On the Determination of $\bar{d}/\bar{u}$ Ratios from Proton-Proton and Proton-Deuteron Drell-Yan processes

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Abstract

By using the convolution formula derived within the framework of relativistic quantum mechanics, we have examined the Fermi Motion effects on the ratios $R_{pd/pp} = \sigma^{pd}/(2\sigma^{pp})$ between the proton-proton (p-p) and proton-deuteron (p-d) Drell-Yan cross sections. We have found that in the small $x_2 < 0.3$ region, the Fermi Motion effect is less than 1% and our results for the ratios $R_{pd/pp} = \sigma^{pd}/(2\sigma^{pp})$ agree well with the data at 800 GeV. In the large Bjorken $x_2 >$ about 0.4 region, the p-d Drell-Yan cross sections can be influenced strongly by the Fermi motion effect. At 120 GeV the predicted Fermi Motion effect can enhance the ratios $R_{pd/pp}$ by about 20 % at $x_2 \sim 0.6$ and about a factor of 2.5 at $x_2 \to 1.0$. Our results suggest that the Fermi Motion effect, along with other possible nuclear effects, must be included, in extracting the $\bar{d}/\bar{u}$ ratios in the proton from the experiments on p-p and p-d Drell-Yan processes at large x_2.

PACS numbers:

I. INTRODUCTION

Since the asymmetry between the anti-up($\bar{u}$) quark and anti-down ($\bar{d}$) quark distributions in the proton was revealed by the New Muon Collaboration[1] (NMC), a series of experiments[2–5] on the di-muons ($\mu^+\mu^-$) production from the Drell-Yan[6] (DY) processes in $p+p$ and $p+d$ collisions had been performed at Fermi National Accelerator Laboratory (Fermi Laboratory). The objective was to extract the $\bar{d}/\bar{u}$ ratio of the parton distribution functions (PDF) in the proton. The information from these experiments and the measurements[1, 7, 8] of deep inelastic scattering (DIS) of leptons from the nucleon have confirmed the NMC's finding, $\bar{d}/\bar{u} > 1$, in the region of low Bjorkin $x \leq$ about 0.3.

The ratio $\bar{d}/\bar{u} > 1$ signals the nonperturbative nature of the sea of the proton. Its dynamical origins have been investigated[9–23] rather extensively. Precise experimental determination of $\bar{d}/\bar{u}$ for higher $x > 0.3$ is needed to distinguish more decisively these models and to develop a deeper understanding of the sea of the proton. This information will soon become available from a forthcoming experiment[24] at Fermi Laboratory.

It is instructive to describe here how the p-d DY data were analyzed. In the analysis of the data of Ref.[5], the leading-order DY cross section of the $p+N$ collision with $N=p$ (proton), n (neutron) is written s

$$\frac{d\sigma^{pN}}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9M^2}\sum_q e_q^2[f_p^q(x_1)f_N^{\bar{q}}(x_2) + f_p^{\bar{q}}(x_1)f_N^{\bar{q}}(x_2)]\,, \tag{1}$$

where the sum is over all quark flavors, $\alpha = 1/137$, e_q is the quark charge, $f_N^q(x)$ is the parton distribution of parton q in hadron N, and M is the virtual photon or di-lepton mass. Here x_1 and x_2 are the Bjorken-x of partons from the beam (p) and target (N), respectively. The DY cross section for $p+d$ is simply taken as

$$\frac{d\sigma^{pd}}{dx_1 dx_2} = \frac{d\sigma^{pp}}{dx_1 dx_2} + \frac{\sigma^{pn}}{dx_1 dx_2}\,. \tag{2}$$

Clearly the effects due to the nucleon Fermi Motion in the deuteron are not included in Eq.(2). The use of Eq.(2) seems valid to a very large extent in the small x_2 region since the available data on nuclei indicate that DY cross section per nucleon is rather independent of the nuclear mass number A. However it is not clear whether it can be used to extract the $\bar{d}/\bar{u}$ ratios in the proton from the upcoming experiments at large x_2.

In the DIS studies[30], it is well recognized that the nuclear effects must be considered in extracting the parton distributions of the nucleon from the data. In particular it is mandatory to include nucleon Fermi motion effects by using various forms of convolution formula to express the DIS cross sections in terms of nucleon momentum distributions and parton distribution functions. In contrast, there exists very limited efforts to include the similar nuclear effects in analyzing the DY data on the deuteron[2–5] and nuclei[25–29]. As a step to improve the situation, it is necessary to investigate under what assumptions Eqs.(1)-(2) can be derived from a formulation within which the effects due to the internal motions of partons in the nucleon and nucleons in the deuteron can be defined rigorously and consistently. This is the main objective of this work.

We will develop convolution formula for calculating the p-d DY cross sections within the relativistic quantum mechanics proposed by Dirac[31], as reviewed by Keister and Polyzou[32]. The same theoretical framework was taken in the studies of electron-deuteron scattering[33, 34], electron-^3He scattering[35], and DIS on deuteron[36]. There exists convolution formula for DY processes on nuclei , as given, for example, in Ref.[25]. However it is not clear how those formula can be related to the relativistic formulation considered in our approach. To explain clearly the content of our approach, we thus will give a rather elementary derivation of our formula with all approximations specified explicitly. We will apply our formula to analyze the available data at 800 GeV[5] and make predictions for the forthcoming experiment[24].

In section II, we start with the general covariant form of the DY cross section and indicate the procedures needed to obtain the well known $q\bar{q} \to \mu^+\mu^-$ cross section $\sigma^{q\bar{q}}$. The same procedures are used in section III to derive formula for calculating the $p+N$ DY cross sections from $\sigma^{q\bar{q}}$ and the properly defined parton distribution functions f^q_N of the nucleon. In section IV, we use the impulse approximation to derive the convolution formula for calculating $p+d$ DY cross sections from $\sigma^{q\bar{q}}$, f^q_N, and the nucleon momentum distribution $\rho_{p_d}(\vec{p}_N)$ of a fast moving deuteron with momentum p_d.

In section V, we describe the procedures for applying the developed formula to perform numerical calculations of $p+p$ and $p+d$ DY cross sections using the available parton distributions[37–41] and realistic deuteron wavefunctions[42–45]. The results are presented in section VI. We will compare our results with the available data at 800 GeV[5] and make predictions at 120 GeV for analyzing the forthcoming experiment[24]. A summary and discussions on necessary future improvements are given in section VII.

II. COVARIANT FORMULA FOR DRELL-YAN CROSS SECTIONS

The formula presented in this section are derived from using the Bjorken-Drell[46] conventions for the Dirac matrices and the field operators for Fermions and photons. To use the formula of Relativistic Quantum Mechanics given in Ref.[32], the plane-wave state $|\vec{k}>$ is normalized as $<\vec{k}|\vec{k}'>=\delta(\vec{k}-\vec{k}')$ and the bound states $|\Phi_\alpha>$ of composite particles, nucleons or nuclei, are normalized as $<\Phi_\alpha|\Phi_\beta>=\delta_{\alpha,\beta}$. To simplify the presentation, spin indices are suppressed; i.e. $|\vec{k}_a>$ represents $|\vec{k}_a,\lambda_a>$ for a particle a with helicity λ_a. Thus the formula presented here are only for the spin averaged cross sections which are the focus of this paper.

We consider the di-muons production from the DY processes of hadron (h) - hadron (T) collisions:

$$h(p_h) + T(p_T) \to \mu^+(k_+) + \mu^-(k_-) + X_h(p_{x_h}) + X_T(p_{x_T})\,, \tag{3}$$

where X_h and X_T are the undetected fragments, and the four-momentum of each particle is given within the parenthesis. In terms of the partonic $q\bar{q} \to \gamma \to \mu^+ + \mu^-$ mechanism, illustrated in Fig.1, the covariant form of the di-muons production cross section can be written as

$$d\sigma = \frac{(2\pi)^4}{4[(p_h\cdot p_T)^2 - m_h^2 m_T^2]^{1/2}} \frac{1}{(2\pi)^6} \frac{d\vec{k}_+}{2E_+} \frac{d\vec{k}_-}{2E_-} \frac{1}{q^4} f^{\mu\nu}(k_+,k_-) F_{\mu\nu}(p_h,p_T,q)\,, \tag{4}$$

where m_h and m_T are the masses for h and T, respectively, $E_\pm = [\vec{k}_\pm^2 + m_\mu^2]^{1/2}$ are the energies of muons $\mu^\pm$, and $q = k_+ + k_-$ is the momentum of the virtual photon. The leptonic tensor is defined by

$$f^{\mu\nu}(k_+,k_-) = (2\pi)^6(2E_+)(2E_-) <\vec{k}_+\vec{k}_-|j^\mu(0)|0><0|j^\nu(0)|\vec{k}_+\vec{k}_->\ . \tag{5}$$

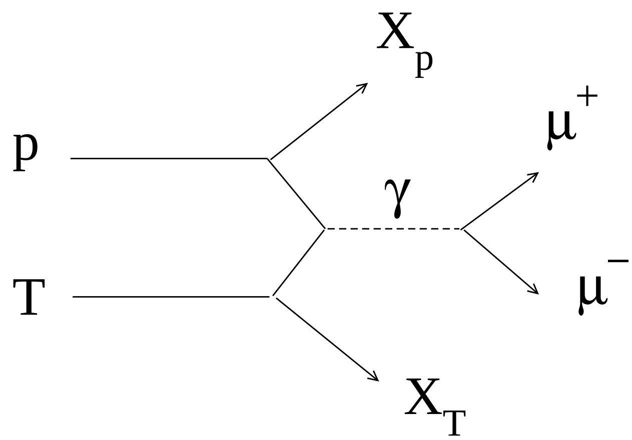

FIG. 1: Drell-Yan process.

