Longitudinal Scaling of Elliptic Flow in Landau Hydrodynamics

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Abstract

This study presents generalization of the Landau hydrodynamic solution for multiparticle production applied to non-central relativistic heavy ion collisions. Obtained results shows longitudinal scaling of elliptic flow v_2 as a function of rapidity shifted by beam rapidity $(y - y_{beam})$ for different energies ($\sqrt{s_{NN}} = 62.4$ GeV and 200 GeV) and for different systems (Au-Au and Cu-Cu). It is argued, that the elliptic flow and its longitudinal scaling is due to the initial transverse energy density distribution and initial longitudinal thickness effect.

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I. INTRODUCTION

Experimentally observed azimuthal asymmetries of particle production in non-central heavy ion collisions are of current high interest, as it provides more information about the early dynamics of the high-energy nuclear reactions. Moreover, RHIC data [1] of elliptic flow for different pseudorapidities shows universal behavior for different nuclei and for different beam energies, as can be seen in figure (1). Not all of the theoretical models are able to reproduce the observed longitudinal scaling of elliptic flow [2], for example: the AMPT (a multiphase transport model, ver 1.11) [3] and UrQMD (ultrarelativistic quantum molecular dynamics, ver 2.3) [4] does not reproduce experimentally observed scaling of elliptic flow. While, the AMPT with string melting and the Buda-Lund model, based on an analytic solution of perfect fluid dynamics, can reproduce experimental data [5].

Landau's approximate hydrodynamic solution for particle production in relativistic heavy ion reactions [6, 7] was formulated in 1953, but even today gets a lot of attention and reproduces the rapidity spectrum of observed particles well in the relativistic nuclear collisions [8]. In recent studies [9, 10] particle production function for different rapidities was slightly modified compared to the original version to the following form:

$$dN/dy \propto \exp\sqrt{(y_{beam}^2 - y^2)}, \tag{1}$$

where $y_{beam} = \ln(\sqrt{s_{NN}}/m_N)$ is the beam rapidity. In order to make a comparison with experimental particle spectra one has to normalize the distribution with total number of particles, which is unknown from the Landau solution. The key reason lies in the fact that the Landau solution does not conserve total energy, thus the total entropy of the system cannot be obtained and that leads to an unknown number of produced particles N_{tot}. Anyway, the solution of perfect fluid dynamics (1) gives the right shape of the rapidity distribution for different relativistic energies. On the other hand, from the definition of the elliptic flow:

$$v_2(y) = \frac{\int d\phi (dN/d\phi dy)\cos(2\phi)}{\int d\phi (dN/d\phi dy)}, \tag{2}$$

one can easily see, that normalization of total particle production is not needed. So the task is to modify the approximate solution of Landau, including the transverse angle ϕ dependence into the solution.

In this study the analytic solution for relativistic hydrodynamic equations will be pre-

