

Nuclear matter and chiral phase transition at large- N_c

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Two aspects of the QCD phase diagrams are studied in the limit of a large number of colors: at zero temperature and nonzero density the (non)existence of nuclear matter, and at zero density and nonzero temperature the chiral phase transition.

1 Introduction and Summary

The limit in which the number of colors N_c is sent to infinity (large- N_c limit) represents a systematic approach¹ to study properties of QCD. The world for $N_c \gg 3$ is simpler because planar diagrams dominate. However, the basic ingredients ‘survive’ in the large- N_c limit: quark-antiquark mesons exist and become weakly interacting, baryons also exist but are formed of N_c quarks. Recently, a lot of effort has been spent to study the properties of the phase diagram of QCD when N_c is varied².

Along the line of zero temperature and nonzero chemical potential, a natural question³ is if nuclear matter binds for $N_c > 3$. We shall find that this is not the case: in view of the peculiar nature of the scalar attraction between nuclei we obtain that nuclear matter ceases to form as soon as $N_c > 3$ is considered. Namely, the scaling behavior of the scalar attraction depends on the nature of the exchanged field with a mass of about 0.6 GeV. Present knowledge in low-energy QCD spectroscopy⁴ shows that this light scalar field is (predominately) not a quark-antiquark field, the alternative possibilities being tetraquark, pion-pion interpolating field, molecular state, etc. In all these interpretations the scalar attraction diminishes in comparison with the vector repulsion, mediated by the well-known vector meson ω , when N_c is increased. As a result, nuclear matter does not take place³: the investigation leading to this result is achieved though a simple effective model of the Walecka type.

When moving along the finite temperature axis while keeping the density to zero, it is interesting to study how different chiral effective models behave at large- N_c . It is quite remarkable that two very well-known models, the quark-based Nambu Jona-Lasinio (NJL) model^{5,6} and the hadron-based σ -model^{7,8}, deliver different result for the critical temperature for chiral restoration T_c . While in the NJL model T_c scales as N_c^0 and is thus, just as the deconfinement phase transition, large- N_c independent, in the σ -model one obtains that $T_c \propto \sqrt{N_c}$. This mismatch can be solved by including in the σ -model one (or more) T -dependent parameter(s): a rather simple modification of the mass term is enough to reobtain the expected scaling $T_c \propto N_c^0$.

The paper^a is organized as follows: in Sec. 2 and Sec. 3 we study nuclear matter and the

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chiral phase transition for $N_c > 3$, respectively. In Sec. 4 we briefly present our conclusions.

2 Nuclear matter at large- N_c

Nuclear matter at large- N_c is studied by means of an effective Walecka-Lagrangian⁹

$$\mathcal{L} = \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - (m_N - g_S S)]\psi + \frac{1}{2}\partial^\mu S\partial_\mu S - \frac{1}{2}m_S^2 S^2 + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + \dots \quad (1)$$

where S represents a scalar field with a mass of about 0.6 GeV and ω the isoscalar vector meson. The large- N_c scaling properties of the latter are well known: $m_\omega \propto N_c^0$, $g_\omega \propto \sqrt{N_c}$. We now examine the possibilities³ for the scalar state S :

- S as quark-antiquark field: $m_S \propto N_c^0$ and $g_S \propto \sqrt{N_c}$. This is the only case in which nuclear matter exists in the large- N_c limit. The binding energy increases with N_c . However, this scenario is –as previously anticipated– unfavored⁴.

- S as tetraquark field¹⁰: $m_S \propto N_c$ and $g_S \propto N_c^0$. Nuclear matter does not bind for $N_c > 3$. On the contrary, for $N_c = 2$ an increased binding is found. This scenario represents a viable possibility in agreement with phenomenology. It might also play an important role at nonzero temperature and density¹¹.

- S as an effective two-pion-exchange effect¹²: $m_S \sim 2m_\pi \propto N_c^0$, $g_S \propto \sqrt{N_c}$. Although the scaling laws are the same as in the quark-antiquark case, no binding is obtained in view of numerical details.

- S as a low-mass scalar glueball¹³: $m_S \propto N_c^0$ and $g_S \propto N_c^0$. No binding for $N_c > 3$ is obtained. Note, this scenario is unfavored by present lattice data which place the glueball at about 1.6 GeV¹⁴.

The result that no nuclear matter exists for large- N_c is stable and does not depend on numerical details. In the framework of the so-called strong anthropic principle it is then natural that we live in a world in which N_c is not large.