Here the leptonic current is

$$j^\mu(x) = e\bar{\psi}_\mu(x)\gamma^\mu\psi_\mu(x)\,, \tag{6}$$

where $\psi_\mu(x)$ is the field operator for muon, and $e = \sqrt{4\pi\alpha}$ with $\alpha = 1/137$. By using the definitions Eqs.(5)-(6), it is straightforward to get the following analytic form of the lepton tensor

$$f^{\mu\nu}(k_+, k_-) \;=\; -4e^2[k_+^\mu k_-^\nu + k_+^\nu k^\mu - g^{\mu\nu}(k_+\cdot k_- + m_\mu^2)]\,. \tag{7}$$

Within the parton model, the hadronic tensor in Eq.(4) is determined by the current $J_\mu(x)$ carried by partons q or $\bar{q}$

$$\begin{aligned} F_{\mu\nu}(p_h, p_T, q) \;=\; & \sum_{x_h, x_T}(2\pi)^6(2E_h)(2E_T)\int d\vec{p}_{x_h}d\vec{p}_{x_T}\delta^4(p_h + p_T - p_{x_h} - p_{x_T} - q) \\ & \times < p_h p_T|J_\mu(0)|\vec{p}_{x_h}d\vec{p}_{x_T} >< \vec{p}_{x_T}\vec{p}_{x_h}|J_\nu(0)|p_h p_T >\,, \end{aligned} \tag{8}$$

where

$$J_\mu(x) = \sum_q [\hat{e}_q e]\bar{\psi}_q(x)\gamma^\mu\psi_q(x)\,. \tag{9}$$

Here $\psi_q(x)$ is the field operator for a quark q with charge $\hat{e}_q e$; i.e $\hat{e}_u = \frac{2}{3}$ and $\hat{e}_d = -\frac{1}{3}$ for the up and down quarks, respectively.

The above covariant expressions are convenient for deriving the formula which can express the hadron-hadron DY cross sections in terms of the elementary partonic $q\bar{q} \to \mu^+\mu^-$ cross sections. To get such formula, we first show how the elementary $q\bar{q} \to \mu^+\mu^-$ cross section can be derived from Eq.(4) with $h = q$ and $T = \bar{q}$. Explicitly, Eq.(4) for the $q(p_q) + \bar{q}(p_{\bar{q}}) \to \mu^+(k_+) + \mu^-(k_-)$ process is

$$d\sigma^{q\bar{q}} = \frac{(2\pi)^4}{4[(p_q\cdot p_{\bar{q}})^2 - m_q^4]^{1/2}}\{\frac{1}{(2\pi)^6}\frac{d\vec{k}_+}{2E_+}\frac{d\vec{k}_-}{2E_-}\frac{1}{q^4}f^{\mu\nu}(k_+, k_-)F_{\mu\nu}^{q\bar{q}}(p_q, p_{\bar{q}}, q)\}\,. \tag{10}$$

The next step is replace the intermediate states $|\vec{p}_{x_h}\vec{p}_{x_T} >$ by the the vacuum state $|0>$ in evaluating the hadronic tensor Eq.(8). We thus have

$$\begin{aligned} F^{q\bar{q}}_{\mu\nu}(p_q, p_{\bar{q}}, q) &= (2\pi)^6(2E_q)(2E_{\bar{q}}) \\ &\times < p_{\bar{q}}p_q|J_\mu(0)|0 >< 0|J_\nu(0)|p_q, p_{\bar{q}} > \delta^4(p_q + p_{\bar{q}} - q)\,. \end{aligned} \tag{11}$$

Substituting parton current Eq.(9) into Eq.(11), the hadronic tensor $F^{q\bar{q}}_{\mu\nu}$ then has a form which is the same as the leptonic tensor $f^{\mu\nu}$ defined by Eqs.(5)-(6) except that the momentum variables and the charges associated with the Fermion field operators are different. By appropriately changing the momentum variables in Eq.(7), we obtain

$$F^{q\bar{q}}_{\mu\nu}(p_q, p_{\bar{q}}, q) = -4[\hat{e}_q e]^2[p^\mu_q p^\nu_{\bar{q}} + p^\nu_q p^\mu_{\bar{q}} - g^{\mu\nu}(p_q \cdot p_{\bar{q}} + m^2_q)]\delta^4(p_q + p_{\bar{q}} - q)\,. \tag{12}$$

By using Eqs.(7) and (12), Eq.(10) for the cross sections of $q(p_q) + \bar{q}(p_{\bar{q}}) \to \mu^+(k_+) + \mu^-(k_-)$ can then be written as

$$\begin{aligned} d\sigma^{q\bar{q}}(p_q, p_{\bar{q}}) &= \frac{(2\pi)^4}{4[(p_q \cdot p_{\bar{q}})^2 - m^4_q]^{1/2}} \frac{1}{(2\pi)^6} \frac{d\vec{k}_+}{2E_+} \frac{d\vec{k}_-}{2E_-} \frac{1}{q^4} \delta^4(p_q + p_{\bar{q}} - q) \\ &\times 8[k_+ \cdot p_q k_- \cdot p_{\bar{q}} + k_- \cdot p_q k_+ \cdot p_{\bar{q}} + m^2_q \frac{(k_+ - k_-)^2}{2} + m^2_\mu \frac{(p_q - p_{\bar{q}})^2}{2}]\,. \end{aligned} \tag{13}$$

It is convenient to express the q-$\bar{q}$ DY cross section in terms of the invariant function $q^2 = (p_q + p_{\bar{q}})^2 = (k_+ + k_-)^2$. After some derivations and accounting for the color degrees of freedom of quarks, we obtain

$$\frac{d\sigma^{q\bar{q}}(p_q, p_{\bar{q}})}{dq^2} = \frac{4\pi\alpha^2}{q^2} \hat{e}^2_q \frac{1}{3N_c} \frac{[q^2 - m^2_\mu/4]^{1/2}}{[q^2 - m^2_q/4]^{1/2}} \delta(q^2 - (p_q + p_{\bar{q}})^2)\,, \tag{14}$$

where N_c is the number of colors. Taking $N_c = 3$ and considering $q^2 >> m^2_\mu$ and $q^2 >> m^2_q$, we then obtain the familiar form

$$\frac{d\sigma^{q\bar{q}}(p_q, p_{\bar{q}})}{dq^2} = \frac{4\pi\alpha^2}{q^2} \hat{e}^2_q \frac{1}{9} \delta(q^2 - (p_q + p_{\bar{q}})^2)\,. \tag{15}$$

The above expression is identical to the commonly used expression, as given, for example, in Ref.[41].

In the next two sections, we will derive formula expressing the p-N and p-d DY cross sections in terms of $d\sigma^{q\bar{q}}(p_q, p_{\bar{q}})/dq^2$ given in Eq.(15). To simplify the presentation, we only present formula for q in the projectile p and $\bar{q}$ in the target N or d. The term from interchanging $q \leftrightarrow \bar{q}$ will be included only in the final expressions for calculations.

III. p-N DY CROSS SECTIONS

Eq.(4) for the $p(p_1) + N(p_2) \to \mu^+(k_+) + \mu^-(k_-) + X_p(p_{X_p}) + X_N(p_{X_N})$ process is

$$d\sigma^{pN} = \frac{(2\pi)^4}{4[(p \cdot p_N)^2 - m^2_p m^2_N]^{1/2}} \{ \frac{1}{(2\pi)^6} \frac{d\vec{k}_+}{2E_+} \frac{d\vec{k}_-}{2E_-} \frac{1}{q^4} f^{\mu\nu}(k_+, k_-) F^{pN}_{\mu\nu}(p_p, p_N, q) \}\,, \tag{16}$$

where the hadronic tensor, defined by Eq.(8), is

$$F_{\mu\nu}^{pN}(p_p,p_N,q) \; = \; (2\pi)^6(2E_p)(2E_N)\{\sum_{X_p,X_N}\int d\vec{p}_{X_p}d\vec{p}_{X_N}\delta^4(p_p+p_N-p_{X_p}-p_{X_N}-q) \times <p_Np_p|J_\mu(0)|p_{X_p}p_{X_N}><p_{X_N}p_{X_p}|J_\nu(0)|p_pp_N>\}\,. \tag{17}$$

Within the parton model, the DY cross sections are calculated from the matrix element $<q\bar{q}|J_\mu(0)|0><0|J_\nu(0)|q\bar{q}>$ which is part of the matrix element describing the annihilation of a q ($\bar{q}$) from the projectile p and a $\bar{q}$ (q) from the target N into a photon. To identify such matrix elements, we insert a complete set of $q\bar{q}$ states

$$1=\int d\vec{p}_qd\vec{p}_{\bar{q}}\;|\vec{p}_q\vec{p}_{\bar{q}}><\vec{p}_{\bar{q}}\vec{p}_q|$$

between $<p_Np_p|$ and $J_\mu(0)$ ($J_\nu(0)$ and $|p_pp_N>$) in Eq.(17) and neglect any electromagnetic contribution from the undetected fragments X_p and X_T. We then have

$$\begin{aligned} F_{\mu\nu}^{pN}(p_p,p_N,q) \; = \; & (2\pi)^6(2E_p)(2E_N)\sum_{X_p,X_N}\int d\vec{p}_{X_p}d\vec{p}_{X_N}\delta^4(p_p+p_N-p_{X_p}-p_{X_N}-q) \\ & \times\int d\vec{p}_qd\vec{p}_{\bar{q}}\int d\vec{p}^{\,'}_qd\vec{p}^{\,'}_{\bar{q}}<\vec{p}_p|\vec{p}_q\vec{p}_{X_p}><\vec{p}_N|\vec{p}_{\bar{q}}\vec{p}_{X_N}><\vec{p}_{X_p}\vec{p}^{\,'}_q|\vec{p}_p><\vec{p}_{X_N}\vec{p}^{\,'}_{\bar{q}}|\vec{p}_N> \\ & \times<\vec{p}_{\bar{q}}\vec{p}_q|J_\mu(0)|0><0|J_\nu(0)|\vec{p}^{\,'}_q\vec{p}^{\,'}_{\bar{q}}>\;. \end{aligned} \tag{18}$$

By momentum conservation, the overlap functions in the above equation can be written as

$$\begin{aligned} <\vec{p}_q\vec{p}_X|\vec{p}_p> \; &= \; <\vec{p}_X|b_{\vec{p}_q}|\vec{p}_p> \\ &= \; \phi_{\vec{p}_p}(\vec{p}_q,\vec{p}_X)\delta(\vec{p}_p-\vec{p}_q-\vec{p}_X)\,, \end{aligned} \tag{19}$$

where $b_{\vec{p}_q}$ is the annihilation operator of quark q. By using the definition Eq.(19), Eq.(18) can be written as

$$\begin{aligned} F_{\mu\nu}^{pN}(p_p,p_N,q) \; = \; & (2\pi)^6(2E_p)(2E_N)\sum_{X_P,X_N}\int d\vec{p}_{X_p}d\vec{p}_{X_N}\delta^4(p_p+p_N-p_{X_p}-p_{X_N}-q) \\ & \times[\int d\vec{p}_q|\phi_{\vec{p}_p}(\vec{p}_q,\vec{p}_{X_p})|^2\delta(\vec{p}_p-\vec{p}_q-\vec{p}_{X_p})] \\ & \times[\int d\vec{p}_{\bar{q}}|\phi_{\vec{p}_N}(\vec{p}_{\bar{q}},\vec{p}_{X_N})|^2\delta(\vec{p}_N-\vec{p}_{\bar{q}}-\vec{p}_{X_N})] \\ & \times<\vec{p}_{\bar{q}}\vec{p}_q|J_\mu(0)|0><0|J_\nu(0)|\vec{p}_q\vec{p}_{\bar{q}}> \end{aligned} \tag{20}$$

