# Deciphering non-femtoscopic two-pion correlations in $p + p$ collisions with simple analytical models

S.V. Akkelin and Yu.M. Sinyukov

*Bogolyubov Institute for Theoretical Physics,*

*Metrolohichna str. 14b, 03680 Kiev, Ukraine*



## Abstract

A simple model of non-femtoscopic particle correlations in proton-proton collisions is proposed. The model takes into account correlations induced by the conservation laws as well as correlations induced by minijets. It gives reasonable description of the two-pion non-femtoscopic correlations of like-sign and unlike-sign pions in proton-proton collision events at $\sqrt{s} = 900$ GeV reported by the ALICE Collaboration. We also argue that the similar non-femtoscopic correlations could appear in hydrodynamic picture with event-by-event fluctuating non-symmetric initial conditions that are typically associated with non-zero higher-order flow harmonics.

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## I. INTRODUCTION

The two-particle femtoscopy of identical particles allows one to analyze the space-time structure of the particle emission from the systems created in heavy ion, hadron and lepton collisions (for reviews see, e.g., Ref. [1]). It is established that specific transverse momentum dependence of the femtoscopy scales - interferometry, or HBT radii - in heavy ion collisions is caused by the collective expansion of the systems and are associated with the homogeneity lengths [2]. As for elementary particle collisions, like $p+p$, where the collective (hydrodynamic) behavior of the matter is open to quest, there is no unambiguous interpretation of $p_T$-dependence of the HBT radii. Moreover, the transverse momentum behavior of the femtoscopy scales extracted from hadron and lepton collisions depends on correlation baseline assumption [3–5] about the strength and momentum dependence of the non-femtoscopic correlations. Such correlations appear also between unlike particles and are, typically, long-range in momentum space. As opposite to the femtoscopic (HBT) correlations, they are not conditioned by the quantum statistics and are not directly related to the spatiotemporal scales of the emitter or to the well studied Coulomb and strong final state interactions. These correlations can appear because of various reasons, e.g., the well known example of the non-femtoscopic correlations is the correlations induced by the energy and momentum conservation laws (see, e.g., Ref. [6]). The correlations do not affect essentially the HBT radii extracted from heavy ion collisions, but are rather noticeable for elementary particle collisions.

Our aim here is to demonstrate with simple analytical models how the minijets, transverse momentum conservation and event-by-event momentum spectra fluctuations (the latter can be conditioned by the asymmetrically fluctuating hydrodynamical densities) contribute to the two-pion correlations in $p+p$ collisions. In particular, we show that simple analytical model with minijets and momentum conservation induced correlations can fit the correlations of unlike-sign pion pairs at $\sqrt{s} = 900$ GeV $p+p$ collisions as well as non-femtoscopic correlations of identical pions obtained from the PHOJET event generator [7] simulations of $p+p$ collision events at $\sqrt{s} = 900$ GeV, the latter has been utilized as the correlation baseline by the ALICE Collaboration [3].

## II. DEFINITIONS AND PARAMETERIZATIONS OF TWO-PARTICLE CORRELATIONS

The two-particle correlation function is defined as

$$C(p_1,p_2)=\frac{P(p_1,p_2)}{P(p_1)P(p_2)}, \tag{1}$$

where $P(p_1,p_2)$ is the probability to observe two particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ while $P(p_1)$ and $P(p_2)$ designate the single-particle probabilities. Experimentally, the two-particle correlation function is defined as the ratio of the distributions of particle pairs from the same event and the pairs from different events. In heavy ion collisions almost all the correlations between identical pions with low relative momentum are due to quantum statistics (QS) and final-state (FS) interactions. Then

$$C(p_1,p_2)=C_F(\mathbf{p},\mathbf{q}), \tag{2}$$

where $\mathbf{p}=(\mathbf{p}_1+\mathbf{p}_2)/2$, $\mathbf{q}=\mathbf{p}_2-\mathbf{p}_1$, and $C_F$ denotes the femtoscopic correlation function. In the case of identical bosons $C_F$ is often parameterized (after corrections for FS correlations) by the Gaussian form which for the one-dimensional parametrization looks like

$$C_F(|\mathbf{p}|,q_{inv})=1+\lambda\exp\left(-R_{inv}^2q_{inv}^2\right). \tag{3}$$

Here $\lambda$ describes the correlation strength, $R_{inv}$ is the Gaussian "invariant" HBT radius, and $q_{inv}=\sqrt{(\mathbf{p}_2-\mathbf{p}_1)^2-(E_2-E_1)^2}$ is equal to the modulus of the momentum difference in the pair rest frame.

In elementary particle collisions additional (non-femtoscopic) correlations, like those arising from jet/string fragmentation and from energy and momentum conservation (see, e.g., Refs. [3–5]) can give also essential contribution. Then, assuming the factorization property,

$$C(p_1,p_2)=C_F(\mathbf{p},\mathbf{q})C_{NF}(\mathbf{p},\mathbf{q}). \tag{4}$$

Here $C_{NF}$ denotes the non-femtoscopic correlation function, and in the simplest case the non-femtoscopic effects can be parameterized as, e.g., 2nd order polynomial

$$C_{NF}(|\mathbf{p}|,q_{inv})=a+bq_{inv}+cq_{inv}^2. \tag{5}$$

This form can be used together with some parametrization of $C_F$ (e.g., with (3)) in order to fit the correlation function $C(p_1,p_2)$ for small systems, as this have been done, for example, by the STAR Collaboration for two-pion correlation functions in $p+p$ collisions at $\sqrt{s} = 200$ GeV [5]. At $c > 0$ the phenomenological parametrization (5) explicitly reproduces the well known effect of positive correlations between particles with large relative momenta $|\mathbf{q}|$ caused by the energy-momentum conservation laws, see the EMCIC model for $C_{NF}$ [6]. Note that $a$, $b$, and $c$ in Eq. (5) depend, in general, on $|\mathbf{p}|$, and they are defined typically by fitting of the related results in complicated models with relatively high number of adjusting parameters.

