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Phenomenological analysis of angular correlations in 7 TeV proton-proton collisions from the CMS experiment

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A phenomenological analysis is presented of recent two-particle angular correlation data on relative pseudorapidity (η) and azimuth reported by the Compact Muon Solenoid (CMS) Collaboration for $\sqrt{s} = 7$ TeV proton-proton collisions. The data are described with an empirical jet-like model developed for similar angular correlation measurements obtained from heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC). The same-side (small relative azimuth), η-extended correlation structure, referred to as the *ridge*, is compared with three phenomenological correlation structures suggested by theoretical analysis. These include additional angular correlations due to soft gluon radiation in $2 \rightarrow 3$ partonic processes, a one-dimensional same-side correlation ridge on azimuth motivated for example by color-glass condensate models, and an azimuth quadrupole similar to that required to describe heavy ion angular correlations. The quadrupole model provides the best overall description of the CMS data, including the ridge, based on χ^2 minimization in agreement with previous studies. Implications of these results with respect to possible mechanisms for producing the CMS same-side correlation ridge are discussed.

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I. INTRODUCTION

The CMS Collaboration at the Large Hadron Collider (LHC) at CERN recently reported the observation of long-range two-particle angular correlations on relative pseudorapidity ($\eta_\Delta \equiv \eta_1 - \eta_2$) for charged particle pairs at mid-rapidity with relative azimuth ($\phi_\Delta \equiv \phi_1 - \phi_2$) $< \pi/2$ (near-side or same-side - SS) in high multiplicity proton-proton (p-p) collisions at $\sqrt{s} = 7$ TeV [1]. The collaboration noted the visual similarity of this same-side "ridge" with two-particle angular correlation structures observed in heavy ion collisions at RHIC [2–5]. For the latter, two-dimensional (2D) angular correlation analysis reveals at least two structures which produce same-side, relative η elongated correlations: (1) a quadrupole, $\cos 2\phi_\Delta$, independent of η (within two units at mid-rapidity) whose amplitude is approximately proportional to the square of the eccentricity of the transverse distribution of colliding nucleons from the impacting nuclei [6], and (2) a 2D peak centered at $(\eta_\Delta, \phi_\Delta) = (0,0)$ whose amplitude and width on relative pseudorapidity increase markedly with more central collisions (i.e. reduced impact parameter) [3]. The quadrupole correlation in heavy ion collisions is conventionally attributed to pressure driven, hydrodynamic flow [7] due to rapid equilibration of a partonic medium [8]. Inconsistencies between this assumed scenario and other RHIC data have been found however [9]; alternate mechanisms for quadrupole correlation production have been studied [10, 11]. The origin of the η-elongated 2D peak at RHIC is not understood although many ideas have been proposed. The appearance of one or both of these ridge-like correlation structures in p-p collisions at the LHC would be quite interesting.

A number of mechanisms have been proposed to explain the η-elongated 2D correlation peak observed in the heavy ion collision data at RHIC. Several of these authors have extended their models to include the CMS p-p results. Dumitru *et al.* [12] proposed that flux tubes of color glass condensate [13] formed immediately after impact produce correlations among the hadron fragments which are transported outward by an assumed radial flow. Shuryak [14] and Voloshin [15] invoked similar ideas except the initial stage correlations result from beam-jets, which are the fragmentation results of forward going nucleon remnants following semi-hard partonic scattering. Hwa *et al.* [16] proposed that the same-side ridge is generated by fast parton (from semi-hard scattering) - soft, "thermal" parton recombination into final-state hadrons. Wong [17] explained the same-side ridge by way of scattering (momentum kick) between fast, transverse scattered partons and soft partons of a medium. Levin and Rezaeian [18] argued that the long-range rapidity and azimuth correlations between charged hadron pairs produced from two BFKL parton showers will contribute to the CMS ridge. Werner *et al.* [19] interpreted the CMS results in terms of energy density fluctuations in the initial collision stage which manifest in the final-state via hydrodynamic expansion; the relevant correlation being a quadrupole.

Within perturbative QCD Field [20] suggested that initial and final state radiation effects in partonic $2 \rightarrow 3$ and $2 \rightarrow 4$ processes may contribute to extending the η range of the jet correlations. Sjöstrand [21] and Trainor [22] independently suggested that fragmenting color flux tubes, as in the LUND model of soft particle production [23], stretching between transverse scattered colored partons and longitudinal colored nucleon remnants from semi-hard collisions could produce η elongation in the jet correlation peak. None of the preceding models have been shown to account for all of the correlation [24] and p_t spectrum [25] features in the heavy-ion data from RHIC which are associated with the η-elongated peak phenomenon.

In a physical model-independent analysis Trainor and Kettler [26] showed that the systematic dependences of the quadrupole correlation amplitude on collision centrality, event multiplicity, and collision energy ($\sim \log\sqrt{s}$) determined from RHIC heavy-ion data [6], when extrapolated to LHC energies for high multiplicity p-p collision events, predicted quadrupole amplitudes which were in reasonable agreement with the CMS near-side ridge. They suggested that the CMS ridge and the quadrupole correlations at RHIC have a common dynamical origin, but not necessarily hydrodynamic. Bożek [27] also showed that the transverse momentum (p_t) selected azimuth projections of the CMS 2D correlations at larger relative pseudorapidity [1] were well described with $\cos\phi_\Delta$ (dipole) and $\cos 2\phi_\Delta$ (quadrupole) terms only.

In the present analysis the 7 TeV p-p angular correlation data from the CMS experiment were fitted in 2D angular space (η_Δ, ϕ_Δ) using a multi-component model similar to that used to describe angular correlations from Au-Au collisions at RHIC [3]. Four data sets were studied corresponding to combinations of event selection (minimum-bias or high multiplicity) and particle p_t selection ($p_t > 0.1$ GeV/c or $1 < p_t < 3$ GeV/c). Additional components (soft gluon radiation, azimuth quadrupole and same-side Gaussian ridge on azimuth) were included in the fitting model in order to phenomenologically represent the correlation structures expected for each class of theoretical models listed above. The purpose of this analysis is to determine which phenomenological description(s) of the p-p CMS correlation data is preferred based on fit quality in an effort to guide ongoing theoretical studies of the CMS ridge.

The paper is organized as follows. The phenomenological fitting model is explained in the next section; fitting results are presented in Sec. III. The relation of this analysis to previous studies of these data and the implications of the fitting phenomenology for theoretical models of η-elongated angular correlations in high energy collisions are discussed in Sec. IV. A summary and conclusions are given in Sec. V.