The evaluation of the expression Eq.(20) needs rather detailed information about the undetected fragments X_p and X_N because of the dependence of $\delta^4(p_p+p_N-p_{X_p}-p_{X_N}-q)$ on their energies $p^0_{X_p}$ and $p^0_{X_N}$. To simplify the calculation, we follow the common practice to neglect the explicit dependence of the energies $p^0_{X_p}$ and $p^0_{X_N}$ of the undetected fragments. This amounts to neglecting the binding effects by setting $p^0_{X_p}\sim\epsilon_1$ and $p^0_{X_N}\sim\epsilon_2$ to write

$$\delta^4(p_p+p_N-p_{X_p}-p_{X_N}-q)\sim\delta^3(\vec{p}_p+\vec{p}_N-\vec{p}_{X_p}-\vec{p}_{X_N}-\vec{q})\delta(p^0_p+p^0_N-\epsilon_1-\epsilon_2-q^0)\,, \tag{21}$$

where ϵ_1 and ϵ_2 are constants.

We now define

$$F^{q}_{\vec{p}_p}(\vec{p}_q)=\sum_{X_p}\int d\vec{p}_{X_p}|\phi_{\vec{p}_p}(\vec{p}_q,\vec{p}_{X_p})|^2\delta(\vec{p}_p-\vec{p}_q-\vec{p}_{X_p}) \tag{22}$$

for the projectile p, and

$$F^{\bar{q}}_{\vec{p}_N}(\vec{p}_{\bar{q}})=\sum_{X_N}\int d\vec{p}_{X_N}|\phi_{\vec{p}_N}(\vec{p}_{\bar{q}},\vec{p}_{X_N})|^2\delta(\vec{p}_N-\vec{p}_{\bar{q}}-\vec{p}_{X_N}) \tag{23}$$

for the target N. These two definitions and the approximation Eq.(21) allow us to cast Eq.(20) into the following form

$$\begin{aligned}F^{pN}_{\mu\nu}(p_p,p_N,q) &= (2\pi)^6(2E_p)(2E_N)\sum_q\int d\vec{p}_q d\vec{p}_{\bar{q}}[F^{q}_{p_p}(p_q)F^{\bar{q}}_{p_N}(p_{\bar{q}})] \\ &\quad\times[<\vec{p}_{\bar{q}}\vec{p}_q|J_\mu(0)|0><0|J_\nu(0)|\vec{p}_q\vec{p}_{\bar{q}}> \\ &\quad\times\delta^3(\vec{p}_q+\vec{p}_{\bar{q}}-\vec{q}))\delta(p^0_p+p^0_N-\epsilon_1-\epsilon_2-q^0)]\end{aligned} \tag{24}$$

We next make a reasonable approximation that the average energy ϵ_1 (ϵ_2) in Eq.(24) is the difference between the energy of the projectile (p) (target N) and the removed parton q ($\bar{q}$); namely assuming

$$\begin{aligned}\delta(p^0_p+p^0_N-\epsilon_1-\epsilon_2-q^0) &= \delta((p^0_p-\epsilon_1)+(p^0_N-\epsilon_2)-q^0) \\ &\sim \delta(p^0_q+p^0_{\bar{q}}-q^0)\,.\end{aligned} \tag{25}$$

Then Eq.(24) can be written as

$$\begin{aligned}F^{pN}_{\mu\nu}(p_p,p_N,q) &= \sum_q\int d\vec{p}_q d\vec{p}_{\bar{q}}[F^{q}_{p_p}(p_q)F^{\bar{q}}_{p_N}(p_{\bar{q}})]\frac{E_pE_N}{E_qE_{\bar{q}}} \\ &\quad\times\{(2\pi)^6(2E_q)(2E_{\bar{q}})<\vec{p}_{\bar{q}}\vec{p}_q|J_\mu(0)|0><0|J_\nu(0)|\vec{p}_q\vec{p}_{\bar{q}}>\delta^4(p_q+p_{\bar{q}}-q)\}\,.\end{aligned} \tag{26}$$

The quantity within the bracket $\{...\}$ in the above equation is just the hadronic tensor $F^{qq}_{\mu\nu}(p_q,p_{\bar{q}})$, defined in Eq.(11), for the $q\bar{q}$ system. We thus have

$$F^{pN}_{\mu\nu}(p_p,p_N,q) = \sum_q\int d\vec{p}_q d\vec{p}_{\bar{q}}[F^{q}_{p_p}(p_q)F^{\bar{q}}_{p_N}(p_{\bar{q}})]\frac{E_pE_N}{E_qE_{\bar{q}}}F^{qq}_{\mu\nu}(p_q,p_{\bar{q}})\,. \tag{27}$$

Note that the above simple expression is due to the use of the approximations Eqs.(21) and (25). The binding effects on the partons in the nucleon and the undetected fragments X_p and X_N are not treated rigorously. If we depart from these two simplifications, we then need the spectral function of the nucleon in terms of parton degrees of freedom to calculate DY cross sections. Such information is not available at the present time.

Substitute Eq.(27) into Eq.(16), we then have

$$\begin{aligned}d\sigma^{pN} &= \sum_q\int d\vec{p}_q d\vec{p}_{\bar{q}}[F^{q}_{p_p}(p_q)F^{\bar{q}}_{p_N}(p_{\bar{q}})]\frac{E_pE_N}{E_qE_{\bar{q}}}\frac{(2\pi)^4}{4[(p_p\cdot p_N)^2-m_p^2m_N^2]^{1/2}} \\ &\quad\times\{\frac{1}{(2\pi)^6}\frac{d\vec{k}_+}{2E_+}\frac{d\vec{k}_-}{2E_-}\frac{1}{q^4}f^{\mu\nu}(k_+,k_-)F^{qq}_{\mu\nu}(p_q,p_{\bar{q}})\}\end{aligned} \tag{28}$$

The quantity in the bracket {..} of the above equation is precisely what is in the bracket {..} of Eq.(10) for the $q\bar{q} \to \mu^+\mu^-$ process. Accounting for the difference in flux factors and extending Eq.(28) to include the $q \leftrightarrow \bar{q}$ interchange term, the full expression of the p-N DY process is

$$\begin{aligned}\frac{d^{pN}(p_p, p_N)}{dq^2} &= \sum_q \int d\vec{p}_q d\vec{p}_{\bar{q}} [F^q_{p_p}(p_q) F^{\bar{q}}_{p_N}(p_{\bar{q}}) + F^q_{p_N}(p_q) F^{\bar{q}}_{p_p}(p_{\bar{q}})] \\ &\quad \times \frac{4[(p_q \cdot p_{\bar{q}})^2 - m_q^4]^{1/2}}{4[(p_p \cdot p_N)^2 - m_p^2 m_N^2)]^{1/2}} \frac{E_p E_N}{E_q E_{\bar{q}}} [\frac{d^{q\bar{q}}(p_q, p_{\bar{q}})}{dq^2}] \end{aligned} \tag{29}$$

where $\frac{d^{q\bar{q}}(p_q,p_{\bar{q}})}{dq^2}$ is the q-$\bar{q}$ DY cross section, as defined by Eq.(15).

We now examine the physical meaning of the functions $F^q_{p_p}(p_q)$ and $F^{\bar{q}}_{p_N}(p_{\bar{q}})$, defined in Eqs.(22)-(23). The probability of finding a quark q with momentum $\vec{p}_q$ in a nucleon state $|\vec{p}_p >$ is defined by

$$P_{\vec{p}_p}(\vec{p}_q) = \frac{< \vec{p}_p | b^\dagger_{\vec{p}_q} b_{\vec{p}_q} | \vec{p}_p | >}{< \vec{p}_p | \vec{p}_p >} . \tag{30}$$

Inserting a complete set of states $1 = \int d\vec{p}_{X_p} |\vec{p}_{X_p} >< \vec{p}_{X_p}|$ into the above equation and using the definition Eq.(19), we then have

$$\begin{aligned} P_{\vec{p}_p}(\vec{p}_q) &= \frac{\sum_{X_p} \int d\vec{p}_{X_p} < \vec{p}_p | b^\dagger_{\vec{p}_q} |\vec{p}_{X_p} >< \vec{p}_{X_p} | b_{\vec{p}_q} |\vec{p}_p | >}{< \vec{p}_p | \vec{p}_p >} \\ &= \frac{\sum_{X_p} \int d\vec{p}_q |\phi_{\vec{p}_p}(\vec{p}_{\bar{q}}, \vec{p}_{X_p})|^2 \delta(\vec{p}_p - \vec{p}_q - \vec{p}_{X_p}) \delta(\vec{p}_p - \vec{p}_q - \vec{p}_{X_p})}{< \vec{p}_p | \vec{p}_p >} . \end{aligned}$$

With our normalization $< \vec{p}_p | \vec{p}_p >= \delta(\vec{p}_p - \vec{p}_p)$, the above equation becomes

$$\begin{aligned} P_{\vec{p}_p}(\vec{p}_q) &= [\frac{\sum_{X_p} \int d\vec{p}_q |\phi_{\vec{p}_p}(\vec{p}_{\bar{q}}, \vec{p}_{X_p})|^2 \delta(\vec{p}_p - \vec{p}_q - \vec{p}_{X_p})] \delta(\vec{p}_p - \vec{p}_p)}{\delta(\vec{p}_p - \vec{p}_p)} \\ &= \sum_{X_p} \int d\vec{p}_q |\phi_{\vec{p}_p}(\vec{p}_{\bar{q}}, \vec{p}_{X_p})|^2 \delta(\vec{p}_p - \vec{p}_q - \vec{p}_{X_p}) \end{aligned} \tag{31}$$

Clearly $F^q_{\vec{p}_p}(\vec{p}_q)$ of Eq.(22) is identical to $P_{\vec{p}_p}(\vec{p}_q)$ given above and is the probability of finding a quark with momentum $\vec{p}_q$ in a nucleon moving with a momentum $\vec{p}_p$. Thus $F^q_{\vec{p}_p}(\vec{p}_q)$ is the parton distribution function in the nucleon.