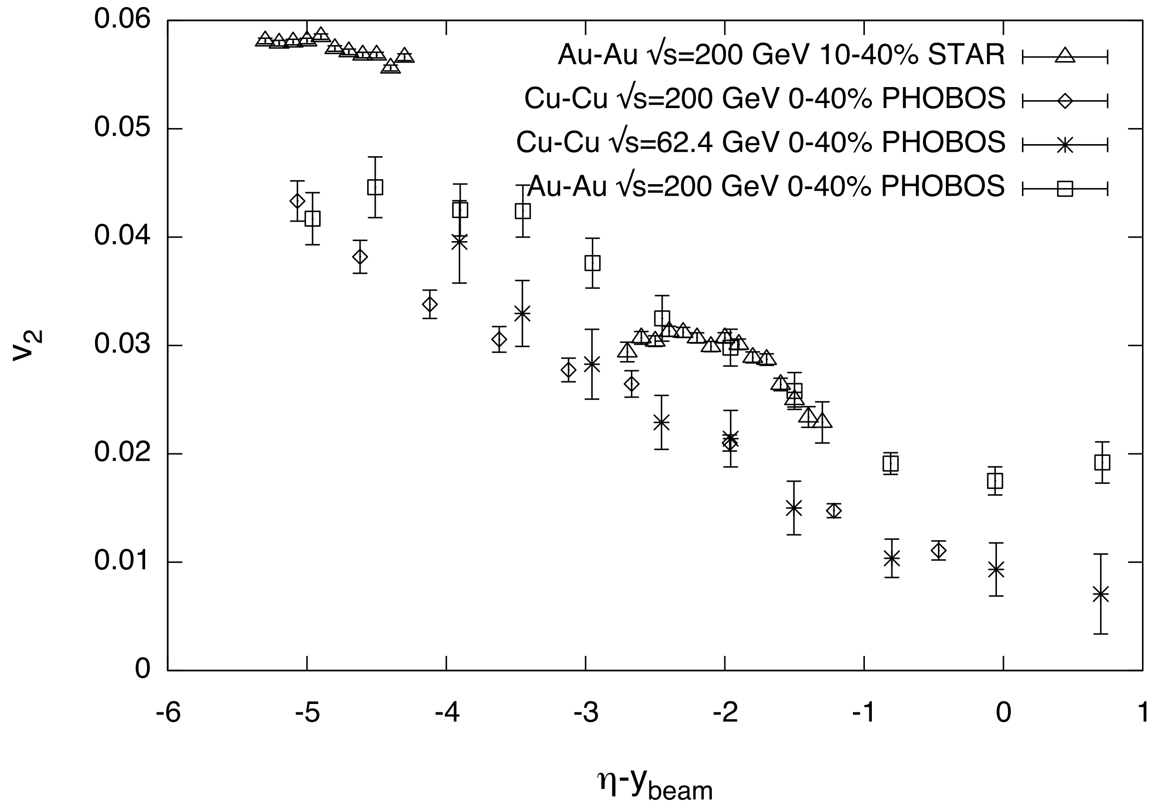

FIG. 1. Elliptic flow dependance on pseudorapidity for Au-Au and Cu-Cu at RHIC experiment for two different collision energies. Data from [11] and [12] .

sented. In the section II the longitudinal expansion will be summarized for the sake of completeness of the study, even though the same results can be found elsewhere. The transverse part of the solution with asymmetrical pressure gradient driven expansion are presented in the section III. The last section IV presents initial state and obtained results.

The presented solution is approximate, but in comparison to the computational hydrodynamics is analytic and transparent. The main assumptions of the model coincides with original Landau approximations and are as follows: i) longitudinal and transverse parts of hydrodynamic equations are solved separately; ii) equation of state of ideal relativistic gas, $P = e/3$, is used to solve transport equations; iii) transverse expansion does not include initial flow and is pressure gradient driven.

II. LONGITUDINAL EXPANSION

In this model we solve the equations of energy-momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0, \tag{3}$$

where the energy-momentum tensor reads as:

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - P g^{\mu\nu}. \tag{4}$$

The solution of local conservation laws (eq. 3) with the equation of state represents the dynamics of the system by relating bulk properties of the matter, such as: energy density, e, local pressure, P and the four-flow of the fluid, $u^\mu = u^0(1, \vec{v})$. The equations of the hydrodynamic longitudinal expansion in 1+1 dimension, along z axis reads as:

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{0z}}{\partial z} = 0, \quad \frac{\partial T^{0z}}{\partial t} + \frac{\partial T^{zz}}{\partial z} = 0. \tag{5}$$

The details on how to solve the above equations can be found in [6], [7], [9], [10], [13], [14], so here we present the derivation shortly including only main equations and the result, as we will need it later.

Solution of the equations of hydrodynamics (5) starts by transforming relativistic velocity field components to rapidity terms, as:

$$u^0 = \cosh y, \quad u^z = \sinh y. \tag{6}$$

From the above transformation, naturally follows, that: $v_z = \tanh y$ and $(u^z)^2 - (u^0)^2 = -1$. In order to solve hydrodynamic equations (5), the following variables are introduced:

$$\kappa = \ln w, \quad w = e + p, \quad \chi = \psi - w u_z z - w u_0 t,$$

where χ is so called Khalatvikov potential, ψ is hydrodynamic potential, which is defined by relation: $w u_i = \frac{\partial \psi}{\partial x^i}$,where ψ is a function of coordinates and time. And, $w = e + P$ is the enthalpy. Using relation for sound velocity, as:

$$c_s^2 = \frac{n}{w}\frac{\partial w}{\partial n},$$

which is $c_s^2 = 1/3$ for the ideal relativistic gas equation of state [15]. After the Legendre transformation to the hodograph plane, the Chaplygin equation for supersonic expansion reads as:

$$c_s^2 \frac{\partial^2 \chi}{\partial \kappa^2} + (1 - c_s^2)\frac{\partial \chi}{\partial \kappa} - \frac{\partial^2 \chi}{\partial y^2} = 0. \tag{7}$$

Detailed investigation on the solution of the 1+1 dimensional perfect fluid hydrodynamics can be found in [16–18]. The solution for above equation reads:

$$t = e^{-\kappa}\Big(\frac{\partial\chi}{\partial\kappa}\cosh y - \frac{\partial\chi}{\partial y}\sinh y\Big), \tag{8}$$

$$z = e^{-\kappa}\Big(\frac{\partial\chi}{\partial\kappa}\sinh y - \frac{\partial\chi}{\partial y}\cosh y\Big). \tag{9}$$

The transformation back to the (t, z) coordinates is shorter with the following new variables:

$$y_+ = \ln((t+z)/\Delta), \tag{10}$$

$$y_- = \ln((t-z)/\Delta), \tag{11}$$

while Δ is the initial thickness of the system in the beam direction, z. Also, Δ is the initial condition after which equation of state assumed to be valid and evolution equations (5) are applied.

