state x_0. From Lemma 3, since the problem is normal and the optimal control exists, then the optimal control of Problem 2 is unique.

Based on the above discussions, there is not an idle channel in the optimal control for a normal problem. As a result, according to Theorem 1, it can be concluded that

$$T_e^*(x_0) < T_o^*(x_0)$$

for any nonzero initial state $x_0 \in \mathbf{R}^n$. □

Replacing the assumption of the normality of Problem 2 in Theorem 2 by that of Problem 1, yields the following corollary:

Corollary 2. If the input redundancy satisfies $B_e = [B_o, b_\alpha]^{\mathrm{T}}$, Problem 1 is normal and the optimal control exists, then

$$T_e^*(x_0) < T_o^*(x_0)$$

for any nonzero initial state $x_0 \in \mathbf{R}^n$, where b_α is one of the columns in the original input matrix B_o.

This corollary indicates that adding identical columns into the input matrix will reduce the optimal time strictly from the same initial state if the original problem is normal..

According to Lemma 2, a time optimal control problem with input constraint Π is normal if every matrix G_j is nonsingular, where $j = 1, \cdots, d(u)$. Obviously, all of G_j are nonsingular means the system is completely controllable, which is a strong condition. Next, it will be pointed out that the completely controllable is not a necessary condition leading to a smaller optimal time after input extension.

Based on the preceding discussions, a critical point to guarantee a smaller optimal time after adding input redundancy is whether or not there exists non-idle channel among the redundant input channels. Since the bang-bang control of Problem 1 (or Problem 2) is deduced by the PMP, which is a necessary but not sufficient condition for the optimal control problem, then any optimal control should satisfy the PMP. As a result, if the optimal control exists and at least one matrix G_κ is nonsingular where $\kappa \in \{r+1, \cdots, r+k\}$, the corresponding redundant input channel $u_\kappa(t) : [0, +\infty) \to \mathbf{R}^1$ is not an idle channel during the optimal time interval, which will lead to a smaller optimal time according to Theorem 1.

Theorem 3. Assume that the time optimal control of Problems 1 and 2 exist, and the optimal control of Problem 2 is unique, if there exists index κ satisfying

$$|G_\kappa| = \left|[b_\kappa \ Ab_\kappa \ \cdots \ A^{n-1}b_\kappa]\right| \neq 0$$

where $\kappa \in \{r+1, \cdots, r+k\}$, then the optimal time of these two problems satisfy:

$$T_e^*(x_0) < T_o^*(x_0)$$

from any nonzero initial state $x_0 \in \mathbf{R}^n$.

Comparing to Theorem 2 and Corollary 2, the normality of Problem 1 or 2 is omitted in this theorem, which means, even if both of the problems are singular, the effect of adding new input redundancy can be evaluated by Theorem 3.

4 Examples

Example 1. Discuss the time optimal control of a double-integral plant without input redundancy:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 \tag{5}$$

and the plant with identical input redundancy:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{6}$$

where the control satisfies the input constraint $-1 \leq u_i(t) \leq 1$ and $i = 1, 2$.

Obviously, the time optimal control problems for both (5) and (6) are normal. Since the eigenvalues of A have non-positive real parts, according to Lemmas 1 and 3, the optimal control for these two plants exist and are unique. With the input constraint Π, the isochrones of (5) and (6), S_1 and S_2, are described in Fig. 2. From the figure, S_1 is strictly contained within S_2, which means that adding the identical input redundancy into a double integral plant will lead to a smaller optimal time, which demonstrates the conclusion in Corollary 2.

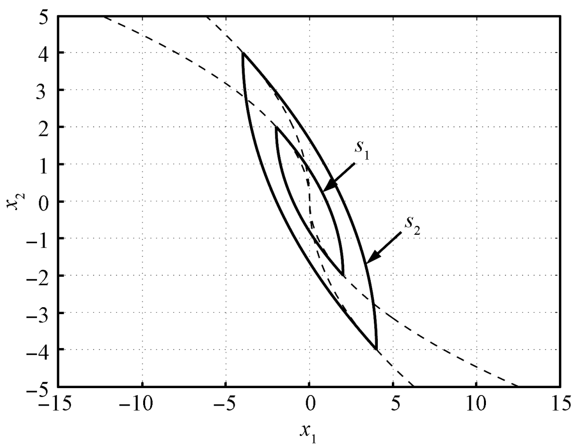

Fig. 2 Isochrones of double-integral plant with and without identical input redundancy when the optimal time is $T^* = 2$

Example 2. Compare with the time optimal control problems of the double-integral plant (DIP) and the extended double-integral plant (EDIP), where DIP is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 \tag{7}$$

and EDIP is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{8}$$

with the input constraint $-1 \leq u_i(t) \leq 1$ and $i = 1, 2$.

On one hand, according to Lemmas 1 and 3, the existence and uniqueness of the time optimal control for (7) and (8) are guaranteed. Moreover, since

$$|G_{IDP}| = |b \ \ Ab| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \neq 0$$

$$|G_{EIDP1}| = |b_1 \ \ Ab_1| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \neq 0$$

$$|G_{EIDP2}| = |b_2 \ \ Ab_2| = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \neq 0$$

from Lemma 2, the time optimal problems with (7) and (8) are normal. According to Theorem 2, the optimal time of these two plants should satisfy $T^*_{\rm EDIP}(x_0) < T^*_{\rm DIP}(x_0)$.