IV. *p*-*d* DY CROSS SECTION

For the $p(p_p) + d(p_d) \to \mu^+(k_+) + \mu^-(k_-)$ DY process, Eq.(4) gives

$$d\sigma^{pd} = \frac{(2\pi)^4}{4[(p_p \cdot p_d)^2 - m_p^2 m_d^2]^{1/2}} \{ \frac{1}{(2\pi)^6} \frac{d\vec{k}_+}{2E_+} \frac{d\vec{k}_-}{2E_-} \frac{1}{q^4} f^{\mu\nu}(k_+, k_-) F^{pd}_{\mu\nu}(p_p, p_d, q) \} . \tag{32}$$

We assume that the p-d DY process occurs only on each of the nucleons in the deuteron, as illustrated in Fig.2. In this impulse approximation, the hadronic tensor for a deuteron target

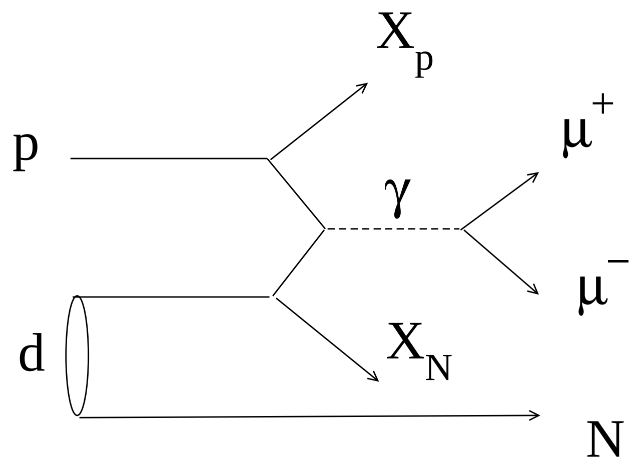

FIG. 2: Impulse approximation of $p + d$ Drell-Yan process.

can be obtained by simply extending Eq.(17) for the $p+N$ to include a spectator nucleon state $|p_s>$ in the sum over the final hadronic states. We thus have

$$F^{pd}_{\mu\nu}(p_p, p_d, q) \; = \; (2\pi)^6(2E_p)(2E_d) \sum_{X_p,X_N} \int d\vec{p}_s d\vec{p}_{X_p} d\vec{p}_{X_N} \delta^4(p_p + p_d - p_{X_p} - p_{X_N} - p_s - q)$$
$$\times[< \Phi_{p_d} p_p|J_\mu(0)|p_{X_p} p_{X_N} p_s >< p_s p_{X_N} p_{X_p}|J_\nu(0)|p_p \Phi_{p_d} >] \, , \tag{33}$$

where $|\Phi_{p_d}>$ is the state of a deuteron moving with a momentum p_d. Within the framework of relativistic quantum mechanics[32], we can expand the deuteron state in terms of two-nucleon plane-wave state $|\vec{p}_N \vec{p}_2>$

$$|\Phi_{p_d}>= \int d\vec{p}_N d\vec{p}_2 \Phi_{p_d}(\vec{p}_N)\delta(\vec{p}_d - \vec{p}_N - \vec{p}_2)|\vec{p}_N \vec{p}_2> \; . \tag{34}$$

Keeping only the contributions which are due to a parton in $|\vec{p}_N>$ of the above expansion and a parton from projectile state $|\vec{p}_p>$, we have

$$< \Phi_{p_d} p_p|J_\mu(0)|p_{X_p} p_{X_N} p_s >$$
$$= \int d\vec{p}_2 \int d\vec{p}_N \Phi^*(\vec{p}_N)\delta(\vec{p}_d - \vec{p}_N - \vec{p}_2) < p p_N|J_\mu(0)|p_{X_p} p_{X_N} >< \vec{p}_2|\vec{p}_s >$$
$$= \int d\vec{p}_N \Phi^*(\vec{p}_N)\delta(\vec{p}_d - \vec{p}_N - \vec{p}_s) < p p_N|J_\mu(0)|p_{X_p} p_{X_N} > \; . \tag{35}$$

By using Eq.(35), Eq.(33) can then be written as

$$F^{pd}_{\mu\nu}(p_p, p_d, q) \; = \; \int d\vec{p}_N |\Phi_{p_d}(p_N)|^2 \frac{(2E_p)(2E_d)}{(2E_p)(2E_N)}$$
$$\times\{(2\pi)^6(2E_p)(2E_N) \times \sum_{X_p,X_N} \int d\vec{p}_{X_p} d\vec{p}_{X_N} \delta^4(p_p + p_N - p_{X_p} - p_{X_N} - q)$$
$$\times[< p_N p_p|J_\mu(0)|p_{X_p} p_{X_N} >< p_{X_N} p_{X_p}|J_\nu(0)|p_p p_N >]\} \, . \tag{36}$$

We see that the quantity within the bracket $\{..\}$ in the above equation is identical to $F^{pN}_{\mu\nu}(p_p, p_N, q)$ of Eq.(17). We then have

$$F^{pd}_{\mu\nu}(p_p, p_d, q) \; = \; \int d\vec{p}_N \rho_{p_d}(\vec{p}_N) \frac{E_p E_d}{E_p E_N} F^{pN}_{\mu\nu}(p_p, p_N, q) \, , \tag{37}$$

where

$$\rho_{p_d}(\vec{p}_N) = |\Phi_{p_d}(p_N)|^2. \tag{38}$$

By using Eq.(34), one can show that $\rho_{p_d}(\vec{p}_N)$ is the nucleon momentum distribution in a deuteron moving with a momentum p_d :

$$\rho_{p_d}(\vec{p}_N) = \frac{< \Phi_{p_d}|b^{\dagger}_{\vec{p}_N} b_{\vec{p}_N}|\Phi_{p_d} >}{< \Phi_{p_d}|\Phi_{p_d} >} . \tag{39}$$

We will present formula for calculating $\rho_{p_d}(\vec{p}_N)$ in section V.

By using Eq.(37), Eq.(32) becomes

$$\begin{aligned} d\sigma^{pd} &= \frac{(2\pi)^4}{4[(p\cdot p_d)^2 - m_p^2 m_d^2]^{1/2}} \sum_{N=p,n} \int d\vec{p}_N \rho_{p_d}(\vec{p}_N) \frac{E_p E_d}{E_p E_N} \\ &\times \{ \frac{1}{(2\pi)^6} \frac{d\vec{k}_+}{2E_+} \frac{d\vec{k}_-}{2E_-} \frac{1}{q^4} f^{\mu\nu}(k_+, k_-) F^{pN}_{\mu\nu}(p_p, p_N, q) \} . \end{aligned} \tag{40}$$

The quantity within the bracket $\{..\}$ of the above equation is exactly what is in the bracket $\{..\}$ of Eq.(16). Accounting for the difference in flux factor, we obviously can write

$$\frac{d^{pd}(p, p_d)}{dq^2} = \sum_{N=p,n} \int d\vec{p}_N \rho_{p_d}(\vec{p}_N) \frac{[(p\cdot p_N)^2 - m_p^2 m_N^2]^{1/2}}{[(p\cdot p_d)^2 - m_p^2 m_d^2)]^{1/2}} \frac{E_p E_d}{E_p E_N} \frac{d^{pN}(p, p_N)}{dq^2} , \tag{41}$$

where $\frac{d^{pN}(p,p_N)}{dq^2}$ is given in Eq.(29). Substituting Eq.(29) into Eq.(41), we have

$$\begin{aligned} \frac{d^{pd}(p, p_d)}{dq^2} &= \sum_{N=p,n} \int d\vec{p}_N \rho_{p_d}(\vec{p}_N) \{ \sum_q \int d\vec{p}_q d\vec{p}_{\bar{q}} \frac{[(p_q\cdot p_{\bar{q}})^2 - m_q^4]^{1/2}}{[(p\cdot p_d)^2 - m_p^2 m_d^2)]^{1/2}} \frac{E_p E_d}{E_q E_{\bar{q}}} \\ &\times [F^q_{p_p}(p_q) F^{\bar{q}}_{p_N}(p_{\bar{q}}) + F^q_{p_N}(p_q) F^{\bar{q}}_{p_p}(p_{\bar{q}})][\frac{d^{q\bar{q}}(p_q, p_{\bar{q}})}{dq^2}]\} . \end{aligned} \tag{42}$$

Clearly the input to our calculations of the p-d DY cross sections are the nucleon momentum distribution $\rho_{p_d}(\vec{p}_N)$, the parton distribution functions $F^q_{\vec{p}}(\vec{p}_q)$ and $F^{\bar{q}}_{\vec{p}}(\vec{p}_q)$, and the elementary partonic $q\bar{q} \to \mu^+\mu^-$ cross section $\frac{d^{q\bar{q}}(p_q,p_{\bar{q}})}{dq^2}$ defined by Eq.(15).

Here we note that within the impulse approximation, as illustrated in Fig.2, the binding on the nucleon in the deuteron is treated accurately if $\rho_{p_d}(\vec{p}_N)$ is calculated from realistic nucleon-nucleon potentials which give high precision fits to the NN scattering data and the deuteron properties. In the considered framework of Relativistic Quantum Mechanics, the nucleon emitting a parton is on its mass-shell, while the interacting two-nucleons system is off the energy-shell of the deuteron: $\sqrt{\vec{p}_1^{\,2} + m_N^2} + \sqrt{\vec{p}_2^{\,2} + m_N^2} \neq \sqrt{\vec{p}_d^{\,2} + m_d^2}$. Precisely because of this choice of formulation, the parton distribution in Eq.(42) is the same as that in the free nucleon. If one chooses a theoretical framework within which the nucleon in the deuteron can be off its mass-shell $p^2 \neq m_N^2$, then this simple relation is lost and one needs addtional theoretical assumptions to extract the $\bar{d}/\bar{u}$ ratios in the proton from p-p and p-d DY data.