The final solution for energy density, $e(y_+, y_-)$, and rapidity, $y(y_+, y_-)$, is:

$$e(y_+, y_-) = e_0 \exp[-4/3(y_+ + y_- - \sqrt{y_+ y_-})], \tag{12}$$

$$y(y_+, y_-) = (y_+ - y_-)/2, \tag{13}$$

while $z = t\tanh y$. The above solution of 1+1-dimensional relativistic hydrodynamics equation (5) will be connected to the solution of transverse expansion, in order to obtain multiplicities of produced particles for different rapidities.

III. TRANSVERSE EXPANSION

In order to solve the transverse part of hydrodynamic equations (3) we will follow original Landau assumptions with some modifications. For simplicity polar coordinates will be used, where four-flow components in polar coordinates are: $u_i = dx_i/dt$, $u_0 = (1 - (\frac{d}{dt}r^2 + r^2\frac{d}{dt}\phi^2))^{-1/2}$, $u_r = u_0 v_r$ and energy-momentum tensor (4) components are as follows:

$$T^{rr} = (e+P)(u^0)^2 v_r^2 + P, \quad T^{\phi\phi} = (e+P)(u^0)^2 v_\phi^2 + P/r^2$$

$$T^{0r} = (e+P)(u^0)^2 v_r, \quad T^{0\phi} = (e+P)(u^0)^2 v_\phi\ .$$

Assuming that the system does not rotate, $v_\phi = 0$, the hydrodynamic equation (3) for the transverse dynamics at the fixed transverse angle ϕ becomes:

$$\frac{\partial T^{0r}}{\partial t} + \frac{\partial T^{rr}}{\partial r} = 0\ . \tag{14}$$

Inserting energy-momentum tensor expressions to the above equation and using ideal gas equation of state, $P = e/3$, one gets:

$$4e(u^0)^2\frac{\partial v_r}{\partial t} + 4e(u^0)^2\frac{\partial v_r^2}{\partial r} + \frac{\partial e}{\partial r} = 0\ . \tag{15}$$

Following original Landau derivation, the fist term in the above equation is an acceleration dependence, and is assumed to be equal to $\partial v_r/\partial t = 2r(t)/t^2$, the second term is set to zero, v_r being comparatively small. To simplify the third term Landau used $\partial e/\partial r \approx -e/R_A$, because the energy density at the center has value e and is zero at the edge of the system, $r = R_A$. In the case of peripheral collisions, we do not expect centrally symmetric energy density distribution, thus the assumption is modified, as:

$$\frac{\partial e}{\partial r} = \frac{e(r = R_\phi) - e(r = 0)}{R_\phi}, \tag{16}$$

where R_ϕ is a transverse radius of the system, which changes with the angle, as the system is not centrally symmetric. We do not know the value of $e(R_\phi)$, so we introduce a new function, as $f(R_\phi) = e(r = R_\phi)/e(r = 0)$, which is a fraction of energy density at the edge of the system with respect to the energy density at the center. In this way, the function, $f(R_\phi)$, must be less than unity for any angle ϕ, as the energy density at the center is higher than at the edge. This modification from the original Landau assumption plays an important role, as it involves transverse asymmetry to the solution by making a different transverse pressure gradient for different angle ϕ. Now from eq. (15), we express the transverse displacement dependence on time, as:

$$r(t) = \frac{(1 - f(R_\phi))t^2}{8(u^0)^2 R_\phi}. \tag{17}$$

The last stage of the model is the so called conic-flight stage, where energy and entropy fluxes stop changing for the fixed cone element $2\pi r dr$. The stage coincides with the kinetic Freeze-Out (FO), as the model does not include any hadronic re-interactions after the evolution stops. With the help of the formula (17) we obtain a hypersurface in space-time, after which hydrodynamics stops and matter streams freely towards detectors. Here again we follow the original Landau model, assuming a fixed transverse distance, $r(t_{FO}) = a = 2R_A$, for the conic-flight to start at the distance of the diameter of the nucleus. Thus, we obtain the value for the time, when the conic-flight starts, which reads as:

$$t_{FO} = 2\cosh y\sqrt{\frac{2aR_\phi}{(1 - f(R_\phi))}}, \tag{18}$$

where relation $u^0 = \cosh y$ was used.