II. ANGULAR CORRELATIONS AND FITTING MODEL

A. Minimum-bias angular correlations

The two-particle angular correlation distributions from CMS [1] and the STAR experiment at RHIC [2–4] represent inclusive sums of all charged particle pairs within the tracking acceptances of the respective detectors. There is no special leading particle or "trigger" particle in this type of analysis and in this sense the CMS angular correlations may be referred to as *minimum-bias*. Correlations are obtained from normalized, binned ratios of real to mixed-event pairs, the latter being constructed by taking the two particles in a pair from different but similar events. If events produce more than one jet within the acceptance all intra-jet charged particle pairs contribute to the correlation peak at small relative angles near (0,0). Inter-jet two-particle correlations are indistinguishable from background for uncorrelated jets. Particle pairs from separate jets of a correlated, back-to-back dijet appear as a broad, opposite-side or away-side ($\phi_\Delta > \pi/2$, AS) ridge on azimuth centered at π. The same-side η-elongated correlations may or may not be associated with jets; proximity of correlation structures on relative angle does not necessarily imply proximity in absolute (η, ϕ) space.

B. Nominal fitting model components

The primary elements of the present fitting function were developed previously by the STAR Collaboration in order to describe 2D angular correlation data for charged particle production from minimum-bias p-p collisions at $\sqrt{s} = 0.2$ TeV [28]. Those studies used p_t cuts and like-sign and unlike-sign pair combinations to reveal simple geometrical shapes in the correlations. Those structures include: (1) a jet-like, 2D Gaussian peak at small relative angles with an accompanying away-side, η_Δ-independent ridge corresponding to approximately back-to-back dijets (e.g. PYTHIA predictions in [1]) or other p_t conserving correlation effects, (2) a narrower 2D exponential peak at zero opening angle representing quantum correlations [29] among identical charged particles, and (3) a 1D Gaussian ridge on η_Δ which is the 2D manifestation of charge-ordering first observed on relative η in collisions at the ISR [30]. The latter is thought to represent longitudinal (beam direction) fragmentation as described for example in the LUND model [23]. With the addition of an η-independent quadrupole this phenomenological model quantitatively describes the 2D angular correlation data from minimum-bias Au-Au collisions at STAR [2, 3].

C. Color coherence in $2 \rightarrow 3$ processes

In addition to the above model components important terms could arise from color coherence effects [31, 32]. Soft gluon radiation amplitudes from color connected partons in hard scattering $2 \rightarrow 2$ processes interfere producing increased yields in the hard scattering plane defined by the incident and scattered partons. Assuming hadronization does not obliterate the soft gluon distribution the resulting final state hadrons will increase the particle yield near the accompanying jet with a larger increase occurring along the beam direction, or on relative pseudorapidity than on relative azimuth. This next-to-leading-order process may therefore contribute to the CMS ridge. For dijets within the acceptance soft gluon radiation will also contribute to the away-side correlation ridge on azimuth.

Color coherence effects can be approximated within the soft gluon momemtum limit ($k \rightarrow 0$) [31] where the squared amplitude for the $2 \rightarrow 3$ process is factored into the product of the probability density for the $2 \rightarrow 2$ hard scattering times the soft gluon radiation probability distributions for each color connected parton pair. The total yield is obtained by summing over all color connected parton pairs. The dominant $2 \rightarrow 3$ reaction considered here for midrapidity hadron production is gluon-gluon scattering given by [31]

$$g(p_1) + g(p_2) \; \rightarrow \; g(p_3) + g(p_4) + g(k), \qquad (1)$$

where the soft gluon yield is approximately

$$P_g \; \approx \; C_A \sum_{i<j=1}^{4} W_{ij}, \qquad (2)$$

the summation includes all parton pairs, gluon color factor $C_A = N_c$ (number of colors), and

$$W_{ij} \; = \; g^2 \frac{p_i \cdot p_j}{p_i \cdot k \, k \cdot p_j}. \qquad (3)$$

In Eq. (3) g is the strong coupling constant and $p_i \cdot p_j$ etc. denote scalar products of 4-momentum vectors. Quantities W_{ij} are singular for massless partons in the limits $k \rightarrow 0$, $\theta_{ik} \rightarrow 0$, or $\theta_{jk} \rightarrow 0$ where θ are the angles between the parton and radiated soft gluon. To regulate the singularities small cut-off masses (5 MeV/c^2) were assumed for each parton and a Gaussian angular cut-off, $F_i = [1 - \exp(-\theta_{ik}^2/2\sigma_g^2)]$ where $\sigma_g = 0.2$ radians, was applied to each amplitude and W_{ij} was weighted with $F_i F_j W_{ij}$. The latter cut-off also respects angular ordering requirements for the subsequent jet fragmentation [31]. The soft gluon radiation yield in the soft limit was approximated by

$$P_g \; \approx \; A_g \frac{g^2 C_A}{E_{T_k}^2} \sin^2\theta_{1k} \sum_{i<j=1}^{4} F_i F_j \frac{\alpha_{ij}}{\alpha_{ik}\,\alpha_{jk}} \; , \qquad (4)$$

where $\alpha_{ij} = 1 - \beta_i \beta_j \cos\theta_{ij}$, $\beta = p/E$ (velocity), gluon 1 defines the beam direction, E_{T_k} is the transverse energy of the radiated gluon, and A_g is a phenomenological amplitude which will be adjusted by the fitting program. The final-state hadron distribution resulting from the soft radiated gluons is assumed to be given by Eq. (4) according to the local parton-hadron duality hypothesis (LPHD) [33].

The soft gluon radiation adds to the final-state hadron distribution associated with minimum-bias jet production, given by $P_{\rm jet,3} + P_{\rm jet,4} + P_g$. The first two yields correspond to hadron distributions from the fragmentation of hard scattered partons 3 and 4 in the reaction in Eq. (1). Each is represented by a 2D Gaussian distribution

$$P_{\rm jet}(\eta,\phi) \; = \; A e^{-(\eta-\eta_{\rm jet})^2/2\sigma_\eta^2} e^{-(\phi-\phi_{\rm jet})^2/2\sigma_\phi^2}, \qquad (5)$$

where η, ϕ are defined within the detector acceptance. The single particle Gaussian widths are directly proportional to the widths of the jet-like angular correlation on η_Δ and ϕ_Δ discussed below. It is expected that $P_g \ll P_{\rm jet}$ and the additional contribution to the 2D angular correlations is dominated by the cross term $P_g(P_{\rm jet,3} + P_{\rm jet,4})$ projected onto relative azimuth and pseudorapidity and averaged over all jet angular positions within the acceptance. The 2D angular autocorrelation in the soft gluon radiation limit for $2 \rightarrow 3$ processes was approximated by