3 Chiral phase transition at large- N_c

The σ -model has been widely used to study the thermodynamics of QCD¹⁵. In one of its simplest forms it reads (as function of N_c):

$$\mathcal{L}_\sigma(N_c) = \frac{1}{2}(\partial_\mu\Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4, \quad (2)$$

where $\Phi^t = (\sigma, \vec{\pi})$ describes the scalar field σ and the pseudoscalar pion triplet $\vec{\pi}$. The quark-antiquark field σ represents the chiral partner of the pion: as mentioned in the previous section, it does not correspond to the resonance $f_0(600)$ with a mass of about 0.6 GeV but to the resonance $f_0(1370)$ with a mass of about 1.3 GeV^{4,8}. The scaling law $\lambda \rightarrow 3\lambda/N_c$ takes into account that the meson-meson scattering amplitude scales as N_c^{-1} . On the contrary, μ^2 contains no dependence on N_c : in this way the quark-antiquark meson masses scales –as desired– as N_c^0 .

The critical temperature T_c for the chiral phase restoration is calculated by using the so-called CJT formalism¹⁶, which is a self-consistent resummation scheme for field theoretical calculations at nonzero temperature. In the Hartree and in the double-bubble approximation T_c is given by the expression

$$T_c(N_c) = f_\pi\sqrt{2\frac{N_c}{3}} \propto \sqrt{N_c}, \quad (3)$$

where $f_\pi = 92.4$ MeV is the pion decay constant. The scaling of T_c is thus in disagreement with the NJL model⁶ where $T_c \propto N_c^0$ and with basic expectations². This result is due to the

fact that for $N_c \gg 3$ a gas of free mesons is realized and thus no transition takes place. In fact, the mechanism responsible for the restoration of chiral symmetry in hadronic models is given by mesonic loops, whose effect vanishes for $N_c \gg 3$. On the contrary, in the NJL model the restoration of chiral symmetry is generated by the quark loops, which do not vanish in the large- N_c limit.

The inconsistency between the NJL model and the σ -model can be easily solved by replacing

$$\mu^2 \rightarrow \mu(T)^2 = \mu^2 \left(1 - \frac{T^2}{T_0^2}\right) \quad (4)$$

(i.e., making it T -dependent) where the parameter $T_0 \simeq \Lambda_{QCD} \propto N_c^0$ introduces a new temperature scale. This is in line with the fact that the σ -model can be obtained by hadronization of the NJL model. In this scheme the parameters of the σ -model turn out to be temperature-dependent. Note also that the here considered T^2 -behavior –although naive at the first sight– has been also obtained in Ref. ¹⁷. In this way the critical temperature is modified to

$$T_c(N_c) = T_0 \left(1 + \frac{1}{2} \frac{T_0^2}{f_\pi^2} \frac{3}{N_c}\right)^{-1/2} \propto N_c^0, \quad (5)$$

which is now large- N_c independent, just as in the NJL case. For $N_c = 3$, using $T_0 = \Lambda_{QCD} \simeq 225$ MeV, the critical temperature T_c is lowered to $T_c \simeq 113$ MeV. Interestingly, in the framework of σ -models with (axial-)vector mesons ⁸, one has to make the replacement $f_\pi \rightarrow Z f_\pi$ with $Z = 1.67 \pm 0.2$. In this way the critical temperature reads $T_c \simeq 157$ MeV, which is remarkably close to the lattice results ¹⁸.

Beyond the phenomenologically motivated modification presented here, one can go further and couple the present σ -model (and generalizations thereof) to the Polyakov loop ¹⁹. Also in this case ²⁰ the critical temperature turns out to be, as desired, independent on N_c . The reason for this behavior can be traced back to the fact that the transition of the Polyakov loop (which describes the confinement-deconfinement phase transition) triggers also the restoration of chiral symmetry.

4 Conclusions

In this work we have investigated the properties of nuclear matter and chiral phase transition in the large- N_c limit.

We have found that present knowledge on the spectroscopy of scalar mesons indicates that nuclear matter does not bind for $N_c > 3$. Namely, the nucleon-nucleon attraction in the scalar-isoscalar channel turns out not to be strong enough to bind nuclei when N_c is increased. Therefore, nuclear matter seems to be a peculiar property of our $N_c = 3$ world.

For what concerns the chiral phase transition at nonzero temperature and zero density, we have found that care is needed when using effective hadronic models of the σ -type. The critical temperature T_c does not scale as expected in the large- N_c limit. It is however possible to introduce simple modifications of chiral hadronic models in such a way that the expected result $T_c \propto N_c^0$ is recovered.

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