Recently the ALICE Collaboration utilized some event generators for an estimate of the correlation baseline (i.e., non-femtoscopic correlation function of identical pions) under the Bose-Einstein peak [3, 4]. It was motivated by the reasonable agreement of the corresponding event generator simulations with the experimental data for the correlation functions of oppositely charged pions in proton-proton collision at LHC energies.[1] Similarly the results for non-identical pion correlation functions, the correlation baseline simulated by the event generators at relatively low $q_{inv}$ decreases with $q_{inv}$ for relatively high $p_T$ and demonstrates approximately flat in $q_{inv}$ behavior for low $p_T$. It was conjectured in Refs. [3, 4] that such a behavior is conditioned by the correlations induced by minijets created in the event generator simulations.

In what follows we propose simple analytical model with the minimal number of parameters for the two-pion correlations induced by minijets and transverse momentum conservation law that can reproduce the above mentioned results and allows one to see clearly the physical mechanisms responding for the peculiarities of $p_T$- and $q_{inv}$- behavior of the unlike-sign pion correlations as well as like-sign non-femtoscopic pion correlations. We make comparison with the two-pion correlation functions in proton-proton collision at $\sqrt{s} = 900$ GeV [3]. For convenience, we compare results with the PHOJET event generator[2] simulations reported in Ref. [3] for like-sign pion pairs. Note that the simulations carried out by the ALICE Collaboration gave similar results for all utilized event generators [3, 4].

[1] Note that unlike-sign pion pairs cannot be directly used for calculation of the correlation baseline for identical pion correlations at least because of the different resonance contributions.

[2] The PHOJET event generator [7] accounts for soft and hard processes, takes into account energy and momentum conservation, and does not include the effects of quantum statistics.

## III. ANALYTICAL MODEL FOR MINIJETS AND MOMENTUM CONSERVATION INDUCED CORRELATIONS

Let us assume that $N$ particles of the same species (say, pions) are produced with momenta $\mathbf{p}_1, ..., \mathbf{p}_N$ in $(N + X)$ multiparticle production events. Then $N$-particle probability density, $P_N(p_1, ..., p_N)$, is symmetrical function as for all $N!$ permutations of the particle momenta $p_i$. For convenience, it is normalized by

$$\int d\Omega_p P_N(p_1, ..., p_N) = 1, \tag{6}$$

where $d\Omega_p = \frac{d^3p_1}{E_1}...\frac{d^3p_N}{E_N}$. Then $N'$-pion probability density, $N' < N$, is defined as

$$P_N(p_1, ..., p_{N'}) = \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*)...E_{N'}^* \delta^{(3)}(\mathbf{p}_{N'} - \mathbf{p}_{N'}^*) P_N(p_1^*, ..., p_N^*). \tag{7}$$

Now, aimed to estimate non-femtoscopic pion correlations, we consider pions as distinguishable, yet equivalent particles with symmetrical probability density functions, and will not make difference between different pion species neglecting, thus, the final state interactions. A distinguishably of equivalent particles means that there is no quantum interference between possibilities that correspond to all $N!$ permutations of the particle momenta $p_i$, then the symmetrized $N$-particle probability density can be defined as

$$P_N(p_1, ..., p_N) = \frac{1}{N!} \sum_{i \neq ... \neq k = 1}^{N} \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_i^*)...E_k^* \delta^{(3)}(\mathbf{p}_N - \mathbf{p}_k^*) \widehat{P}_N(p_1^*, ..., p_N^*), \tag{8}$$

where non-symmetrized $N$-particle probability density, $\widehat{P}_N(p_1, ..., p_N)$, is normalized to unity, and $N!$ in the denominator is required to guarantee the normalization condition (6). Then, taking into account Eq. (7), we see that the single-particle probability, $P_N(p_1)$, and the two-particle probability, $P_N(p_1, p_2)$, can be written as

$$P_N(p_1) = \frac{1}{N} \sum_{i=1}^{N} \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_i^*) \widehat{P}_N(p_1^*, ..., p_N^*), \tag{9}$$

$$P_N(p_1, p_2) = \frac{1}{N(N-1)} \sum_{i \neq j = 1}^{N} \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_i^*) E_j^* \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_j^*) \widehat{P}_N(p_1^*, ..., p_N^*). \tag{10}$$

The non-symmetrized $N$-pion probability density in such events reads

$$\widehat{P}_N(p_1, ..., p_N) = \frac{1}{K} \sum_X \int d\Omega_k \delta^{(4)}(p_a + p_b - \sum_{i=1}^{N} p_i - \sum_{j=1}^{X} k_j) |M_{N+X}(p_1, ..., k_X)|^2, \tag{11}$$

where $M_{N+X}(p_1, ..., k_X)$ is non-symmetrized $(N+X)$-particle production amplitude, $p_a$ and $p_b$ are 4-momenta of colliding particles (protons), and $K$ is the normalization factor,

$$K = \sum_X \int d\Omega_k d\Omega_p \delta^{(4)}(p_a + p_b - \sum_{i=1}^{N} p_i - \sum_{j=1}^{X} k_j)|M_{N+X}(p_1, ..., k_X)|^2. \tag{12}$$