$$A_{\rm jet,soft-g}(\eta_\Delta,\phi_\Delta) \; \approx \; \frac{1}{8\Omega(\eta_\Delta)} \frac{1}{2\pi\Delta\eta} \int_{-\pi}^{\pi} d\phi_{\rm jet,3} \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta_{\rm jet,3} \int_{-\Omega(\eta_\Delta)}^{\Omega(\eta_\Delta)} d\eta_\Sigma \int_{-\pi}^{\pi} d\phi_\Sigma$$
$$\times \; \{P_g(\eta_1,\phi_1)\,[P_{\rm jet,3}(\eta_2,\phi_2) + P_{\rm jet,4}(\eta_2,\phi_2)] + P_g(\eta_2,\phi_2)\,[P_{\rm jet,3}(\eta_1,\phi_1) + P_{\rm jet,4}(\eta_1,\phi_1)]\} \qquad (6)$$

where $\Delta\eta$ is the (symmetric) pseudorapidity acceptance, $\Omega(\eta_\Delta) = |\Delta\eta - |\eta_\Delta||$, $\eta_\Sigma = \eta_1 + \eta_2$ and $\phi_\Sigma = \phi_1 + \phi_2$. Jets 3 and 4 were for simplicity assumed to be uniformly distributed within the $2\pi\Delta\eta$ acceptance and back-to-back in the p-p collision c.m. system (CMS lab frame), where $\phi_{\rm jet,4} = \phi_{\rm jet,3} \pm \pi$ and $\eta_{\rm jet,4} = -\eta_{\rm jet,3}$. $P_g(\eta,\phi)$ was obtained from Eq. (4) for back-to-back dijets at angular positions $(\eta_{\rm jet,3}, \phi_{\rm jet,3})$ and $(\eta_{\rm jet,4}, \phi_{\rm jet,4})$ with the soft gluon (resulting final-state hadron) at (η,ϕ).

The resulting angular correlations from Eq. (6) for the high multiplicity, $1 < p_t < 3$ GeV/c data set are shown in Fig. 1 for different values of cut-off parameters and for jet widths $\sigma_\eta = 0.20$ and $\sigma_\phi = 0.22$. The parton and soft gluon cut-off masses and collinear cut-off σ_g assumed in panels (a) - (d) were 5 MeV and 0.2 radians, 1 MeV and 0.2 radians, 5 MeV and 0.05 radians, and 5 MeV and 0.35 radians, respectively. In each panel the amplitude was arbitrarily adjusted to unit height difference between the positions on $(\eta_\Delta, \phi_\Delta)$ at $(0,0)$ and $(0,\pi/2)$. The away-side ridge corresponds to correlations between soft-gluons emitted near jet-3 and the final-state hadrons from jet-4 and vice versa. The increased amplitude of the away-side ridge at larger $|\eta_\Delta|$ results from the increased radiative yields when the angles between incident and outgoing

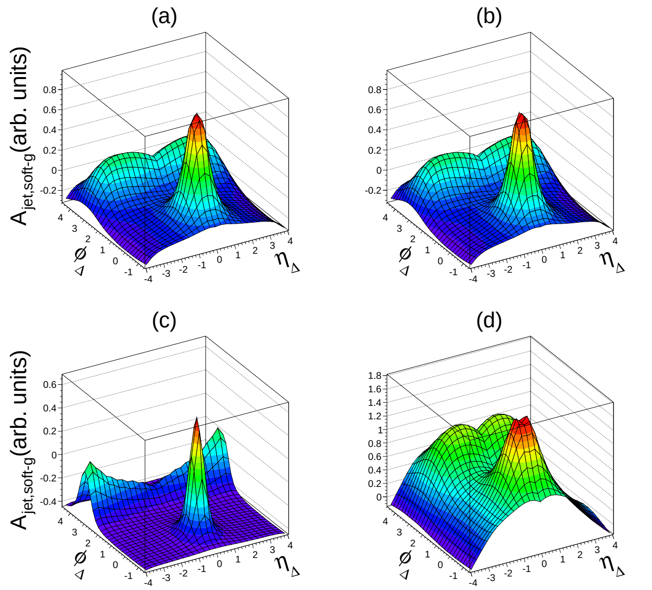

FIG. 1: (Color online) Perspective views of two-dimensional charge-independent angular correlations from Eq. (6), assuming arbitrary normalization, for soft-gluon radiation from back-to-back Gaussian jets estimated in the soft limit ($k \rightarrow 0$) on relative azimuth and pseudorapidity (η_Δ, ϕ_Δ) for the high multiplicity, $1 < p_t < 3$ GeV/c data from CMS. From upper-left, to lower-right panels (a)-(d) show angular correlations assuming nominal cut-off masses 5 MeV and angle 0.2 rad; 1 MeV, 0.2 rad; 5 MeV, 0.05 rad; and 5 MeV, 0.35 rad, respectively. The single particle jet distribution widths on η and ϕ were 0.20 and 0.22 respectively.

partons decrease. The fall-off at the largest $|\eta_\Delta|$ results from both the collinear cut-off σ_g and the $\sin^2\theta_{1k}$ factor in Eq.(4).

The angular correlations are weakly sensitive to the assumed cut-off mass (panels a and b), but for masses of order the pion mass the correlations become similar to that in panel (d). The angular correlations are however strongly affected by the collinear cut-off. For the fits to data $\sigma_g = 0.2$ was selected (panel a) in consideration of angular ordering while minimizing the suppression at larger $|\eta_\Delta|$. For the fits to the four 7 TeV data sets cut-off parameters 5 MeV and 0.2 radians were always used. The jet single particle Gaussian widths on η and ϕ were (0.35,0.52), (0.22,0.26), (0.30,0.30) and (0.20,0.22) for the minimum-bias low and higher p_t data, and the high multiplicity, low p_t and higher p_t data sets, respectively.

D. Ridge components

Recent theoretical predictions [19, 34] and phenomenological analysis of the 7 TeV data from CMS [26, 27] warrant including the quadrupole component. Although the presence of the quadrupole correlation in the charged particle yields from relativistic heavy ion collisions is often cited as evidence for hydrodynamic evolution, other theoretical studies [10, 11] show that a quadrupole correlation may result directly from QCD.

Models assuming outward, collective flow with initial-state energy fluctuations (*e.g.* CGC, beam-jets) or recombination suggest including a same-side 1D ridge on azimuth with no accompanying away-side structure other than that already present in the model. Momentum conservation induced correlations associated with this class of models would manifest as an away-side ridge on η_Δ and are therefore already included in the fitting function.