V. NUMERICAL PROCEDURES

In this section, we develop numerical procedures for applying the formula presented in previous sections to investigate the Fermi motion effect on the ratio $R_{pd/pp} = \sigma^{pd}/(2\sigma^{pp})$ between the p-d and p-p DY cross sections. Our first task is to relate our momentum variables p_p, p_T and q to the variables used in the analysis[5] of the available data. This will be given in the first subsection V.A. The procedures for calculating DY cross sections are given for p-p in subsection V.B and p-d in subsection V.C. The calculations of momentum distributions $\rho_{p_d}(\vec{p}_N)$ of a moving deuteron are described in subsection V.D.

A. Kinematic variables for DY cross sections

It is common[5] to use the co-linear approximation to define the parton momentum:

$$p_{q_p} = x_1 p_p \,, \tag{43}$$
$$p_{q_T} = x_2 p_T \,, \tag{44}$$

where p_{q_p} (p_{q_T}) is the momentum of a parton in the projectile (target), and x_1 and x_2 are scalar numbers. The momentum q of the virtual photon in the $q\bar{q} \to \gamma \to \mu^+\mu^-$ can then be written as

$$\begin{aligned} q &= p_q + p_{\bar{q}} \\ &= x_1 p_p + x_2 p_T \,. \end{aligned} \tag{45}$$

In the considered very high energy region $E_p > 100$ GeV, the masses of projectile ($p_p^2 = m_p^2$) and target ($p_T^2 = m_T^2$) can be neglected and we have

$$\begin{aligned} &s = (p_p + p_T)^2 \sim 2p_p \cdot p_T \,, \\ &p_T \cdot q \sim x_1 p_p \cdot p_T \,, \\ &p_p \cdot q \sim x_2 p_p \cdot p_T \,. \end{aligned}$$

The above relations give

$$x_1 \sim \frac{2q \cdot p_T}{s} \,, \tag{46}$$
$$x_2 \sim \frac{2q \cdot p_p}{s} \,. \tag{47}$$

It is most convenient to perform calculations in terms of x_1 and x_2 in the center of mass (c.m.) system in which the projectile is in the z direction and the target in -z direction:

$$p_p = (\sqrt{p^2 + m_p^2}, 0, 0, p) \sim (p, 0, 0, p) \,, \tag{48}$$
$$p_T = (\sqrt{p^2 + m_T^2}, 0, 0, -p) \sim (p, 0, 0, -p) \,. \tag{49}$$

With the choices Eqs.(48)-(49), we

$$\begin{aligned} s &= (p_p + p_T)^2 \sim 4p^2 \,, \\ M^2 &= q^2 = (x_1 p_p + x_2 p_T)^2 \sim 4x_1 x_2 p^2 \,. \end{aligned}$$

The above two equations leads the simple relation

$$x_1 x_2 \sim \frac{M^2}{s} \,. \tag{50}$$

By using Eqs.(46)-(47) and Eqs.(48)-(49), we can define a useful variable x_F

$$\begin{aligned} x_F &= x_1 - x_2 \sim \frac{2(p_T - p_p)\cdot q}{s} \\ &\sim \frac{2\sqrt{s}\hat{z}\cdot\vec{q}}{s} = \frac{2\sqrt{s}\hat{p}_p\cdot\vec{q}}{s} \,. \end{aligned} \tag{51}$$

In the notation of Ref.[5], we write

$$x_F \sim \frac{p_{\parallel}^{\gamma}}{\sqrt{s}/2} \,, \tag{52}$$

where $p_{\parallel}^{\gamma} = \hat{p}_p \cdot \vec{q}$ is clearly the longitudinal momentum of the intermediate photon with respect to the projectile in the c.m. frame. Experimentally, s, M, x_F, and $d\sigma/(dMdx_F)$ are measured[47]. With the relation Eq.(50), we certainly can determine the corresponding x_1, x_2 and $d\sigma/(dx_1dx_2)$. We thus will only give the expression of $d\sigma/(dx_1dx_2)$ in the following subsections.

Before we proceed further, it is necessary to discuss how our formula can be expressed in terms of x_1 and x_2. As given in Eqs.(43)-(44), these variables define the fraction of the hadron momentum carried by the emitted parton. For the projectile proton, this is unambiguous $x_1 = p_q/p_p = p_q^z/p_p^z$ since we choose $\vec{p}$ in z-direction. For the deuteron target, x_2 could be identified with the deuteron or the nucleons inside the deuteron since in the above derivations hadron masses are all neglected. Since the impulse approximation is used in our derivations, we identify x_2 with the mean nucleon momentum $< p_N >$ in the deuteron. As will be seen from the realistic nucleon momentum distribution to be presented in Fig.3, $< p_N > \sim |\vec{p_d}|/2$ as expected naively.

B. Calculation of p-p DY cross sections $d\sigma^{pp}/dx_1dx_2$

We now note that with the variables x_1 and x_2 defined above the flux factor associated with Eq.(29) become 1. Substituting Eq.(15) into Eq.(29), the DY cross section for $p(p_p) + p(p_N) \to \mu^+\mu^-$ is then calculated from

$$\begin{aligned} \frac{d^{pN}(p_p,p_N)}{dq^2} &= \sum_q \int d\vec{p}_q d\vec{p}_{\bar{q}} [F^q_{p_p}(p_q)F^{\bar{q}}_{p_N}(p_{\bar{q}}) + F^q_{p_N}(p_q)F^{\bar{q}}_{p_p}(p_{\bar{q}})] \\ &\quad \times \frac{4\pi\alpha^2}{9q^2} e_q^2 \delta(q^2 - (p_q + p_{\bar{q}})) \,. \end{aligned} \tag{53}$$

In the chosen center of mass frame, defined by Eqs.(48)-(49), let us consider $\bar{q}$ in the target proton moving with a momentum $p_N = (p_N^z, 0, 0, p_N^z)$. In the precise co-linear approximation, only the z-component of the $\bar{q}$ momentum is defined by p_N^z. We thus write $\vec{p_{\bar{q}}} = (\vec{p}_{\bar{q},T}, p_{\bar{q}}^z)$ where

$$p_{\bar{q}}^z = x_2 p_N^z \,, \tag{54}$$

and the transverse component $\vec{p}_{\bar{q},T}$ can be arbitrary. The integration over the $\bar{q}$ momentum distribution in the target N can then be written as

$$\begin{aligned}\int d\vec{p}_{\bar{q}} F^{\bar{q}}_{p_N}(\vec{p}_{\bar{q}}) &= \int dp^z_{\bar{q}} \int d\vec{p}_{\bar{q},T} F^q_p(p^z_{\bar{q}}, \vec{p}_{\bar{q},T}) \\ &= p^z_N \int dx_2 \int d\vec{p}_{\bar{q},T} F^q_p(x_2 p^z_N, \vec{p}_{\bar{q},T}) \\ &= \int dx_2 f^{\bar{q}}_{p_N}(x_2)\,, \end{aligned} \tag{55}$$

with

$$f^{\bar{q}}_{p_N}(x_2) = p^z_N \int d\vec{p}_{\bar{q},T} F^q_{p_N}(x_2 p^z_N, \vec{p}_{\bar{q},T})\,. \tag{56}$$

We identify $f^{\bar{q}}_{p_N}(x_2)$ with the parton distribution functions (PDF) determined by several groups[37–41]. To compare with the results of Ref.[5], we use the PDF of CETEQ5m[41].

Similarly, we can define for the projectile proton

$$\int d\vec{p}_q F^q_{p_p}(\vec{p}_q) = \int dx_1 f^q_{p_p}(x_1)\,, \tag{57}$$

where $f^q_{p_p}(x_1)$ is defined by the same Eq.(56) with q replaced by $\bar{q}$ and x_2 by x_1. By using Eqs.(56) and (57), Eq.(53) can be written as

$$\begin{aligned}\frac{d^{pN}(p_p, p_N)}{dq^2} &= \sum_q \int dx_1 dx_2 [f^q_{p_p}(x_1) f^{\bar{q}}_{p_N}(x_2) + f^q_{p_N}(x_1) f^{\bar{q}}_{p_p}(x_2)] \\ &\quad \times \frac{4\pi\alpha^2}{9q^2} e^2_q \delta(q^2 - (p_q + p_{\bar{q}})^2)\,. \end{aligned} \tag{58}$$

By integrating over dq^2, the above equation leads to

$$\frac{d^{pN}(p_p, p_N)}{dx_1 dx_2} = \sum_q \frac{4\pi\alpha^2}{9(p_q + p_{\bar{q}})^2} e^2_q [f^q_{p_p}(x_1) f^{\bar{q}}_{p_N}(x_2) + f^q_{p_N}(x_1) f^{\bar{q}}_{p_p}(x_2)]\,. \tag{59}$$

which is the same as Eq.(1) used in the analysis of Ref.[5] since $(p_q + p_{\bar{q}})^2 = q^2 = M^2$ for the partonic process $q\bar{q} \to \gamma$.

We only consider up (u) and down (d) quarks in the proton. Eq.(59) for the p-p DY cross section then obviously takes the following form

$$\begin{aligned}\frac{d^{pp}(p, p_p)}{dx_1 dx_2} &= \frac{4\pi\alpha^2}{9M^2} [\frac{4}{9}(f^{\bar{u}}_p(x_1) f^u_{p_p}(x_2) + f^u_p(x_1) f^{\bar{u}}_{p_p}(x_2)) \\ &\quad + \frac{1}{9}(f^{\bar{d}}_p(x_1) f^d_{p_p}(x_2) + f^d_p(x_1) f^{\bar{d}}_{p_p}(x_2))]\,. \end{aligned} \tag{60}$$

C. Calculation of pd DY cross sections of $d\sigma^{pd}/dx_1 dx_2$

From Eq.(42), we see that the calculations of p-d DY cross sections involve the convolution of the parton distribution associated with a nucleon with momentum $\vec{p}_N$ over the nucleon momentum distribution $\rho_{p_d}(\vec{p}_N)$ in a moving deuteron. To see this more clearly, we consider the contribution from an anti-quark in a nucleon N of the deuteron target and a quark in the projectile proton. By using Eq.(15), this particular contribution to Eq.(42) is

$$\frac{d^{pd}(p,p_d)}{dq^2} = \int d\vec{p}_q \int d\vec{p}_{\bar{q}} \{ \frac{[(p_q\cdot p_{\bar{q}})^2 - m_q^4]^{1/2}}{[(p\cdot p_d)^2 - m_p^2 m_d^2)]^{1/2}} \frac{E_N(p)E_d(p_d)}{E_q(p_q)E_q(p_{\bar{q}})} \} \times \int d\vec{p}_N \rho_{p_d}(\vec{p}_N)[f_p^q(\vec{p}_q) f_{p_N}^{\bar{q}}(\vec{p}_{\bar{q}}] \frac{4\pi\alpha^2}{9q^2} e_q^2 \delta(q^2 - (p_q+p_{\bar{q}}))\,. \quad (61)$$