Expression (11) for $\widehat{P}_N(p_1, ..., p_N)$ is rather complicated because, in particular, it depends on $X$ particles that are produced in addition to $N$ pions. It means also that one can hardly expect that total energy or momentum of the pion subsystem are constants in the system's center of mass, instead, one can expect that they fluctuate in event-by-event basis. Here we assume that the total transverse momentum of $N$ pions is equal to zero in the system's center of mass (keeping, however, in mind that this constraint is, in fact, too strong and can be weakened if necessary), and neglect the constrains conditioned by the conservation of energy and longitudinal momentum supposing that the system under consideration is barely $N$-pion subsystem in a small midrapidity region of the total system. Then, motivated by Eq. (11), we assume that a non-symmetrized $N$-pion probability density can be written as

$$\widehat{P}_N(p_1, ..., p_N) = \frac{1}{K}\delta(p_1, ..., p_N)F_N(p_1, ..., p_N), \tag{13}$$

where $F_N(p_1, ..., p_N)$ is a non-symmetrized function of pionic momenta, $\delta(p_1, ..., p_N)$ denotes average constraints on the $N$-pion states that appear due to energy and momentum conservations in multiparticle production events, and we assume that

$$\delta(p_1, ..., p_N) = \delta^{(2)}(\mathbf{p}_{T1} + \mathbf{p}_{T2} + ... + \mathbf{p}_{TN}), \tag{14}$$

where $\mathbf{p}_{T1}, \mathbf{p}_{T2}, ...\mathbf{p}_{TN}$ are transverse components of the momenta of the $N$ particles. Then the normalization factor is

$$K = \int d\Omega_p \delta(p_1, ..., p_N)F_N(p_1, ..., p_N). \tag{15}$$

If the only correlations are the correlations that associate with transverse momentum conservation, we have

$$F_N(p_1, ..., p_N) = f(p_1)f(p_2)...f(p_{N-1})f(p_N), \tag{16}$$

and calculations of single-particle and two-particle probability densities in large $N$ limit result in the special case of the EMCIC parametrization [6] of the correlations induced by the energy and momentum conservation laws. However, such a simple prescription is not enough to reproduce the non-femtoscopic unlike-sign pion correlations as well as generated by the event generators like PHOJET like-sign pion correlations. Based on the physical background of such generators, one can conjecture that the corresponding non-femtoscopic correlations can be caused by the jet-like and energy-momentum conservation induced correlations. Then, to describe the non-femtoscopic pion correlations in a simple analytical model, let us assume that there are no other correlations in the production of $N$-pion states except the correlations induced by transverse momentum conservation and cluster (minijet) structures in momentum space. For the sake of simplicity we assume here that the only two-particle clusters appear. Then one can write for fairly large $N \gg 1$

$$F_N(p_1, ..., p_N) = f(p_1)...f(p_N)Q(p_1, p_2)...Q(p_{N-1}, p_N), \quad (17)$$

where $Q(p_i, p_j)$ denotes the jet-like correlations between momenta $\mathbf{p}_i$ and $\mathbf{p}_j$; existence of such correlations means that $F_N$ cannot be expressed as a product of one-particle distributions. Then, utilizing the integral representation of the $\delta$-function by means of the Fourier transformation, $\delta^{(2)}(\mathbf{p}_T) = (2\pi)^{-2} \int d^2 r_T \exp(i\mathbf{r}_T\mathbf{p}_T)$, and accounting for Eqs. (9), (13), (14), (17), the single-particle probability reads

$$P_N(p_1) = \frac{1}{(2\pi)^2 K} \int d^2 r_T G_N(\mathbf{p}_1, \mathbf{r}_T), \quad (18)$$

where

$$G_N(\mathbf{p}_1, \mathbf{r}_T) = \int d\Omega_{p^*} E_1^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) e^{i\mathbf{r}_T(\mathbf{p}_{T1}^* + ... + \mathbf{p}_{TN}^*)} F_N(p_1^*, ..., p_N^*). \quad (19)$$

A possibility of different cluster configurations of particles means, in particular, that registered particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ can belong either to different minijets or to the same minijet. Then, taking into account Eqs. (10), (13), (14), (17), we get

$$P_N(p_1, p_2) = \frac{N}{N(N-1)} P_N^{1jet}(p_1, p_2) + \frac{N(N-1) - N}{N(N-1)} P_N^{2jet}(p_1, p_2), \quad (20)$$

where

$$P_N^{1jet}(p_1, p_2) = \frac{1}{(2\pi)^2 K} \int d^2 r_T G_N^{1jet}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T), \quad (21)$$

$$P_N^{2jet}(p_1, p_2) = \frac{1}{(2\pi)^2 K} \int d^2 r_T G_N^{2jet}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T), \quad (22)$$

and

$$G_N^{1jet}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T) = \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) E_j^* \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_2^*) e^{i\mathbf{r}_T(\mathbf{p}_{T1}^* + ... + \mathbf{p}_{TN}^*)} F_N, \quad (23)$$

$$G_N^{2jet}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T) = \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) E_j^* \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_3^*) e^{i\mathbf{r}_T(\mathbf{p}_{T1}^* + ... + \mathbf{p}_{TN}^*)} F_N, \quad (24)$$

here $F_N \equiv F_N(p_1^*, ..., p_N^*)$. The first term in the right hand side of Eq. (20) is associated with events where the two registered particles appear from the different minijets, and second term corresponds to events when the particles belong to the same minijet. Evidently, the latter happens relatively rare, however notice that the second term can be significant for small systems with not very large $N$.

## IV. RESULTS AND DISCUSSION

Now let us check whether this model can reproduce with reasonable parameters the non-femtoscopic correlation functions of unlike-sign pions measured by the ALICE Collaboration [3] and like-sign pions that are generated in PHOJET simulations and utilized as the correlation baseline by the ALICE Collaboration [3]. Calculations within the model will be deliberately as simple as possible just to demonstrate its viability. We do not use here the approximate methods like the saddle point approach, instead we utilize appropriate analytical parameterizations of the functions in interest, namely,

$$f(p_i) = E_i \exp\left(-\frac{\mathbf{p}_{i,T}^2}{T_T^2}\right) \exp\left(-\frac{\mathbf{p}_{i,L}^2}{T_L^2}\right), \tag{25}$$

and

$$Q(p_i, p_j) = \exp\left(-\frac{(\mathbf{p}_i - \mathbf{p}_j)^2}{\alpha^2}\right), \tag{26}$$