E. Specific modifications for the CMS data

Other modifications of the fitting model in [3] were required for the CMS data. At RHIC energies the away-side correlation ridge (non-quadrupole) for lower p_t particles is well described by a dipole, $\cos\phi_\Delta$, which is understood to represent the limit of a periodic away-side azimuth ridge (Gaussian) as its width increases [35]. Preliminary studies showed that the 7 TeV CMS correlation data were better described with a periodic Gaussian. The measured correlations also display significant away-side η_Δ-dependence. Analysis of Au-Au quadrupole correlations with large η acceptance indicates that reduction in amplitude with increasing $|\eta_\Delta|$ is possible [36]. Consequently, the away-side 1D Gaussian, the same-side 1D Gaussian, and the quadrupole components all include symmetric η_Δ-dependent modulations.

F. Fitting model

The complete fitting function used here is given by [37],

$$
\begin{aligned}
F(\eta_\Delta, \phi_\Delta) &= A_0 + A_Q f(\eta_\Delta)\cos(2\phi_\Delta) \\
&+ A_1 e^{-\frac{1}{2}\left\{\left(\frac{\eta_\Delta}{\sigma_{\eta_\Delta}}\right)^{2\alpha} + \left(\frac{\phi_\Delta}{\sigma_{\phi_\Delta}}\right)^{2\beta}\right\}} \\
&+ A_2 e^{-\left\{\left(\frac{\eta_\Delta}{w_\eta}\right)^2 + \left(\frac{\phi_\Delta}{w_\phi}\right)^2\right\}^{1/2}} + A_3 e^{-\frac{1}{2}\left(\frac{\eta_\Delta}{\sigma_0}\right)^{2\gamma}} \\
&+ A_4\left[g(\eta_\Delta)F_{AS}(\phi_\Delta) - \left(F_{AS}(0) + F_{AS}(\pi)\right)/2\right] \\
&+ A_5\left[f(\eta_\Delta)F_{SS}(\phi_\Delta) - \left(F_{SS}(0) + F_{SS}(\pi)\right)/2\right], \\
F_{AS}(\phi_\Delta) &= \sum_{k=odd-int} e^{-\frac{1}{2}\left(\frac{\phi_\Delta - k\pi}{\sigma_{AS}}\right)^2}, \\
F_{SS}(\phi_\Delta) &= \sum_{k=even-int} e^{-\frac{1}{2}\left(\frac{\phi_\Delta - k\pi}{\sigma_{SS}}\right)^2},
\end{aligned}
\quad (7)
$$

where $f(\eta_\Delta) = 1 + \delta\eta_\Delta^2$, $g(\eta_\Delta) = 1 + \epsilon\eta_\Delta^2 + \zeta\cos(2\pi\eta_\Delta/\Delta\eta)$, and the summations for the periodic away-side and same-side 1D Gaussian ridges (A_4 and A_5 terms) were accurately truncated with sums $(-3, -1\cdots 5)$ and $(-4, -2\cdots 4)$, respectively. The quadrupole (A_Q term) and the same-side periodic 1D

Gaussian (A_5 term) were applied alternately, not simultaneously in the fitting. Terms periodic in ϕ_Δ for the A_1 and A_2 components [i.e. dependent on $\phi_\Delta - (\pm 2\pi)$] were included but were negligible for the azimuth widths used here. The constant offsets for the AS and SS Gaussian ridge components were included in order that coefficients A_4 and A_5 only (approximately) affect the amplitudes of the ϕ_Δ-dependent oscillations and not the overall offset.

Function $F(\eta_\Delta, \phi_\Delta)$ was fitted to measured quantity $R(\eta_\Delta, \phi_\Delta)$ from CMS [1]. The fitting parameters include: A_0 (normalization offset), A_Q or A_5 and σ_{SS} with δ, same-side jet parameters A_1, σ_{η_Δ}, σ_{ϕ_Δ}, α and β, 2D exponential parameters A_2, w_η and w_ϕ, 1D Gaussian parameters A_3, σ_0 and γ (only required for the minimum-bias, $p_t > 0.1$ GeV/c data), and away-side 1D Gaussian ridge parameters A_4, σ_{AS}, ϵ and ζ. Due to the similar magnitudes of the width parameters for the A_1 and A_2 terms the fitting model may not provide an accurate separation between jet fragment correlations and quantum correlations in each case.

The soft gluon radiation component was computed for each case using the Gaussian width parameters for the jets obtained by fitting the data without the soft radiation component and with the cut-off parameters discussed in the above subsection. The single particle jet width parameters in Eq. (5) are related to the SS 2D correlation peak Gaussian widths by $\sigma_\eta = \sigma_{\eta_\Delta}/\sqrt{2}$ and $\sigma_\phi = \sigma_{\phi_\Delta}/\sqrt{2}$ owing to the definition of the difference variables η_Δ and ϕ_Δ. The function computed in Eq. (6) was arbitrarily adjusted in overall magnitude by the fitting program in order to obtain the best fit.

G. Fitting errors

Best fits were selected by χ^2 per degree of freedom (dof), residuals, the 2D χ^2 distributions, and parameter fit errors. The χ^2/dof values were fairly large indicating that residual, systematic effects are statistically significant. Such effects may include experimental artifacts and/or physical processes which are not accurately described by the fitting model. The fit parameter error covariance matrix was estimated by the following matrix inversion,

$$\delta^2 P_{jk} \approx \left[\sum_i \frac{1}{\varepsilon_i^2} \frac{\partial F_i}{\partial P_j} \frac{\partial F_i}{\partial P_k} \right]^{-1} \tag{8}$$

where the summation includes all unique $(\eta_\Delta, \phi_\Delta)$ bins, ε_i is the statistical error, and $\partial F_i / \partial P_j$ are partial derivatives of the model function with respect to fit parameter P_j at the χ^2 minimum. This estimate assumes that near the χ^2 minimum the fitting function dependence on parameter P_j can be accurately expanded to the leading order (linear) terms in a Taylor series. Parameter fit errors, $\sqrt{\delta^2 P_{ii}}$, are listed along with the best fit values in Table I. The CMS collaboration quotes an overall 15% systematic uncertainty in the data near the same-side ridge resulting in an additional 15% systematic uncertainty in the quadrupole and same-side ridge amplitudes A_Q and A_5.