Bu using the variables x_1 and x_2 defined in subsection V.A, the ratio between flux factors in the {..} bracket of the above equation is close to 1. We thus only need to consider

$$\frac{d^{pd}(p,p_d)}{dq^2} = \int d\vec{p}_q \int d\vec{p}_{\bar{q}} \int d\vec{p}_N \rho_{p_d}(\vec{p}_N)[f_p^q(\vec{p}_q) f_{p_N}^{\bar{q}}(\vec{p}_{\bar{q}}] \times \frac{4\pi\alpha^2}{9q^2} e_q^2 \delta(q^2 - (p_q+p_{\bar{q}})^2) \quad (62)$$

We now define the momentum fraction x_2^N of a parton associated with a nucleon with momentum p_N in the deuteron

$$p_{\bar{q}} = x_2^N p_N^z\,. \quad (63)$$

By using the definitions Eq.(63) for x_2^N and Eqs.(56)-(57) for parton distributions, Eq.(62) can be written as

$$\frac{d^{pd}(p,p_d)}{dq^2} = \int dx_1 \int dx_2^N \int d\vec{p}_N \rho_{p_d}(\vec{p}_N)[f_p^q(x_1) f_{p_N}^{\bar{q}}(x_2^N)] \times \frac{4\pi\alpha^2}{q^2} e_q^2 \frac{1}{9} \delta(q^2 - (p_q+p_{\bar{q}})^2) \quad (64)$$

Similar to the pp case, $\vec{p}_d = (p_d^z, \vec{p}_{d,T} = 0)$ is chosen to be on the z-direction of the c.m.frame. As discussed above, we need to find a way to relate x_2^N to x_2 which is determined by experimental variables M, s and x_f through the relations: $x_1 x_2 = M^2/s$ and $x_f = x_1 - x_2 = p_{\|}^{\gamma}/(\sqrt{s}/2)$. In the impulse approximation we have used to derive the p-d DY cross section, it is assumed that the parton is emitted from one of the nucleons in the deuteron, as illustrated in Fig.2. Thus it is reasonable to choose $p_{\bar{q}} = x_2 p_{ave}^z$ where p_{ave}^z is the averaged nucleon momentum in deuteron. Combining this assumption and Eq.(63), we then have the following relation

$$x_2^N = x_2 \frac{p_{ave}^z}{p_N^z}\,. \quad (65)$$

In section VI, we will determine p_{ave}^z from investigating $\rho_{p_d}(\vec{p}_N)$ generated from realistic deuteron wavefunctions.

By using Eq.(65) to change the integration variable, we can write Eq.(64) as

$$\frac{d^{pd}(p,p_d)}{dq^2} = \int dx_1 \int dx_2 \int d\vec{p}_N \rho_{p_d}(\vec{p}_N) \frac{p_{ave}^z}{p_N^z} [f_p^q(x_1) f_{p_N}^{\bar{q}}(\frac{x_2 p_{ave}^z}{p_N^z})] \times \frac{4\pi\alpha^2}{q^2} e_q^2 \frac{1}{9} \delta(q^2 - (p_q+p_{\bar{q}})^2) \quad (66)$$

Integrating over q^2 on both sides of the above equation, we obtain

$$\frac{d^{pd}(p,p_d)}{dx_1dx_2} = [f^q_p(x_1)\Gamma^{\bar{q}}_{p_d,N}(x_2)]\frac{4\pi\alpha^2}{9q^2}e^2_q\,, \tag{67}$$

where the $\bar{q}$ contribution is isolated in

$$\Gamma^{\bar{q}}_{p_d,N}(x_2) = \int d\vec{p}_N\rho_{p_d}(\vec{p}_N)[\frac{p^z_{ave}}{p^z_N}f^{\bar{q}}_{p_N}(\frac{x_2p^z_{ave}}{p^z_N})]\,. \tag{68}$$

The derivation of Eq.(68) can be extended to have q in deuteron and $\bar{q}$ in the projectile proton. We finally obtain

$$\frac{d^{pd}(p,p_d)}{dx_1dx_2} = \sum_q\{f^q_p(x_1)[\sum_{N=p,n}\Gamma^{\bar{q}}_{p_d,N}(x_2)] + f^{\bar{q}}_p(x_1)[\sum_{N=p,n}\Gamma^{q}_{p_d,N}(x_2)]\}\frac{4\pi\alpha^2}{9q^2}e^2_q\,. \tag{69}$$

We use the charge symmetry to calculate the parton distribution functions for the neutron from that of proton: $f^d_n = f^u_p$, $f^u_n = f^d_p$, $f^{\bar{d}}_n = f^{\bar{u}}_p$, $f^{\bar{u}}_n = f^{\bar{d}}_d$. Furthermore $\rho_{p_d}(p_N)$ is the same for neutron and proton. Including the charges for u and d quarks appropriately, Eq.(69) can be written as

$$\begin{aligned}\frac{d^{pd}(p,p_d)}{dx_1dx_2} &= \frac{4\pi\alpha^2}{9q^2}[\frac{4}{9}f^u_p(x_1) + \frac{1}{9}f^d_p(x_1)][\Gamma^{\bar{u}}_{p_d,p}(x_2) + \Gamma^{\bar{d}}_{p_d,p}(x_2)] \\ &\quad +[\frac{4}{9}f^{\bar{u}}_p(x_1) + \frac{1}{9}f^{\bar{d}}_p(x_1)][\Gamma^{u}_{p_d,p}(x_2) + \Gamma^{d}_{p_d,p}(x_2)]\,. \end{aligned} \tag{70}$$

D. Calculation of $\rho_{p_d}(\vec{p}_N)$

To calculate the p-d DY cross section Eq.(42), we need to generate the nucleon momentum distribution $\rho_{P_d}(\vec{p}_N)$ defined by Eq.(39). Since the available realistic nucleon-nucleon potentials only provide the wavefunctions in the rest frame, we boost these wavefunctions within the relativistic quantum mechanics proposed by Dirac, as reviewed in Ref.[32]. We note here that the parton distributions defined by Eqs.(22)-(23) or Eqs.(56)-(57) are also within the same framework. We thus have a consistent relativistic description of both the nucleonic structure of the deuteron and the partonic structure of the nucleon. Such a consistency is essential in investigating the Fermi motion effect on DIS and DY processes.

Starting with a mass operator (Hamiltonian in the rest frame) which defines the mass and the wavefunction $\chi_d(\vec{k})$ of the deuteron in its rest frame, one can generate the wavefunction Φ_{p_d} of a moving deuteron with momentum p_d using one of the three possible forms[31, 32] of the relativistic quantum mechanics. Furthermore, one can identify $\chi_d(\vec{k})$ with the deuteron wavefunctions generated from the available realistic nucleon-nucleon potentials, as discussed in Refs.[33, 34]. Here we consider the Instant-Form (IF) and Light-Form (LF). The formula for calculating $\rho_{p_d}(\vec{p}_N)$ can be derived by using the procedures presented in Ref.[32] and will not be detailed here. We will simply write down the formula used in our calculations.

1. Instant Form

The deuteron wavefunction $\chi_d(\vec{k})$ in the $\vec{p}_d = 0$ rest frame is normalized as

$$\int d\vec{k}|\chi_d(\vec{k})|^2 = 1\,.$$

If we neglect the interaction in the Lorentz Boost and follow the procedures detailed in Ref.[32], it is straightforward to show that Eqs.(34) and (39) lead to

$$\begin{aligned}
\rho_{p_d}(\vec{p}) &= \frac{< \Phi_{p_d}|b^\dagger_{\vec{p}} b_{\vec{p}}|\Phi_{p_d} >}{< \Phi_{p_d}|\Phi_{p_d} >} \\
&= |\frac{\partial(\vec{p}_d, \vec{k})}{\partial(\vec{p}, \vec{p}_2)}||\chi_d(\vec{k})|^2 , \qquad (71)
\end{aligned}$$

with

$$|\frac{\partial(\vec{p}_d, \vec{k})}{\partial(\vec{p}, \vec{p}_2)}| = \frac{\omega_m(\vec{k})\omega_m(\vec{k})\omega_{M_0}(\vec{p}_d)}{\omega_m(\vec{p})\omega_m(\vec{p}_2)M_0} , \qquad (72)$$

$$(73)$$

where m is the nucleon mass, $\omega_m(\vec{k}) = \sqrt{m^2 + \vec{k}^{\,2}}$, and

$$\begin{aligned}
\vec{p}_2 &= \vec{p}_d - \vec{p} , && (74) \\
\vec{k} &= \vec{p} + \frac{\vec{p}_d}{M_0}[\frac{\vec{p}_d \cdot \vec{p}}{M_0 + H_0} - \omega_m(\vec{p})] , && (75) \\
H_0 &= \omega_m(\vec{p}) + \omega_m(\vec{p}_2) , && (76) \\
M_0 &= [H_0^2 - \vec{p}_d^{\,2}]^{1/2} . && (77)
\end{aligned}$$

2. Light Form

The Light-Form (LF) momentum is defined by

$$\tilde{p} = (p^+, \vec{p}_\perp) ,$$

where

$$\begin{aligned}
p^+ &= \omega_m(p) + p^3 , \\
\vec{p}_\perp &= (p^1, p^2) .
\end{aligned}$$

By using the above definitions for all momenta in Eqs.(34) and (39) and following the procedures detailed in Ref.[32], one can show that the nucleon momentum distribution in LF is

$$\begin{aligned}
\rho^{LF}_{\tilde{p}_d}(\tilde{p}) &= \frac{< \Phi_{\tilde{p}_d}|b^\dagger_{\tilde{p}} b_{\tilde{p}}|\Phi_{\tilde{p}_d} >}{< \Phi_{\tilde{p}_d}|\Phi_{\tilde{p}_d} >} \\
&= |\frac{\partial(\tilde{p}_d, \vec{k})}{\partial(\tilde{p}, \tilde{p}_2)}||\chi_d(\vec{k})|^2 , \qquad (78)
\end{aligned}$$

where

$$|\frac{\partial(\tilde{p}_d, \vec{k})}{\partial(\tilde{p}, \tilde{p}_2)}| = \frac{\omega_m(\vec{k})\omega_m(\vec{k})p_d^+}{p^+ p_2^+ M_0} \qquad (79)$$