where $T_T$, $T_L$ and $\alpha$ are some parameters, and in what follows we assume that $T_L \gg T_T$. In accordance with ALICE baseline obtained from the PHOJET event generator simulations, we assume that only $q_{inv}$ is measured for each $\mathbf{p}_T$ bin. Assuming that longitudinal components of the registered particles are equal to zero, $p_{1L} = p_{2L} = 0$, we approximate $q_{inv}^2$ as

$$q_{inv}^2 \approx \mathbf{q}_T^2 \left(\frac{m^2 + \mathbf{p}_T^2 \sin^2\phi}{m^2 + \mathbf{p}_T^2}\right), \tag{27}$$

where $\phi$ denotes unregistered angle between $\mathbf{p}_T$ and $\mathbf{q}_T$, $\mathbf{p}_T\mathbf{q}_T = |\mathbf{p}_T||\mathbf{q}_T|\cos\phi$. Then

$$C_{NF}(|\mathbf{p}_T|, q_{inv}) = \frac{\int_0^{2\pi} d\phi P_N(p_1, p_2)}{\int_0^{2\pi} d\phi P_N(p_1) P_N(p_2)} \tag{28}$$

and, taking into account Eq. (20), we get

$$C_{NF}(|\mathbf{p}_T|, q_{inv}) = \frac{N-2}{N-1}\left(C_N^{2jet}(|\mathbf{p}_T|, q_{inv}) + \frac{1}{N-2} C_N^{1jet}(|\mathbf{p}_T|, q_{inv})\right), \tag{29}$$

where

$$C_N^{2jet}(|\mathbf{p}_T|, q_{inv}) = \frac{\int_0^{2\pi} d\phi P_N^{2jet}(p_1, p_2)}{\int_0^{2\pi} d\phi P_N(p_1) P_N(p_2)}, \tag{30}$$

$$C_N^{1jet}(|\mathbf{p}_T|, q_{inv}) = \frac{\int_0^{2\pi} d\phi P_N^{1jet}(p_1, p_2)}{\int_0^{2\pi} d\phi P_N(p_1) P_N(p_2)}. \tag{31}$$

It is well known, see e.g. Ref. [6], that influence of exact conservation laws on single-particle and two-particle momentum probability densities at the $N$-particle production process depends on a value of $N$ and disappears at $N \to \infty$. Since one considers a subsystem of $N$ pions but not total system, to weaken influence of total transverse momentum conservation on pions we shall consider $C_M^{1jet}$ and $C_M^{2jet}$ with $M > N$ instead of $C_N^{1jet}$ and $C_N^{2jet}$ in Eq. (29). It is the simplest way to account for a weakened conservation law in our model. At the same time the factor $1/(N-2)$ in (29) is associated with combinatorics of distribution of particles between clusters in momentum space ("minijets"), which happens no matter if one weakens or not the total momentum conservation law. Also, for more exact fitting of the data points in each average transverse momentum bin, we utilize the auxiliary factors, $\Lambda$, when compared results of our calculations with ALICE two-pion correlation and simulation data, these proportionality factors differ slightly from unit in our calculations (nearly 0.9). Then Eq. (29) gets the form

$$C_{NF}(|\mathbf{p}_T|, q_{inv}) = \Lambda(\mathbf{p}_T)(C_M^{2jet}(|\mathbf{p}_T|, q_{inv}) + \frac{1}{N-2} C_M^{1jet}(|\mathbf{p}_T|, q_{inv})). \tag{32}$$

The results of our calculations of the non-femtoscopic correlation functions $C_{NF}$ are shown in Figs. 1-10 in comparison with the non-femtoscopic correlation functions reported by the ALICE Collaboration [3] for different transverse momentum of pion pairs (actually, we performed calculations for the mean value in each bin). The data for unlike-sign pion correlations as well as for PHOJET simulations of identical two-pion non-femtoscopic correlation functions at midrapidity for total charged multiplicity $N_{ch} \geq 12$ bin in $p+p$ collisions at $\sqrt{s} = 900$ GeV are taken from Refs. [3] and [8]. Our results are obtained for $M = 50$, $T_T = \alpha = 0.65$ GeV (to minimize the number of fit parameters, we fixed $T_T = \alpha$ for all calculations)[3], and the fitted values of $N$ are different for like-sign and unlike-sign pion pairs, namely, $N^{\pm\pm} = 20$ for the former and $N^{+-} = 11$ for the latter. The relatively high value of

[3] Note that with these parameter values the mean transverse momentum, $\langle p_T \rangle$, is about 0.58 GeV.

$M$ can be interpreted as residual effect to pion subsystem of total energy-momentum conservation in multiparticle production process. The relation, $N^{+-} < N^{\pm\pm}$, between fitted $N$ values means that the magnitude of minijet induced correlations for unlike-sign pion pairs is higher than for like-sign ones. It is natural for minijet induced two-pion correlations because there is no local charge conservation constraint for production of oppositely-charged pion pair and, therefore, one can expect less identically-charged pion pairs from the fragmenting minijets than oppositely-charged ones. It is worth to note that such a difference in the strength of the two-pion correlations cannot be attributed to resonances: although it is easier to produce an oppositely-charged pion pair from a resonance than an identically-charged one because, again, of the charge conservation constraint, a resonance decay results, typically, in back-to-back correlations of the produced particles according to local energy-momentum conservation laws, see e.g. Ref. [9]. It is unlike to one-side correlations from the minijet fragmentation. One can see from the figures that the behavior of the non-femtoscopic correlation functions of pions, $C_{NF}$, are reproduced well despite a simplicity of our model. This is a result of the competition of the two trends: an increase of the correlation function with $q_{inv}$ because of the momentum conservation and a decrease of it due to the fragmentation of one minijet into the registered pion pair. Figures 5 and 10 demonstrate also the relative contribution of the first and second terms in Eq. (32) to the non-femtoscopic correlation functions.