III. RESULTS

The fitting results for the high event multiplicity data with $1 < p_t < 3$ GeV/c from the CMS experiment are discussed first. Initially the data were fit using the A_0, A_1, A_2 and A_4 terms in Eq. (7) where 1D Gaussian component A_3 was not required. The same-side peak structure was accurately described by the combination of approximate 2D Gaussian and 2D exponential terms where the fitted exponents of the former term equaled 2 (Gaussian) within 2% (but note the caveat in Sec. II F). The away-side ridge was well described by the periodic Gaussian (A_4 term) with moderate η_Δ modulation. Although the SS peak and AS ridge were well described the χ^2 was rather large due to the inability of the model to account for the η_Δ-extended same-side ridge. Including the soft gluon radiation correlations with adjustable, non-negative amplitude failed to improve the fit quality in this instance. χ^2/dof was improved significantly however (by 12 units) when either the quadrupole or SS periodic Gaussian ridge (A_5 term) was included. Fits including the quadrupole obtained the lowest χ^2 (see Table I). The data, the best fit (with quadrupole), and the residuals (model - data) are shown in Fig. 2 in the right-most column of panels. The soft gluon radiation correlations afforded very little improvement in fit quality for the SS Gaussian ridge model, but significantly improved the χ^2/dof from 4.18 to 4.04 when the quadrupole was included.

Similar results were found for the high multiplicity, $p_t > 0.1$ GeV/c data except that the χ^2 improvement obtained with either the additional quadrupole or SS Gaussian was less dramatic (about 4 units for χ^2/dof) than was obtained for the higher p_t data. The SS 2D peak function (A_1) relaxed to a non-Gaussian geometry with both pseudorapidity and azimuth exponents less than 1 (see Table I). Statistically significant evidence for both the quadrupole and SS Gaussian ridge was found. The best fit was obtained assuming the additional quadrupole component. The soft gluon radiation correlations provided no significant improvement in the fit quality. The data, best fit (quadrupole model), and residuals are shown in Fig. 2 in the third column of panels from the left.

The quadrupole and same-side Gaussian ridge can have very similar shapes and amplitudes for the same-side azimuth range. The two are of course very different on the away-side. Evidently the χ^2/dof reduction afforded by the quadrupole for both high event multiplicity data sets is due, at least in part, to improvements in the fit quality for the away-side data. Visual evidence of an away-side quadrupole contribution to the CMS high multiplicity data is not apparent, unlike the angular correlation data for mid-central relativistic heavy ion collisions [3, 6]. Quantitative analysis of the type presented here and in Refs. [26, 27] is required to differentiate between these two descriptions of the SS and AS ridges. Both the quadrupole and SS Gaussian amplitudes increase for the higher p_t selected particle cuts. A modest, negative η_Δ^2 modulation was obtained for both the quadrupole and SS Gaussian which had only minor effects.

The magnitudes of the residuals for the high event multiplicity data are less than 0.1 except near the origin and the η_Δ acceptance edge and are about 1% of the magnitudes of the principal correlation structures. Although the residuals increase at large $|\eta_\Delta|$ the increasing statistical errors at large $|\eta_\Delta|$ prevent these bins from making significant χ^2 contributions. The band structures in the residuals on η_Δ and ϕ_Δ are not understood and could be of experimental and/or dynamical origin. In any case the residuals are small relative to the principal correlation structures including the quadrupole and SS Gaussian ridge.

Both sets of minimum-bias correlation data were well described with just the A_0, A_1, A_2, A_3 (only for the $p_t > 0.1$ GeV/c data) and A_4 components. χ^2 for the $p_t > 0.1$ GeV/c data was dominated by the single (0,0) angular bin which was subsequently omitted in the χ^2 minimization procedure. No evidence for a non-negative quadrupole or SS Gaussian ridge structure was found. However, including the soft gluon radiation correlations improved the χ^2/dof by about 1 unit. The data, best fits (no quadrupole or SS Gaussian ridge), and residuals are shown in the first two columns of panels in Fig. 2. The magnitude of the residuals is less than 0.03 except near the origin and the η_Δ acceptance edge and is about 1% of the magnitudes of the principal correlation structures.

For each case the same-side peak structure was well described with a combination of approximate 2D Gaussian and 2D exponential functions. The exponent fit parameters α and β were generally close to unity corresponding to a 2D Gaussian-like peak shape. For each data set the five parameters of the 2D Gaussian-like component were not significantly affected by including or not including the quadrupole and SS Gaussian ridge. Trainor and Kettler [26] showed that the minimum-bias jet correlation structure in the CMS data is similar to that at RHIC energies when scaled by $\log(\sqrt{s})$. The total number of jet-like correlated pairs per final-state particle, estimated by the volume of the same-side 2D Gaussian-like peak (see Table I, but note the caveat in Sec. II F), increases for high multiplicity events because those events have increased probability of minimum-bias jet production within the acceptance. However, this volume decreases for the higher p_t ($1 < p_t < 3$ GeV/c) particle selection cut. Studies of 2D charged particle (p_{t1}, p_{t2}) correlations from $\sqrt{s} = 0.2$ TeV minimum-bias NSD p-p collisions at RHIC [28] suggest that the $p_t \in [1, 3]$ GeV/c cut-range may exclude a large fraction of the correlated pairs produced by minimum-bias jets. This may account for some of the reduction. The widths of the same-side jet-like peak decrease with higher p_t selection as expected for jet fragmentation with uniform transverse momentum distributions relative to the jet thrust axis [38]. Further analysis of the minimum-bias jet-like correlations in p-p collisions at the LHC will likely require different p_t selection cuts which are optimized for that purpose.

In general the away-side azimuth ridge structure was better described with an η_Δ-dependent, periodic Gaussian than with an η_Δ-dependent dipole. A periodic series of 1D Gaussians, as in Eq. (7), approaches an azimuth dipole plus offset for sufficiently large width σ_{AS} (*e.g.* higher-order multipole contributions are $< 1\%$ for $\sigma_{AS} > 1.75$ [35]). The fitted widths here are less than 1.2 indicating that the dipole limit is not reached. The amplitude A_4 approximately follows the volume trends of the SS 2D peak as expected if this correlation structure is dominated by inter-jet charged particle fragment pairs for dijets within the tracking acceptance. The majority of the η_Δ dependence (fall-off) of the away-side ridge (see Fig. 2) is described by the negative $\epsilon\eta_\Delta^2$ modulation. Weaker dependences were approximated by the $\zeta \cos(2\pi\eta_\Delta/\Delta\eta)$ modulation. Model descriptions of the AS η_Δ dependence did not significantly affect the fitted parameters of the SS 2D peak or the SS ridge structure. The away-side Gaussian decreased in amplitude and increased in width when the quadrupole component was included in the fitting.