The solution in the conic-flight stage is straightforward, as the energy and entropy does not change at a fixed cone element. The transverse and longitudinal solutions are matched at the time $t = t_{FO}$. Knowing, that $dS = su^0dz$ at a given time within element dz and entropy density, $s = ce^{3/4}$, we express entropy change over rapidity from the energy density formula (12), as:

$$\frac{dS}{dy} = ce_0^{3/4}\exp[-(y_+ + y_- - \sqrt{y_+y_-})]\frac{t}{\cosh y}. \tag{19}$$

Inserting the solution for the freeze-out time equation (18) to the entropy equation above and assuming that the number of produced particles is directly proportional to the entropy, $dN \propto dS$, one can obtain number of particles for different rapidities for a fixed angle ϕ. What is still needed, is the function $f(R_\phi)$ and initial thickness Δ.

IV. INITIAL CONDITIONS AND RESULTS

As it is natural for hydrodynamics, the initial state is based on the predictions from other models. In this case the widely accepted and analytically simple Wounded Nucleon (WN) model [19] will be used to parametrize initial conditions. It is based on the Woods-Saxon nuclear density parametrization [20], as follows:

$$\rho_A(\mathbf{r}) = \frac{\rho_0}{1+\exp(\frac{\mathbf{r}-R_A}{d})}, \tag{20}$$

which is continuous and can be connected to the Landau equations straightforwardly. The main requirement for the initial conditions and new function $f(R_\phi)$ is that for the central collision case, $b = 0$, the result must be equal to the original Landau one. Moreover, WN model connects impact parameter b with number of participating nucleons, N_{part}, and number of binary collisoins, N_{coll}, what makes easy comparison with the experimental data.

The density of wounded nucleons in the transverse plane and in polar coordinates, (r, ϕ), can be obtained by:

$$\begin{aligned} n_{WN}(r,\phi) = T_A(r,\phi)\left[1-\Big(1-\frac{\sigma T_B(r,\phi)}{B}\Big)^B\right] \\ + T_B(r,\phi)\left[1-\Big(1-\frac{\sigma T_A(r,\phi)}{A}\Big)^A\right]. \end{aligned}$$

Here, and everywhere else vector r starts at the center of the almond shaped system of interest. The thickness functions are then expressed, as:

$$T_A(r,\phi) = T_A(x - b/2, y) = \int dz\ \rho_A(\mathbf{r}),$$

$$T_B(r,\phi) = T_B(x + b/2, y) = \int dz\ \rho_B(\mathbf{r}),$$

using Woods-Saxon parametrization (20) with $R_A = 1.12A^{1/3} - 0.86A^{-1/3}$ [fm], $d = 0.54$ [fm] and $n_0 = 0.17fm^{-3}$.

Now assuming, that energy density is proportional to the WN density [21]: $e(r,\phi;b) \propto n_{WN}(r,\phi;b)$, the function $f(R_\phi)$ can be obtained. It is by definition, the ratio of energy density at the edge of the system with energy density at the center, for the fixed impact parameter b and reads as:

$$f(R_\phi) = \frac{n_{WN}(R_\phi,\phi;b) - \min(n_{WN}(R_\phi,\phi;b))}{n_{WN}(0,0;b)}. \tag{21}$$

The radius of the system, R_ϕ, is dependent on the angle ϕ and is obtained from the geometry on how two circles overlap, as: $R_\phi^2 + R_\phi b\cos\phi + \frac{b^2}{4} - R_A^2 = 0$. The term $\min(n_{WN}(R_\phi,\phi;b))$ is a minimal density at the edge of the system and is used in order to have the original Landau solution for $R_\phi = R_A$, so that $f(R_A) = 0$. Because originally Landau used "sharp sphere" picture, assuming, that energy density is zero at the edge of the system while solving (15). In the case of Woods-Saxon, the density (20) at the edge of the nuclei at $r = R_A$ is not zero, so the minimal value is subtracted. Now the acceleration term in (15) is the same in central collision and in Landau, but for peripheral collisions acceleration does depend on the angle ϕ.