Note, however, that the lower magnitude of the non-femtoscopic correlations for like-sign pion pairs as compared to the correlations of unlike-sign pions is mainly caused by the assumed mechanism of pions production through minijets fragmentation, and it can be not the case for another production mechanisms. For example, hydrodynamic models are very successful in description of heavy ion collisions and give reasonable description of elementary particle collisions (see, e.g., Ref. [10] where it was demonstrated that EPOS model,[4] that includes hydrodynamical stage, can describe $p+p$ collisions at LHC energies), typically do not demonstrate noticeable minijets production at relatively low $p_T$. The choice of model for non-femtoscopic correlations is important for interpretation of physics in $p+p$ collisions. Indeed, the non-femtoscopic correlations of like-sign pion pairs obtained in PHOJET and similar event generators are utilized by the ALICE Collaboration as baseline for HBT analysis of

[4] EPOS model calculates flux tubes that are utilized as initial conditions for hydrodynamic expansion, and the later rare hadronic stage is calculated by means of hadronic cascade model (UrQMD).

the femtoscopic correlations, see Eq. (4), and this correlation baseline significantly affects the interferometry radii [3, 4].

Then the question arises: whether the non-femtoscopic effects in two-pion correlations are ultimately caused by minijets and conservation laws only, or the similar behavior can be attributed to hydrodynamics also? On the other hand, if such a behavior is ultimately related with minijets, it strongly restricts the area of applicability of the corresponding correlation baseline to the processes where the matter, in grate extent, is produced through emission of minijets. Note that if thermalization takes place in $p+p$ collisions and hydrodynamical evolution forms the soft spectra, then the PHOJET and similar models cannot give adequate description of particles production in this momentum region and an utilization of the corresponding correlation baseline for $p+p$ collisions can be in doubt.

Below we demonstrate that non-femtoscopic correlation functions in $p+p$ collisions can also appear in hydrodynamic models if one accounts for event-by-event fluctuating initial conditions for hydrodynamic stage (in hybrid models this stage is matched with subsequent hadronic cascade stage). Let us give here the illustrative example as for such a possibility. First, note that there are no correlations induced by the exact global energy momentum conservation in hydrodynamic/hybrid models, and corresponding conservation laws are satisfied only in average for particles that are produced at some hypersurface where hydrodynamics is switched off. Then, one can expect that the only source of the non-femtoscopic correlations in such models is event-by-event fluctuations of initial conditions for hydrodynamical stage. These fluctuations result in fluctuations of the two-particle and single-particle momentum spectra, and, as usual, effect of fluctuations is more pronounced for small systems. Then

$$\widehat{P}_N(p_1,p_2,...,p_N)=\sum_i w(u_i)\widehat{P}_N(p_1,p_2,...,p_N;u_i), \tag{33}$$

where $\widehat{P}_N(p_1,p_2,...,p_N;u_i)$ is $N$-particle probability density for some $u_i$-type of the initial conditions and $w(u_i)$ denotes distribution over initial conditions, $\sum_i w(u_i)=1$. Let us assume, for the sake of simplicity, uncorrelated particle emissions for each specific initial condition. Then, accounting for $\delta(p_1,...,p_N)=1$ in Eq. (13), one can write

$$\widehat{P}_N(p_1,p_2,...,p_N;u_i)=f(p_1;u_i)f(p_2;u_i)...f(p_{N-1};u_i)f(p_N;u_i), \tag{34}$$

where we normalize $f(p;u_i)$ as follows, $\int \frac{d^3p}{E}f(p;u_i)=1$, then $K=1$, see Eq. (15). Two-particle non-femtoscopic correlation function, $C_{NF}$, then reads

$$C_{NF}(p_1, p_2) = \frac{\sum_i w(u_i) f(p_1; u_i) f(p_2; u_i)}{\sum_i w(u_i) f(p_1; u_i) \sum_j w(u_j) f(p_2; u_j)}. \tag{35}$$

Evidently, the different type of fluctuations, i.e., the form of distribution $w(u_i)$, leads to the different behavior of the non-femtoscopic correlations. To illustrate that fluctuations can lead to the non-femtoscopic correlation functions that are similar to ones induced by minijets, let us consider the toy model where

$$w(\mathbf{u}_T) = \frac{\alpha^2}{\pi} \exp(-\mathbf{u}_T^2 \alpha^2), \tag{36}$$

$$f(p; \mathbf{u}_T) = \frac{\beta^2 \gamma}{\pi^{3/2}} E \exp(-(\mathbf{p}_T - \mathbf{u}_T)^2 \beta^2) \exp(-p_L^2 \gamma^2), \tag{37}$$

and normalization is chosen in such a way that $\int d^2 u_T w(\mathbf{u}_T) = 1$ and $\int \frac{d^3p}{E} f(p; \mathbf{u}_T) = 1$. Main feature of such a model is that event-by-event single-particle transverse momentum spectra have maximum for event-by-event fluctuating $\mathbf{p}_T$ values. Such momentum spectrum fluctuations could take place, e.g., in hydrodynamics with highly inhomogeneous initial energy density profile without cylindrical or elliptic symmetry. One can easily see that in this case $C_{NF}$ decreases with $q_T^2$,

$$C_{NF}(p, q) \sim \exp(-\frac{\beta^4}{2(\alpha^2 + \beta^2)} q_T^2), \tag{38}$$

and it means (after taking into account (27) and (28)) that $C_{NF}$ decreases with $q_{inv}^2$ too, that is similar to the behavior of $C_{NF}$ if the non-femtoscopic correlations are induced by minijets. At the same time, unlike the latter, the hydrodynamical fluctuations lead to the similar (up to the resonance contributions) correlations for like-sign and unlike-sign pion pairs.