IV. DISCUSSION

The CMS collaboration noted the similarity between the same-side η_Δ elongated correlations and that reported for Au-Au collisions at RHIC by the STAR [2–4] and PHOBOS [5] collaborations for total energy per nucleon-nucleon (N-N) pair $\sqrt{s_{NN}} = 0.2$ TeV. What was observed in minimum-bias collisions at RHIC using all charged particles with $|\eta| \leq 1.0$ and $p_t > 0.15$ GeV/c was a 2D approximately Gaussian peak centered at $(\eta_\Delta, \phi_\Delta) = (0, 0)$ whose amplitude and η width increase substantially and monotonically with decreasing impact parameter between the colliding ions (increasing centrality). In addition analysis of these data provided evidence for an η_Δ-independent quadrupole which smoothly varies with collision centrality and is approximately proportional to the square of the spatial eccentricity of the overlapping, colliding ions. In similar correlation studies using higher momentum "trigger" particles with lower momentum associated particles the same-side peak evolved to a non-Gaussian structure [4]. It is plausible that jet fragmentation could be modified in the dense environment produced in relativistic heavy ion collisions [39].

Field [20] pointed out that initial and final state radiative processes ($2 \to 3$, $2 \to 4$) accompanying transverse partonic scattering preferentially produce hadronic fragments in the hard-scattering plane defined by the beam and jet axes. Single gluon radiation in $2 \to 2$ hard processes was calculated in the soft gluon limit in [31, 32].

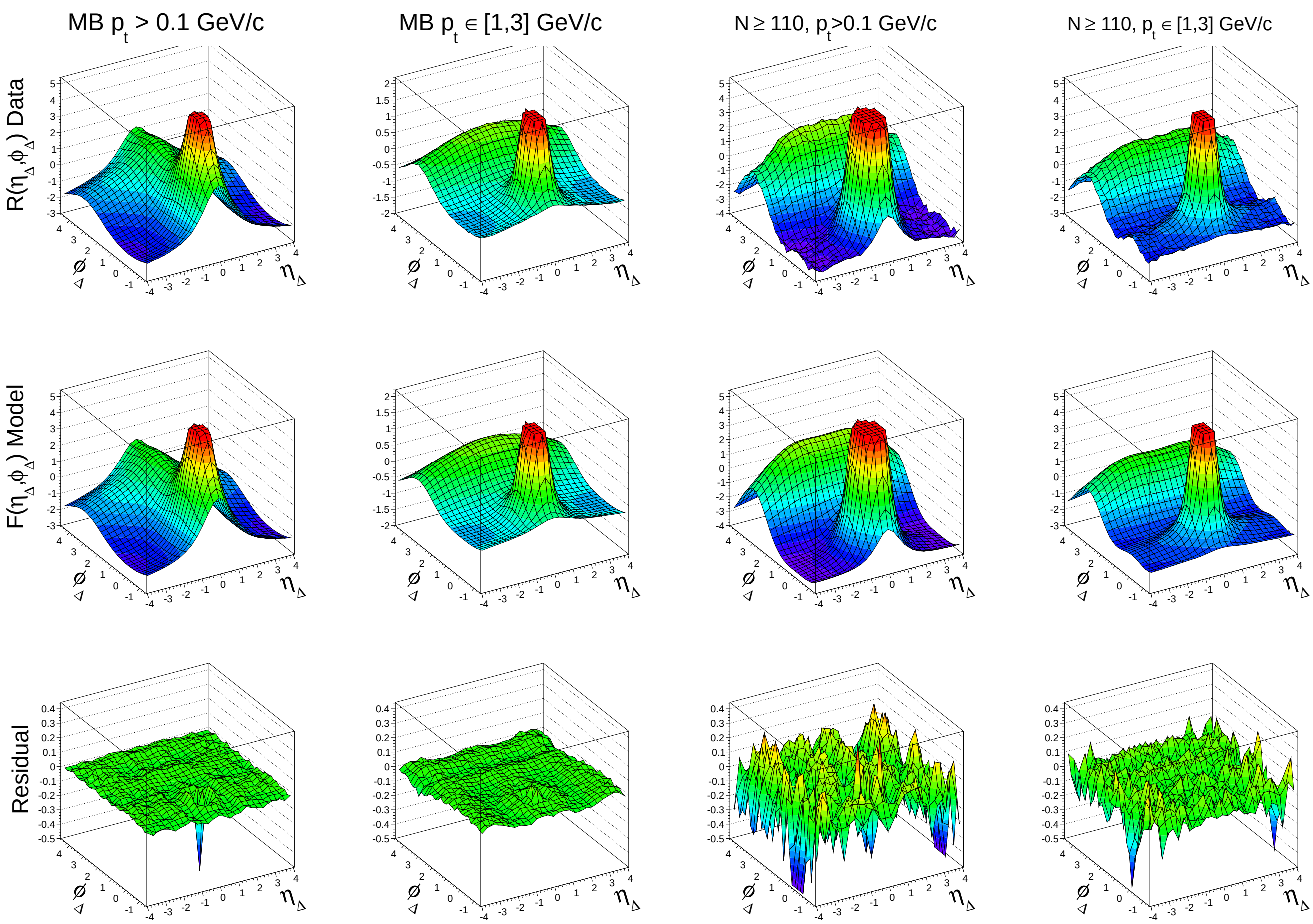

FIG. 2: (Color online) Perspective views of two-dimensional charge-independent correlations for p-p minimum-bias and high event multiplicity ($N \geq 110$) collisions at $\sqrt{s} = 7$ TeV from the CMS collaboration [1]. Upper, middle and lower rows of panels display data, model fits, and residuals (model - data), respectively. The columns of panels from left to right correspond to minimum-bias events with charged particle selection $p_t > 0.1$ GeV/c and $1 < p_t < 3$ GeV/c, and high multiplicity events with $p_t > 0.1$ GeV/c and $1 < p_t < 3$ GeV/c, respectively. The fitting models selected for display are described in the text. All residuals are shown using the same, expanded scale.

Using the $gg \rightarrow ggg$ distribution from [31] additional angular correlations for the p-p collision system were calculated and included in the present fitting model. Although the additional correlations made statistically significant improvements in the fit quality for the minimum-bias collision data, they could not explain the same-side η_Δ-extended ridge in the high event multiplicity data.

Sjöstrand [21] and Trainor [22] suggested that color-flux tube connections between transverse (scattered) and longitudinal (beam direction) partons could, in analogy to the LUND color-string fragmentation model [23] for soft particle production, produce an η_Δ extended, exponentially decreasing enhancement in the jet-like correlations. The present analysis does not find evidence of increasing η width (i.e. increased value of σ_{η_Δ}) or evolution toward an exponential shape [exponent parameter α in Eq. (7)] in conjunction with the appearance of the same-side ridge, in agreement with previous findings by Trainor and Kettler [26]. Nevertheless, this suggested jet fragmentation mechanism should be further considered with respect to the η_Δ-elongated correlations observed in A-A collisions at RHIC and the LHC [40].