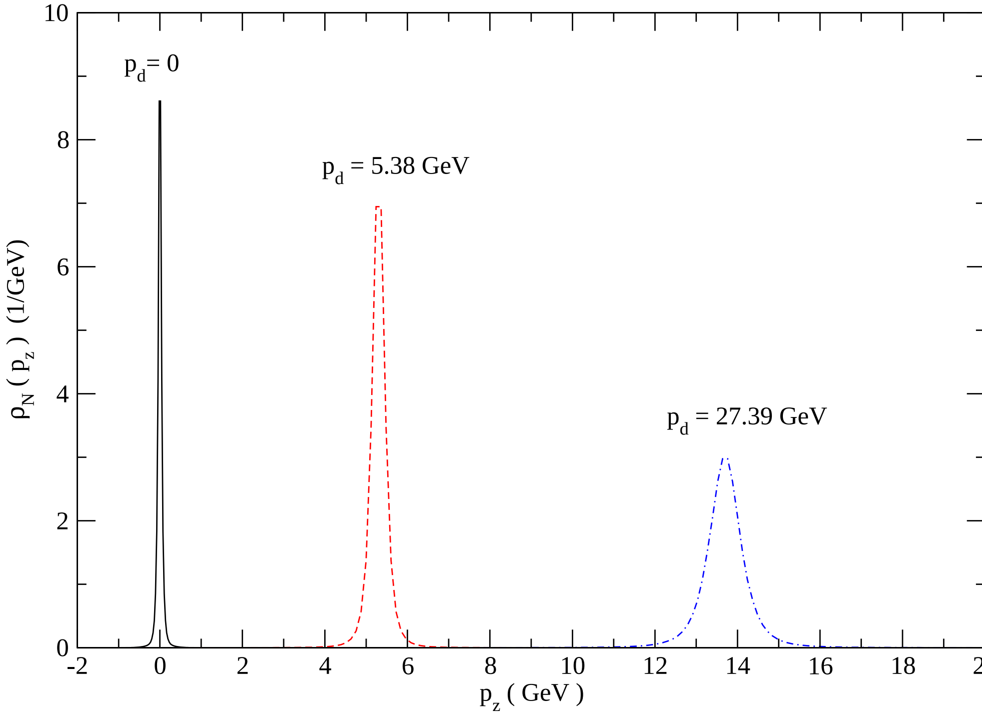

FIG. 3: Nucleon momentum distribution $\rho_{p_d}(p)$ of a deuteron moving with a momentum $p_d = |\vec{p}_d|$ in p-d collisions at the proton laboratory energies $E=$ 0 (solid), 120 (dashed), 800 (dash-dotted) GeV

.

with

$$\tilde{p}_2 = \tilde{p}_d - \tilde{p}\,, \tag{80}$$

$$\vec{k}_\perp = \frac{p_2^+}{p_d^+}\vec{p}_\perp - \frac{p^+}{p_d^+}\vec{p}_{2\perp}\,, \tag{81}$$

$$k^3 = \frac{1}{2}(p^+ - p_2^+)\sqrt{\frac{m^2 + \vec{k}_\perp^{\,2}}{p^+ p_2^+}}\,, \tag{82}$$

$$M_0 = 2\omega_m(\vec{k})\,. \tag{83}$$

To use LF results in the calculation of DY cross section Eq.(42), we need to transform the LF variables into the usual variables. We thus have

$$\rho_{p_d}(\vec{p}) = (1 + \frac{p^3}{\omega_m(\vec{p})})\rho^{LF}_{\tilde{p}_d}(\tilde{p}) \tag{84}$$

VI. NUMERICAL RESULTS

To calculate p+d DY cross sections, we need to first determine the average nucleon momentum p^z_{ave} in Eq.(68). This is done by examining the dependence of the nucleon momentum distribution $\rho_{p_d}(p)$ on the deuteron p_d using the formula given in subsection

V.D. In Fig.3, we show $\rho_{p_d}(p_z) = \int \rho_{p_d}(\vec{p}) dp_x dp_y$ of the Instant Form (IF) which are used in our calculations for the p-d DY cross sections at $E = 0, 120, 800$ GeV. The Argonne-V18 potential[42] is used here to generate the deuteron wavefunction $\chi_d(\vec{k})$. We see that as p_d increases, the spreading of the momentum distribution is wider. This is of course the consequence of Lorentz contraction of the density distribution in coordinate space. In the same figure, we also indicate the deuteron momentum p_d for each case. We observed that their peaks are near $p_d/2$ at each energy, and we hence set $p^z_{ave} = p^z_d/2$ in the calculation of DY cross sections using Eqs.(68) and (70).

We have also carried out calculations for the Light-Form (LF) and find that the results are indistinguishable from that shown in Fig.3. This turns out to be due to the fact that in the region where $|\chi(k)|^2$ dominant, the Jacobin $|\frac{\partial(\vec{p}_d,\vec{k})}{\partial(\vec{p},\vec{p}_2)}|$ of IF and $|\frac{\partial(\tilde{p}_d,\vec{k})}{\partial(\tilde{p},\tilde{p}_2)}|$ of LF are almost the same numerically.

We use the CETEQ5m[41] parton distributions to perform calculations using Eq.(60) for p-p and Eq.(68)and Eq.(70) for p-d DY cross sections. If we neglect the p^z_N-dependence in the parton distribution function $f^q(x_2 p^z_{ave}/p^z_N)$ in Eq.(68) and set $p^z_N \to p^z_{ave}$ such that $f^q(x_2 p^z_{ave}/p^z_N) \to f^q(x_2)$, we then have $\Gamma^q_{p_d,N}(x_2) \to f^q_N(x_2)$ since $\int d\vec{p}_N \rho_{p_d}(\vec{p}_N) = 1$ as defined by the normalization of deuteron wavefunction χ_d. The calculations based on this simplification are then identical to those from using Eqs.(1)-(2) of Ref.[5]. Clearly the results from using this frozen-nucleon approximation do not include the Fermi Motion (FM) effects. Our interest is to see how these "No FM" results are compared with the "With FM" results calculated from using the convolution formula Eqs.(68) and (70).

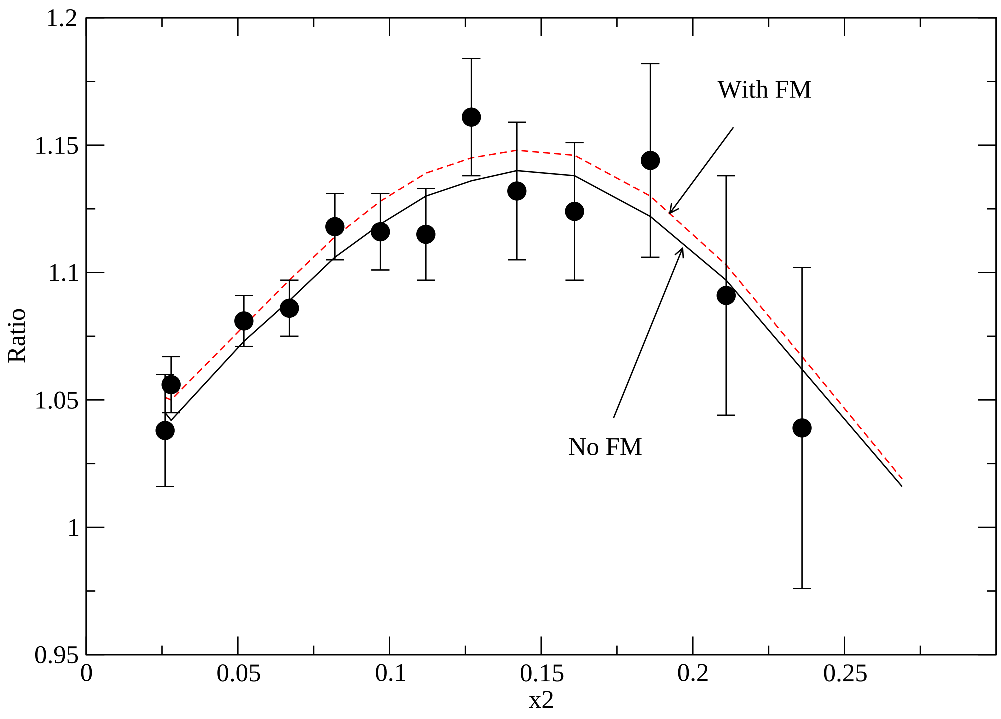

FIG. 4: Ratio $\sigma^{pd}/\sigma(pp)$ at $E = 800 GeV$. The data are from Ref.[5].

We first investigate the data at 800 GeV of Ref.[5] by calculating the ratio

$$R_{pd/pp} = \frac{d^{pd}(p, p_d)}{dx_1 dx_2} / (2 \times \frac{d^{pp}(p, p_p)}{dx_1 dx_2}) \,. \tag{85}$$

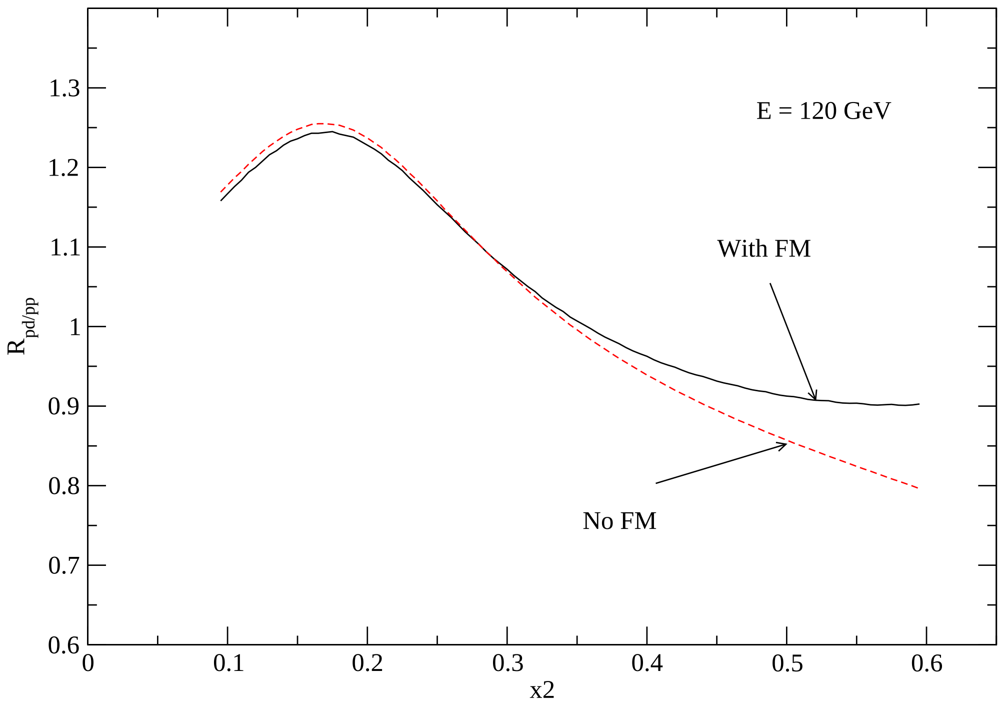

FIG. 5: Ratio $\sigma^{pd}/\sigma(pp)$ at $E = 120GeV$ and $x_1 = 0.6$.