## V. CONCLUSIONS

We can conclude that noticeable non-femtoscopic two-pion correlations can appear for small systems as a result of the cluster (minijet) structures in final momentum space of produced particles, or as a result of event-by-event fluctuating initial conditions for hydrodynamical stage; another source of the non-femtoscopic correlations is the global energy-momentum conservation constraints. The latter typically results in an increase with $q_{inv}$ for fairly high $q_{inv}$ of the non-femtoscopic two-pion correlation functions of small systems, whereas the former mostly determines a decrease of the ones at relatively low $q_{inv}$. We presented here the simple analytical model that takes into account correlations induced by the total transverse momentum conservation as well as minijets, and show that the model gives reasonable description of the two-pion non-femtoscopic correlations of identical and non-identical pions in proton-proton collision events at $\sqrt{s}$ = 900 GeV reported by the ALICE Collaboration [3].

Although details of particle production processes affect the non-femtoscopic correlations, the different types of multiparticle production mechanism could result in qualitatively similar non-femtoscopic correlation functions. We presented some heuristic arguments that the two-pion non-femtoscopic correlation functions calculated in hydrodynamics with event-by-event fluctuating initial conditions can be qualitatively similar at relatively low $q_{inv}$ to ones calculated in PHOJET like generators where the non-femtoscopic correlations for low $q_{inv}$ are mainly caused by the minijets. Then non-symmetrical fluctuations of initial conditions lead not only to non-zero $v_3$ and higher flow harmonics (see, e.g., Ref. [11]), but can also influence on the behavior of the HBT radii in inclusive measurements.[5] It is worthy noting the important difference between the non-femtoscopic correlations induced by minijets and hydrodynamical fluctuations: while the former lead to higher magnitude of the non-femtoscopic correlations for unlike-sign pion pairs as compared to like-sign pions, the latter result in similar strength of the non-femtoscopic correlations for identical and non-identical pions. This can result in the different predictions for the correlation baselines.

## VI. ACKNOWLEDGMENTS

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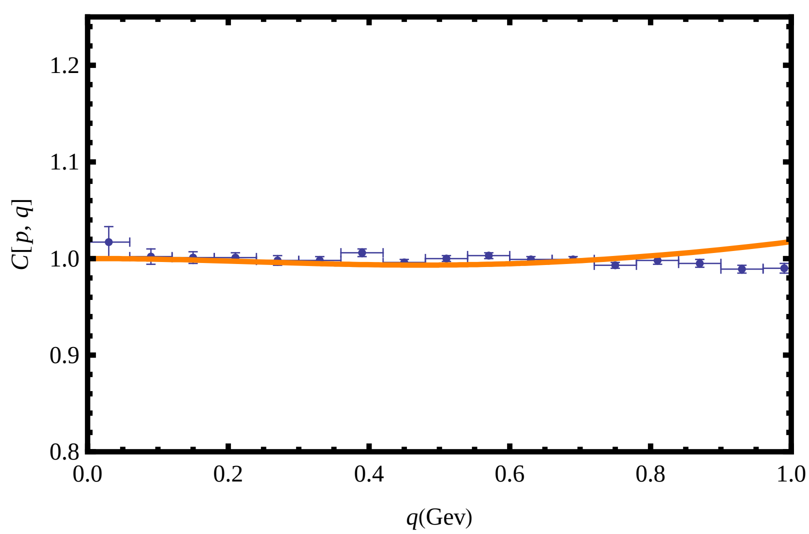


FIG. 1. The non-femtoscopic correlation functions of like-sign pions in $0.1 < p_T < 0.25$ GeV bin from a simulation using PHOJET [3, 8] (full dots) and that calculated from the analytical model: minijets + momentum conservation (full line), see the text for details.

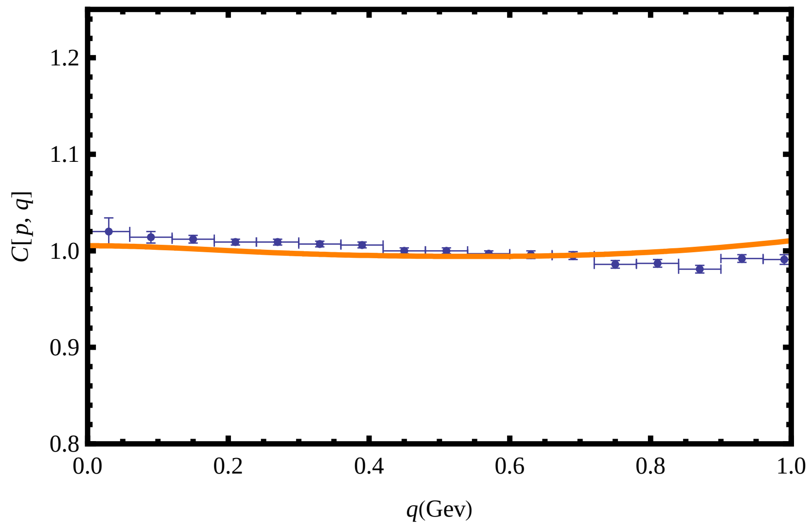


FIG. 2. The same as Fig. 1 but in $0.25 < p_T < 0.4$ GeV bin.

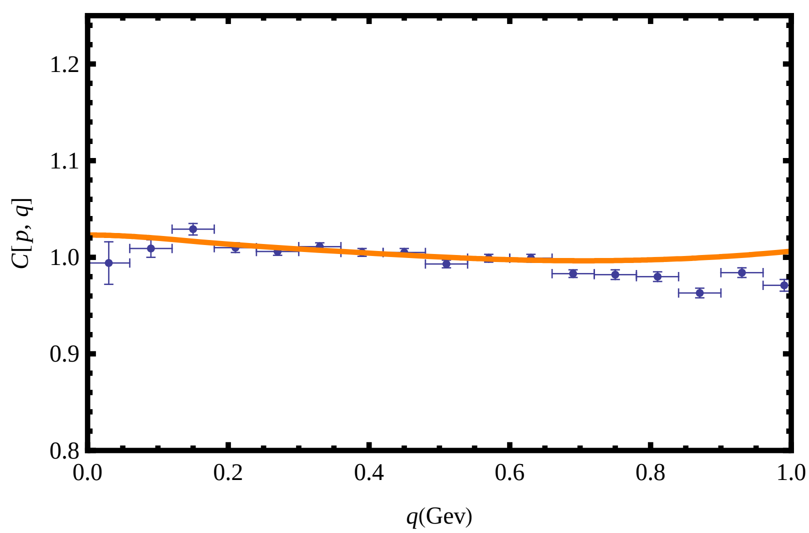


FIG. 3. The same as Fig. 1 but in $0.4 < p_T < 0.55$ GeV bin.