Two classes of theoretical models purport to explain the same-side η_Δ-elongated ridge correlations observed in the RHIC heavy ion data. The first assumes event-by-event energy and/or momentum density fluctuations in the initial collision stage immediately after impact which propagate outward via pressure driven radial expansion. The initial stage fluctuations are described as color-glass condensate (CGC) [13] flux tubes (glasma) [12], localized "hot spots" in a quark-gluon plasma [19, 41, 42], or beam-jet remnants from hard scattering processes [14, 15]. The second class of models assumes hard, transverse parton − soft, longitudinal parton interactions via recombination [16] or strong rescattering [17] to induce the η_Δ-elongated correlations.

So far, most proponents have only attempted to address a subset of the experimental features associated

TABLE I: Best fit model parameters with fitting statistical errors (in parentheses) for p-p minimum-bias and high event multiplicity ($N \geq 110$) collisions at $\sqrt{s} = 7$ TeV.

Parameter	MB $p_t > 0.1$ GeV/c	MB $p_t \in [1,3]$ GeV/c	$N \geq 110, p_t > 0.1$ GeV/c		$N \geq 110, p_t \in [1,3]$ GeV/c	
	Jet+ASG	Jet+ASG	Quad	SSG	Quad	SSG
A_1	1.915(19)	4.40(3)	11.63(15)	11.90(17)	16.52(15)	16.17(14)
σ_{η_Δ}	0.501(3)	0.313(1)	0.426(4)	0.422(4)	0.285(1)	0.285(1)
α	1.021(5)	1.032(4)	0.938(8)	0.916(7)	0.979(5)	0.990(5)
σ_{ϕ_Δ}	0.737(7)	0.370(1)	0.422(7)	0.475(6)	0.310(1)	0.318(2)
β	0.883(9)	1.035(4)	0.740(6)	0.794(7)	1.003(6)	1.012(7)
A_2	8.07(3)	6.88(3)	35.0(2)	34.8(2)	14.3(2)	14.68(15)
w_η	0.753(2)	0.462(1)	0.446(2)	0.446(3)	0.418(3)	0.421(3)
w_ϕ	0.807(5)	0.429(1)	0.458(3)	0.435(2)	0.377(2)	0.363(2)
A_3	1.532(24)	–	–	–	–	–
σ_0	0.826(3)	–	–	–	–	–
γ	0.931(3)	–	–	–	–	–
A_4	2.125(12)	1.446(1)	5.62(2)	6.23(3)	3.08(2)	3.80(2)
σ_{AS}	1.196(2)	0.956(1)	0.947(8)	0.862(4)	1.156(31)	0.841(5)
ϵ	-0.021(1)	-0.0326(1)	-0.0231(3)	-0.0242(3)	-0.020(1)	-0.0208(3)
ζ	-0.055(3)	0.0080(6)	-0.040(1)	-0.035(1)	-0.046(2)	-0.042(1)
A_Q	–	–	0.486(23)[a]	–	0.657(24)[a]	–
A_5	–	–	–	0.556(40)[a]	–	0.846(27)[a]
σ_{SS}	–	–	–	0.550(28)	–	0.583(15)
δ	–	–	-0.056(2)	-0.017(6)	-0.032(1)	-0.021(3)
A_0	-1.344(5)	-0.124(1)	-1.038(18)	-0.571(17)	-0.505(19)	0.164(11)
χ^2/dof	14.3	16.8	9.78	10.9	4.18	4.57
Vol(2DG)	4.64	3.13	15.5	17.2	9.23	9.20
$A_4\sigma_{AS}$	2.54	1.38	5.32	5.37	3.56	3.20
v_2	–	–	0.046	–	0.053	–

[a]Additional 15% systematic uncertainty should be included due to quoted systematic error in the ridge signal [1].

with the same-side angular correlations, *e.g.* either the width or amplitude dependence on collision centrality. Accompanying correlation structures including the away-side ridge, charge-dependence (like - unlike charged pair difference), and the 2D two-particle (p_{t1}, p_{t2}) correlation [24] have not yet been addressed by these theoretical models. A recent analysis [43] of (p_{t1}, p_{t2}) correlation predictions for CGC flux tube induced correlations [44] shows that such initial stage models, even including strong radial flow, are irrelevant for describing the same-side η_Δ-extended correlations. Nevertheless, these models motivated the periodic, same-side Gaussian ridge component adopted in the present phenomenological study. Fits to the minimum-bias data provide no significant evidence for this component while fits to the high event multiplicity, $N \geq 110$ data do.

The quadrupole angular correlation continues to be extensively studied in relativistic heavy ion collisions [9] and was recently reported for Pb-Pb collisions at the LHC by the ALICE collaboration [45]. The most compelling explanation for the quadrupole correlation is that it is a consequence of anisotropic particle emission of the form $[1 + 2v_2 \cos(2(\phi - \psi))]$, where ψ is a random, event-wise angle for maximum azimuth particle density. Averaging over an event ensemble produces two-particle correlation component, $2v_2^2 \cos(2\phi_\Delta)$. The anisotropic emission presumably reflects the initial geometry (i.e. eccentricity ϵ) of the transverse overlap region between the colliding constituents of the two nuclei. Measurements show that observed v_2 is approximately proportional to initial eccentricity estimated using a Glauber model [6]. Naive superposition of independent N-N collisions, even if the spatial distribution of N-N collision positions is non-isotropic, produces on average an isotropic final-state particle distribution on azimuth resulting in $v_2 = 0$. What causes the azimuth modulation (v_2) is a mystery although theories are abundant (e.g. [7] but note discussion in [9]).

Trainor and Kettler [26] and Bożek [27] both showed that the p-p high multiplicity 7 TeV CMS correlation data could be well described with a quadrupole component. Trainor and Kettler described the 7 TeV CMS data with a similar, 2D fitting model as in Eq. (7) but without the 2D exponential, same-side Gaussian ridge, and η_Δ modulations assumed here. They showed that the quadrupole amplitude inferred for the 7 TeV high multiplicity p-p correlation data is consistent with the simple, linear trends on collision energy [proportional to $\log(\sqrt{s_{NN}})$], eccentricity and event multiplicity reported for Au-Au minimum-bias collisions at RHIC [6]. Bożek [27] fitted the 1D azimuth projections for $|\eta_\Delta|$ from 2.0 to 4.8 for sixteen combinations of event multiplicity and p_t interval [1] using $\cos(\phi_\Delta)$ and $\cos(2\phi_\Delta)$ terms only. Good fits were obtained although contributions of the same-side jet peak for $|\eta_\Delta| > 2$ occur and the azimuth projection averages over the strong η_Δ dependence of the away-side ridge beginning at $|\eta_\Delta| > 2$. The quadrupole amplitude A_Q in Eq. (7) for the CMS correlation quantity R (see Eq. (1) in [1]) is related to azimuth anisotropy parameter v_2 defined above by,

$$A_Q \;=\; 2\left[\langle N_{trk}^{offline}\rangle - 1\right] v_2^2. \qquad (9)$$

Inferred values of v_2 from the present analysis (see Table I) and Eq. (9) are $v_2 = 0.046$ and 0.053 for the $N \geq 110$, $p_t > 0.1$ GeV/c and $1 < p_t < 3$ GeV/c data, respectively, in fair agreement with ~ 0.04 obtained in both Trainor and Kettler [26] and Bożek [27]. The best fits obtained in this analysis were those with the quadrupole component as discussed in Sec. III.