Finally, to calculate the elliptic flow (2) one should merge equations (18, 19, 21) and initial longitudinal thickness Δ, which for the peripheral collisions is expressed , as:

$$\Delta(\phi) = \kappa_\phi R_A/\gamma, \tag{22}$$

where $\kappa_\phi = \sqrt{n_{WN}(R_\phi,\phi;b)/\max(n_{WN}(R_\phi,\phi;b))}$. The following means, that longitudinal expansion (5) starts with azimuthally asymmetric initial thickness, which is wider where initial nuclear density is higher. The term $\max(n_{WN}(R_\phi,\phi;b))$ is used to have no effect of the modification for central collision case, $\kappa_\phi(b = 0) = 1$. Obtained results are shown in figure (2) for Au-Au at $b = 6$fm and Cu-Cu at $b = 3$fm reactions for two different beam energies, used at RHIC.

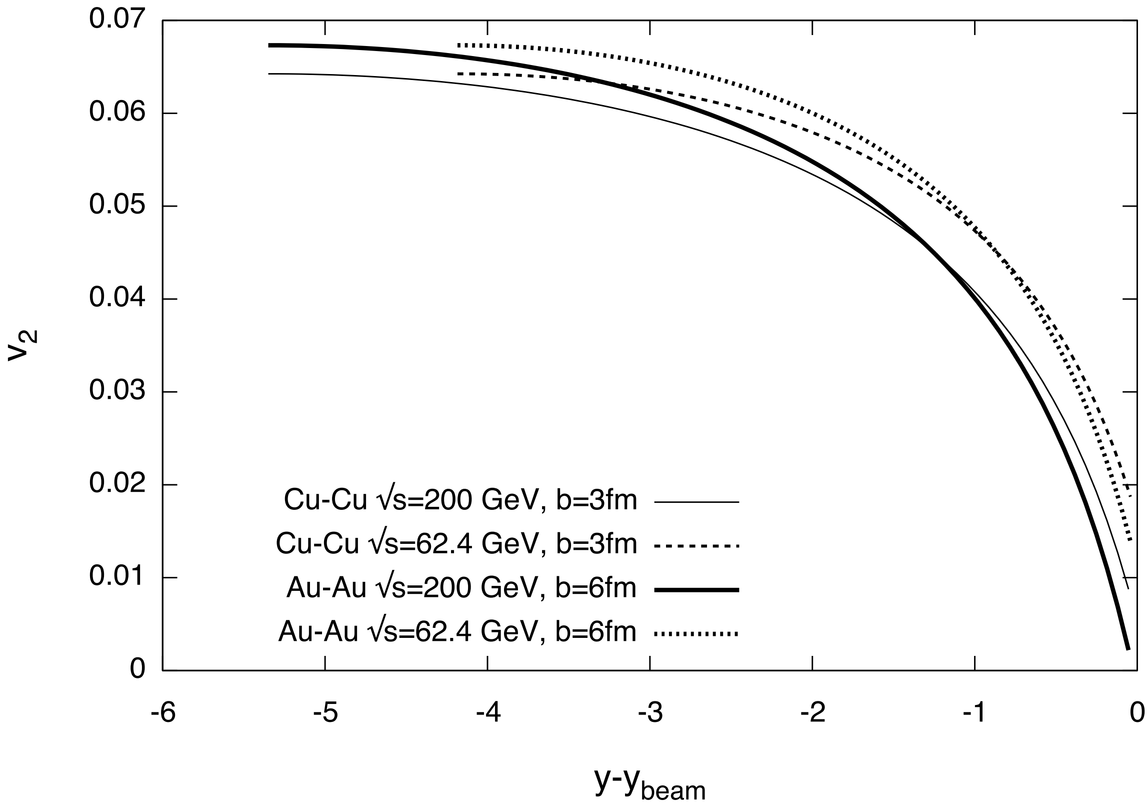

FIG. 2. Elliptic flow dependance on rapidity for Au-Au at $b = 6$ fm and Cu-Cu at $b = 3$ fm for two different collision energies $\sqrt{s_{NN}} = 62.4$ GeV and 200 GeV.

Initial state is chosen to be as simple and transparent as possible, in order to show pure hydrodynamic effects for the system with transverse asymmetry. The initial energy density distribution $f(R_\phi)$ and initial longitudinal thickness $\Delta(\phi)$ might be based on more sophisticated models, but for now we can conclude, that Landau hydrodynamic solution and it's assumptions work not only for particle multiplicity spectra, but for elliptic flow and elliptic flow longitudinal scaling as well.

The generalized solution of Landau hydrodynamics incorporates transverse asymmetries into initial configuration and can be compared to the wide amount of experimental data, giving deeper insight into the early dynamics of the system.

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