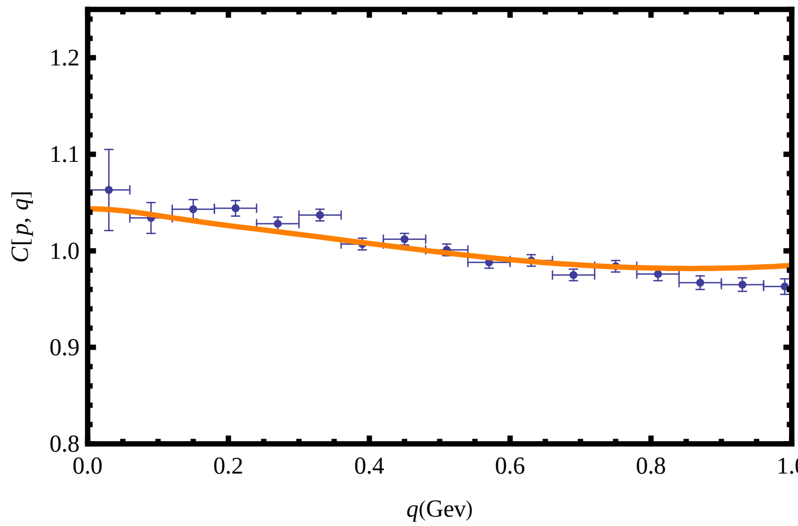


FIG. 4. The same as Fig. 1 but in $0.55 < p_T < 0.7$ GeV bin.

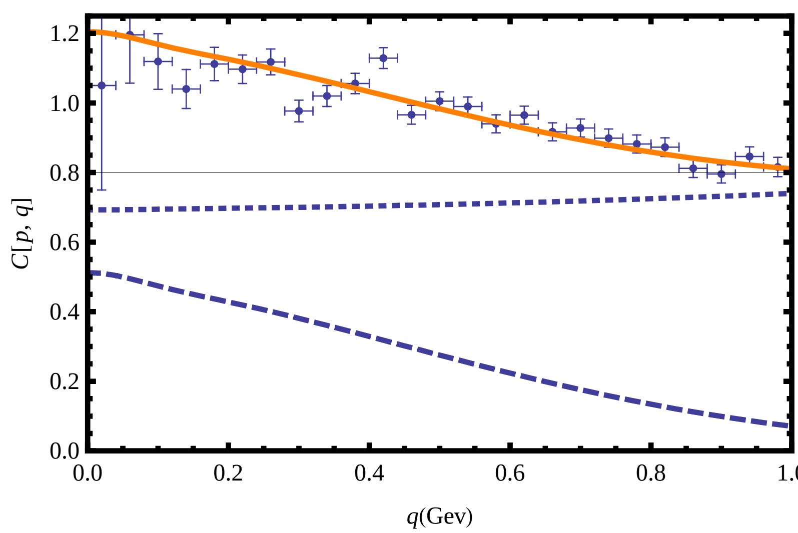


FIG. 5. Above thin horizontal line: the non-femtoscopic correlation functions of like-sign pions in $0.7 < p_T < 1.0$ GeV bin from a simulation using PHOJET [3, 8] (full dots) and that calculated from the analytical model (full line). Below thin horizontal line: relative contribution to non-femtoscopic correlation function by first (dotted line) and second (dashed line) terms of Eq. (32)

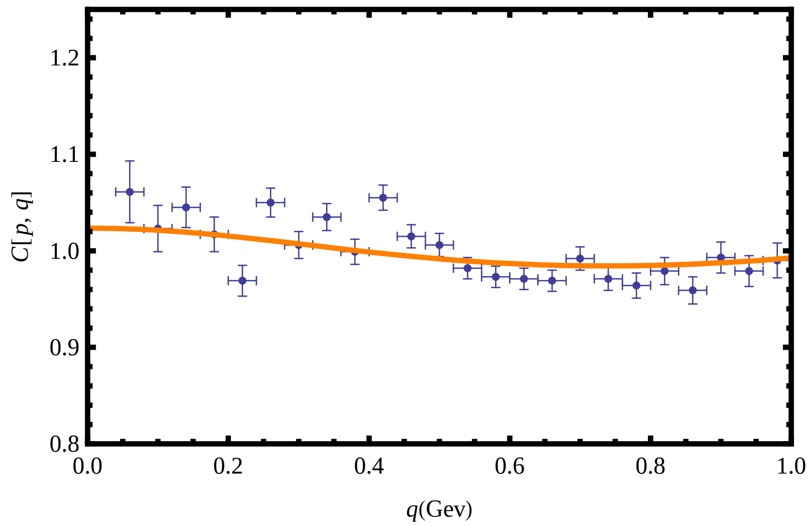


FIG. 6. The non-femtoscopic correlation functions of unlike-sign pions in $0.1 < p_T < 0.25$ GeV bin from Refs. [3, 8] (full dots) and that calculated from the analytical model: minijets + momentum conservation (full line), see the text for details.