The CMS experiment also reported strong variation in the large $|\eta_\Delta|$ ($2.0 < |\eta_\Delta| < 4.8$) same-side ridge amplitude for high multiplicity events ($N > 90$) in four p_t ranges from 0.1 to 4 GeV/c (see Fig. 9 in [1]). The larger values occurred in the $p_t \in [1, 2]$ and $[2, 3]$ GeV/c bins. A similar increasing then decreasing trend with p_t was reported by the STAR experiment [6] for the Au-Au quadrupole correlation in more central Au-Au collisions. CMS also found that the same-side ridge correlations for like-sign and unlike-sign charged particle pairs were the same within uncertainties as shown in Fig. 10 of [1]. Quadrupole correlations in Au-Au collisions at RHIC are also charge-independent within errors [2, 3, 6]. Non-zero charge-dependent angular correlations in Au-Au collisions at RHIC extend out to only 2 units on relative η [46]. The CMS results in Figs. 9 and 10 of Ref. [1] are qualitatively consistent with quadrupole phenomenology for Au-Au collisions at RHIC energies.

The conventional assumption in the heavy ion physics literature is that strong interactions among the (partonic) constituents lead to rapid thermal equilibration within about 1 fm/c after impact [8]. This results in pressure gradients which drive collective expansion as described by particle transport models [47] or hydrodynamic evolution [7]. Hadronization models are invoked in the final stages of expansion which enable predictions to be compared with experiment. Event-wise fluctuations in energy and momentum density in the initial stage of the collision are expected to propagate to the final state producing correlations. Critical to this interpretation is the requirement of very rapid thermal equilibration; delays beyond a few fm/c result in insufficient hydrodynamic pressure to account for observed v_2 values. It has been shown [48] that rapid thermal equilibration cannot occur via pQCD processes or by non-perturbative instanton mechanisms in the brief amount of time available (of order 10 fm/c) during the heavy ion collision process. That thermal equilibration might occur during a p-p collision is even more problematic due to the reduced spatial and temporal scales involved. Kovchegov [48] concluded that if rapid thermal equilibration does occur some as yet unidentified, non-perturbative mechanism would be required. Therefore, if the p-p CMS correlation data do in fact display a quadrupole correlation as concluded in [26, 27] and indicated in the present results via χ^2 selection, a production mechanism for the quadrupole correlation other than rapid thermal equilibration is implied.

Three alternate mechanisms to the hydrodynamic scenario for generating the azimuth modulation in heavy ion collisions have been proposed. Boreskov *et al.* [10] assumed a non-local N-N inclusive collision vertex model with final-state emission oriented relative to the momentum transfer. Convolution of this vertex function with the transverse nucleus-nucleus overlap distribution, which includes important anisotropic density gradients, produced anisotropic final-state particle distributions with v_2 values of reasonable magnitude. Kopeliovich *et al.* [11] used a color-dipole model and predicted v_2 for p-p collisions at 200 GeV of order a few percent, which is a reasonable magnitude given the present results and those in [26, 27] and the empirical energy dependence of the quadrupole correlation reported by Kettler [6]. Trainor [9] suggested that interacting color currents in the nucleus-nucleus overlap region could emit gluon multipole radiation in analogy to classical electrodynamics.

V. SUMMARY AND CONCLUSIONS

Theoretically motivated phenomenological fitting models were applied to the two-particle 2D angular correlation data for p-p collisions at $\sqrt{s} = 7$ TeV reported by the CMS Collaboration. The dominant features of the data were well described by jet-like correlation structures projected onto relative η, ϕ subspace, consisting of a 2D Gaussian-like peak at the origin with accompanying away-side ridge at $\phi_\Delta = \pi$ representing pair correlations between dijets within the acceptance. For the high multiplicity events with $N \geq 110$ a new, extended correlation ridge along η_Δ was observed at small relative azimuth. The same-side jet-like component was allowed to have (1) an extended width along η_Δ which is required to describe similar angular correlation data from Au-Au collisions at RHIC, (2) non-Gaussian distortions which are suggested by possible higher-order fragmentation processes [21, 22], and (3) additional soft radiated gluon - jet particle correlations [31, 32]. None of these extensions enabled the jet related components to describe the same-side correlation ridge.

However, the same-side ridge was well described when either an additional same-side Gaussian ridge on azimuth or an azimuth quadrupole $[\cos(2\phi_\Delta)]$ was included in the fitting. The former structure was predicted for dense, strongly interacting systems where initial energy/momentum density fluctuations are propagated to the final hadronic state via pressure driven radial flow. The latter (quadrupole) has been observed in angular correlations for relativistic heavy ion collisions.

Based on the present analysis neither the quadrupole or same-side Gaussian ridge phenomenological descriptions can be ruled out, although the quadrupole model results in smaller χ^2/dof and residuals for both the lower p_t ($p_t > 0.1$ GeV/c) and higher p_t ($1 < p_t < 3$ GeV/c) high event multiplicity data. No evidence for either the quadrupole or same-side Gaussian ridge was found in this analysis for the 7 TeV minimum-bias p-p correlation data. Alternative mechanisms for producing the quadrupole correlation have been proposed [9–11] which do not rely on rapid thermal equilibration and pressure driven collective flow. It would be interesting to apply those models to the CMS p-p correlation data.

The present analysis shows that the CMS ridge could be a manifestation of the quadrupole correlation, well-known from relativistic heavy ion collisions, and possibly appearing in p-p collisions for the first time in these data from CMS. The phenomenological results presented here and in [26, 27] together with the problematic assumption of rapid equilibration in p-p collision systems, warrant further study of the underlying mechanism(s) producing the quadrupole correlation in high energy collisions.

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$2e^{-2\sigma^2}\cos(2\phi_\Delta)\cdots]$ for $\sigma \gg 1$.

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