Our results are shown in Fig.4. We see that the Fermi motion has less than 1% effect on the ratio $\sigma^{pd}/(2\sigma^{pp})$ in the low $x_2 < 0.25$ region covered by this experiment. Our "No FM" results are similar to that presented in Ref.[5] and agree with the data.

In Fig.5, we show the Fermi motion effect at 120 GeV which will be considered in the upcoming experiment at Fermi Laboratory. The results are for $x_1 = 0.6$. We see that at small x_2, the FM effect is also very small as in Fig.4 for 800 GeV. However the FM effects become increasingly larger with x_2; about 25% as $x_2 \to 0.6$. This is similar to what is known in the DIS studies.

To see the Fermi motion effects more clearly, we present results for $x_1 = 0.9$ in Fig.6. We see that the ratio $\sigma^{pd}/(2\sigma^{pp}) \to$ about 2.5 as $x_2 \to 0.9$. This can be understood from Fig.7 where we show the differences between the convoluted parton distribution $\Gamma^{\bar{u}}_{p_d,N}(x_2)$, defined by Eqs.(68), and $f^{\bar{u}}_N(x_2)$ of CETEQ5m. We see that their differences can be as large as a factor of about 2 as $x_2 \to 0.9$. Similar differences are also for other parton distributions.

The results presented in Figs.3-7 are from using the deuteron wavefuncition $\chi(\vec{k})$ generated from Argonne-V18 NN potential. Similar results have also been obtained by using the wavefunctions from Nijmegan[43], CD-Bonn[44], and Paris[45] NN potentials.

VII. SUMMARY AND DISCUSSIONS

Within a formulation based on relativistic quantum mechanics, we have developed convolution formula for calculating the p-d Drell-Yan cross sections using the available parton distribution functions[41] and the nucleon momentum distributions generated from realistic nucleon-nucleon potentials[42–45]. When the Fermi motion in the deuteron is neglected, our formula are reduced to the factorized form Eq.(2) which was used in the previous analysis.

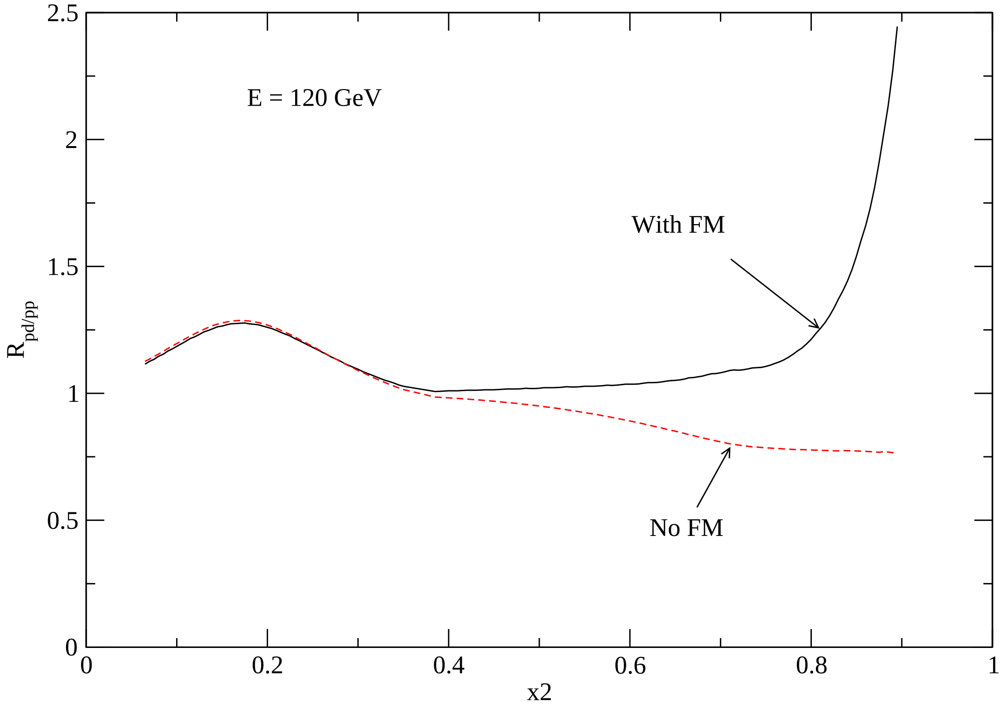

FIG. 6: Ratio $\sigma^{pd}/\sigma(pp)$ at $E = 120 GeV$ and $x_1 = 0.9$.

By comparing the results from the convoluted and factorized calculations, we have shown that in the small $x_2 < 0.3$ region, the Fermi motion effect is very small and our results for the ratios $R_{pd/pp} = \sigma^{pd}/(2\sigma^{pp})$ agree well with the data at 800 GeV of Ref.[5]. On the other hand, the Fermi motion effect can influence significantly the p-d Drell-Yan cross sections in the large Bjorken $x_2 >$ about 0.4 region. At 120 GeV the predicted Fermi Motion effect can enhance the ratios $R_{pd/pp}$ by about 20 % at $x_2 \sim 0.6$ and about a factor of 2.5 at $x_2 \to 1.0$. Our results suggest that the Fermi motion effect must be included in extracting the $\bar{d}/\bar{u}$ ratio in the nucleon from the upcoming experiment at Fermi Laboratory.

In this work, we have only investigated the most trivial nuclear effects on DY processes. To extract the $\bar{d}/\bar{u}$ ratios precisely, we need to extend our formulation to account for other nuclear effects. By using the approximations Eqs.(21) and (25) for the time-components of δ-functions in our derivations, the binding effects on the partons in the nucleon and the undetected fragments X_p and X_N are not treated rigorously. To improve this incomplete treatment of binding effects, one must depart from the simple convolution formula by performing calculations using the spectral functions of the nucleon in terms of parton degrees of freedom. Such information is not available at the present time.

The use of impulse approximation, as illustrated in Fig.2 , only accounts for the leading-order term of the full DY amplitude. It is also necessary to investigate the accuracy of this approximation. For example, we perhaps need to investigate the double scattering mechanisms which are considered in Ref.[48] to describe the shadowing effects on DIS. We also need to investigate the effects due to pion excess, as discussed in Refs.[25, 50].

Finally, we mention that there exists other relativistic formulations of DIS and DY processes. The differences between different approaches are mainly in describing the structure of the considered composite systems, nucleons or nuclei, in terms of interactions between

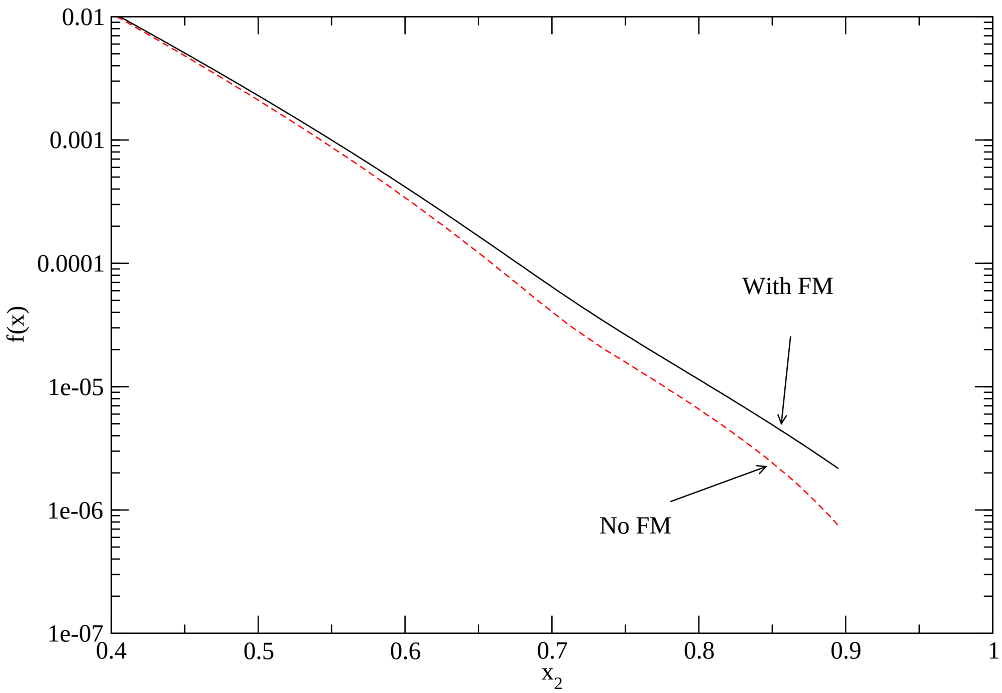

FIG. 7: Convoluted and factorized $\bar{u}$ distributions at $E = 120GeV$ and $x_1 = 0.9$. The results for $\bar{d}$ are very similar and are therefore not shown.

their constituents. In the formulation considered in this work, all particles are on their mass shells, but the systems can be off the energy shell during interactions. Thus it is also necessary to examine how our formulation differs from other formulations within which the nucleons in the deuteron can be off their mass-shell $p_N^2 \neq m_N^2$, such as the covariant formulation given in Ref.[49]. Similar theoretical questions also exist in many-years investigations of nuclear dynamics at intermediate and high energies; such as the differences[51] between different nucleon-nucleon potentials constructed using different three-dimensional reductions of Bethe-Salpeter equations.

Acknowledgments

The author would like to thank D.F. Geesaman, R.J. Holt, and J.-C. Peng for their helpful discussions. This work is supported by the U.S. Department of Energy, Office of Nuclear Physics Division, under Contract No. DE-AC02-06CH11357.

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