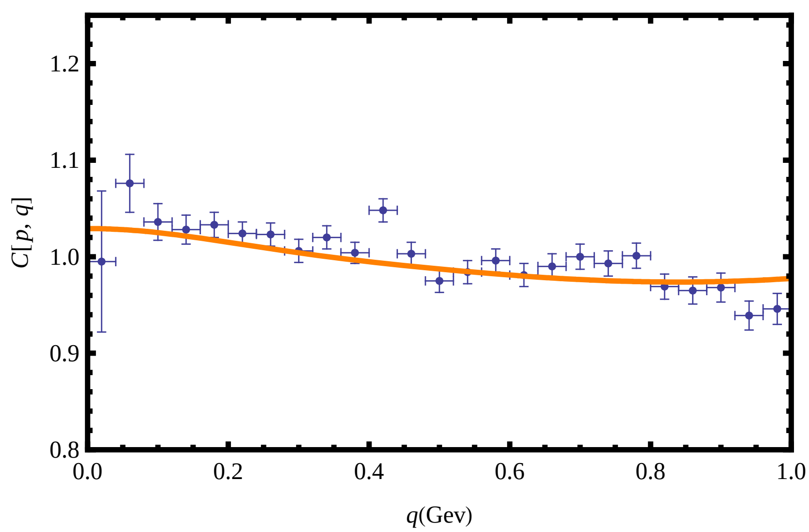


FIG. 7. The same as Fig. 6 but in $0.25 < p_T < 0.4$ GeV bin.

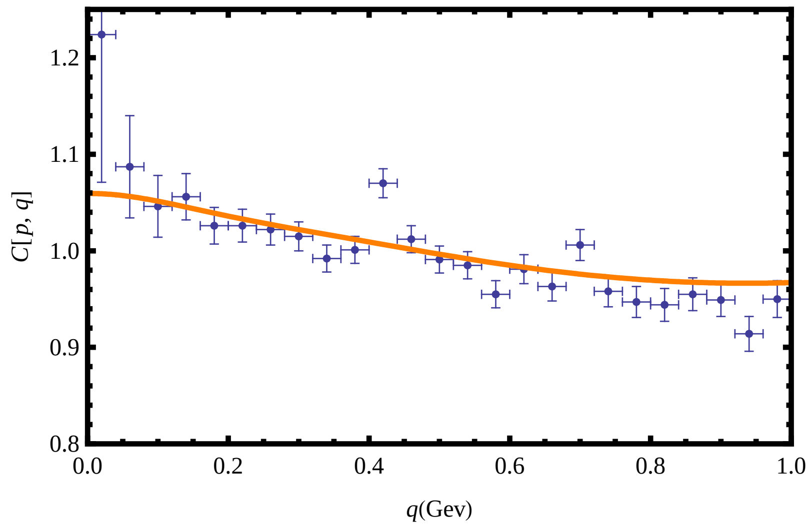


FIG. 8. The same as Fig. 6 but in $0.4 < p_T < 0.55$ GeV bin.

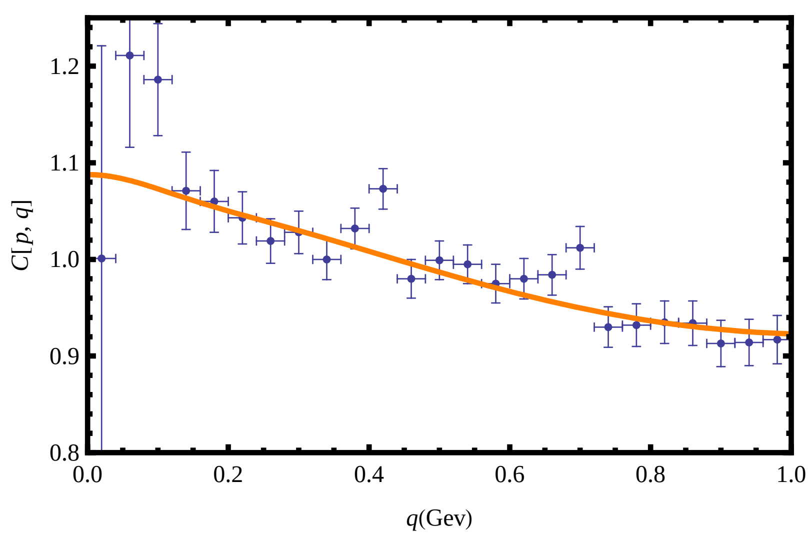


FIG. 9. The same as Fig. 6 but in $0.55 < p_T < 0.7$ GeV bin.

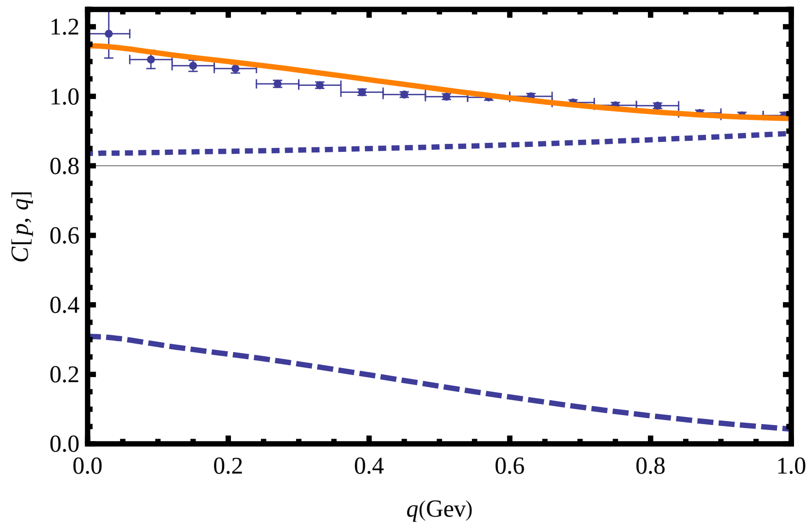


FIG. 10. Above thin horizontal line: the non-femtoscopic correlation functions of unlike-sign pions in $0.7 < p_T < 1.0$ GeV bin from Refs. [3, 8] (full dots) and that calculated from the analytical model (full line). Below thin horizontal line: relative contribution to non-femtoscopic correlation function by first (dotted line) and second (dashed line) terms of Eq. (32).