On the other hand, the numerical simulations of the optimal trajectories for the EDIP from different initial states are illustrated in Fig. 3, where the dashed line represents a switching curve. The optimal control switches from $u^* = [+1,+1]^{\rm T}$ to $u^* = [-1,+1]^{\rm T}$ on the dashed line if $x_1 < 0$, while on the rest part of the dashed line, the optimal control switches from $u^* = [-1,-1]^{\rm T}$ to $u^* = [+1,-1]^{\rm T}$. Meanwhile, the dash-dotted line is another switching curve, where the optimal control changes from $u^* = [-1,+1]^{\rm T}$ to $u^* = [-1,-1]^{\rm T}$ on it if $x_2 > 0$, otherwise, the optimal control changes from $u^* = [+1,-1]^{\rm T}$ to $u^* = [+1,+1]^{\rm T}$ on it. Finally, the optimal trajectories go along either the dash-dotted line or the horizontal part of the dashed line to the origin point.

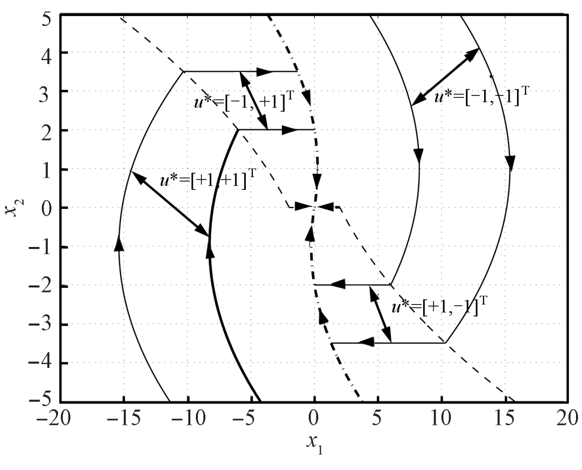

Fig. 3 The optimal trajectories for the EDIP

Fig. 4 illustrates the isochrones of the EDIP and the DIP when the optimal time is $T^* = 2\,{\rm s}$. In this figure, the isochrone of the EDIP strictly contains the one of the DIP, which means that the optimal time of the EDIP is smaller than the one of the DIP for any nonzero initial states x_0, i.e. $T^*_{\rm EDIP}(x_0) < T^*_{\rm DIP}(x_0)$.

As a result, the numerical results in this section demonstrate the theoretical conclusions in the preceding section.

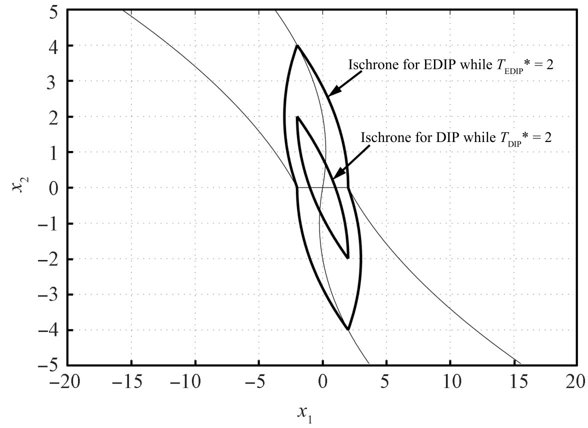

Fig. 4 The comparison of isochrones between the DIP and the EDIP while the optimal time is $T^* = 2\,{\rm s}$

5 Conclusions

In this paper, the constrained time optimal control problem with input redundancy is investigated. Several sufficient conditions lead to a smaller optimal time for the system with input redundancy than the one without redundancy. A concept of idle channel is introduced, which represents the channel never being used during the entire optimal time interval. Then we prove that the optimal time for the redundant system is shorter than the one of the original system if there exists non-idle channel among the redundant input channels. According to a widely discussed input constraint Π, the normality of the extended system will guarantee a smaller optimal time for the system with input redundancy if the optimal control exists and it is unique. Moreover, if the input redundancy only consists of the columns from the original input matrix B_o and the original system is normal, then the optimal time becomes smaller and smaller as the input redundancy increases. According to Lemma 2, the time optimal control problem with input constraint Π is normal if and only if all the matrices G_j are nonsingular, which is a strong condition. To weaken this condition, we point out that only one redundant input channel is completely controllable will also guarantee a smaller optimal time. At last, two numerical examples about the double integral plants with and without input redundancy have been employed to demonstrate the main results of the paper, where the isochrones of the original systems are strictly contained by the ones of the systems with input redundancy.

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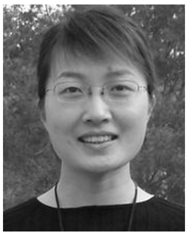

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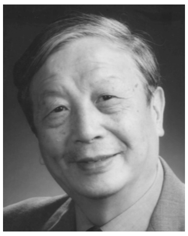

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