

# QHD equation of state for strongly magnetized neutron stars

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We investigate the quantum hadrodynamical equation of state for neutron stars (with and without including hyperons) in the presence of strong magnetic fields. The deduced masses and radii are consistent with recent observations of high mass neutron stars even in the case of hyperonic nuclei for sufficiently strong magnetic fields. The calculated adiabatic index and the moments of inertia for magnetized neutron stars exhibit rapid changes with density. This may provide some insight into the mechanism of star-quakes and flares in magnetars.

PACS numbers:

## I. INTRODUCTION

Soft  $\gamma$ -ray repeaters (SGRs) and anomalous  $X$ -ray pulsars (AXPs) are believed to be evidence for magnetars, i.e. neutron stars with surface magnetic fields of  $10^{14} \sim 10^{15}$  G [1]. In the interior of these magnetic neutron stars, the magnetic field strength could be as much as  $10^{18}$  G according to the scalar virial theorem. Such strong magnetic fields may affect the properties of neutron stars such as the relative populations of various particles, the equation of state (EOS), and the mass-radius relation. Many studies of dense nuclear matter in the presence of strong magnetic fields have been reported [2–9]. These works have considered the electromagnetic interaction, the Landau quantization of charged particles, and the baryon anomalous magnetic moments (AMMs). However, a detailed analysis of neutron stars with strong magnetic fields is still an area of active research.

Recently [10], a giant  $\gamma$ -ray flare, SGR 1806-20, has been observed. The total flare energy was estimated to be as much as  $2 \times 10^{46}$  erg. This is  $\sim 10^2$  times higher than the two previously observed giant flares [11, 12], and is believed to have been released during a reconfiguration of magnetic fields of the neutron star. In Ref. [13], the similarities between SGR events and star-quakes relating to the sudden change of pulsar periods were discussed, and the possibility that SGRs may be powered by starquakes was suggested. These phenomena might be explained by sudden changes in the magnetic pressure.

Although many uncertainties remain regarding star-

quakes and the strength of magnetic fields in the interior of neutron stars, it has been suggested [4] that neutron star matter might be unstable in the presence of strong magnetic fields because of the onset of discrete Landau level energy quantization. This can cause rapid changes in the pressure response to changes in density. This instability could be one source for star-quakes. Thus, one may conjecture the following unifying scenario from star-quakes to pulsar glitches. If a strong magnetic field can cause star-quakes, it may crack the surface of the star. Magnetic field energy may then be released through the cracks. The released magnetic energy and associated reconnection may be observed as a SGR or a giant flare. In addition, the release of the magnetic field energy density may affect the equation of state (EOS) of the neutron star. This, in turn, could give rise to a change of the moment of inertia, and thus, the neutron-star spin period

In Ref. [4] properties of magnetic stars were studied, but only in the context of a cold free  $n, p, e$  Fermi gas. It could not be determined in that work whether the features of interest would persist with a realistic nuclear equation of state. It is useful, therefore, to reconsider the structure and dynamics of magnetic neutron stars in the context of a realistic nuclear equation of state. Therefore, in this paper we calculate the populations of particles, the instability in the adiabatic index  $\Gamma$ , the EOS, and the moment of inertia for various magnetic field strengths by using the techniques of quantum hadrodynamics (QHD). We then discuss the possibility that star-quakes and SGR flares might be explained by the rapid change of the adiabatic index  $\Gamma$  with density due to the population of Landau levels. We find that, similar to the Fermi gas approximation [4], the magnetic QHD model also shows

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an instability, *i.e.* a sudden change of the adiabatic index as the density increases. As in the free nucleon gas this is attributable to the discrete excitation of Landau levels as the density increases, but also to the appearance of hadronic species at high density. These changes in the adiabatic index lead to a sudden change in the pressure response of the star. Therefore, star-quakes and an associated release of magnetic field energy may take place. Moreover, when we assume that magnetic fields as large as  $10^{18}\text{G}$  exist inside the star, we find that a release of magnetic field energy could decrease of the moment of inertia leading to an increase in the the spin rate of the star.

In section II, we briefly introduce our theoretical framework for magnetized dense matter based upon the QHD approach. A method for calculating the possible change in the moment of inertia by the abrupt variations of the adiabatic index is also presented. Discussions of star-quakes and the neutron-star spin are presented along with numerical results in section III. A summary and conclusions are given in section IV.

## II. THEORY

### A. Relativistic mean field Lagrangian with strong magnetic fields

The Lagrangian of the QHD model for dense nuclear matter in the presence of a magnetic field can derived by introducing of a vector potential  $A^\mu$ . The resulting Lagrangian can be written in terms of the baryon octet, leptons, and five meson fields as follows

$$\begin{aligned}
\mathcal{L} = & \sum_b \bar{\psi}_b \left[ i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - M_b^*(\sigma, \sigma^*) \right. \\
& - g_{\omega b} \gamma_\mu \omega^\mu - g_{\phi b} \gamma_\mu \phi^\mu \\
& \left. - g_{\rho b} \gamma_\mu \vec{\tau} \cdot \rho^\mu - \frac{1}{2} \kappa_b \sigma_{\mu\nu} F^{\mu\nu} \right] \psi_b \\
& + \sum_l \bar{\psi}_l \left[ i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l \right] \psi_l \\
& + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) \\
& + \frac{1}{2} \partial_\mu \sigma^* \partial^\mu \sigma^* - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} \\
& + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu \\
& - \frac{1}{4} R_{i\mu\nu} R_i^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (1)
\end{aligned}$$

where the indices  $b$  and  $l$  denote the baryon octet and the leptons ( $e^-$  and  $\mu^-$ ), respectively. The effective baryon mass,  $M_b^*$ , is simply given by  $M_b^* = M_b - g_{\sigma b} \sigma - g_{\sigma^* b} \sigma^*$ , where  $M_b$  is the free mass of a baryon in vacuum and the  $g_{\sigma b}$  are associated coupling constants. The  $\sigma$ ,  $\omega$  and  $\rho$  meson fields describe the nucleon-nucleon ( $N-N$ ) and nucleon-hyperon ( $N-Y$ ) interactions. The  $Y-Y$  interaction is mediated by the  $\sigma^*$  and  $\phi$  meson fields.  $U(\sigma)$

is the self interaction of the  $\sigma$  field given by  $U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$ .  $W_{\mu\nu}$ ,  $R_{i\mu\nu}$ ,  $\Phi_{\mu\nu}$ , and  $F_{\mu\nu}$  denote the field tensors for the  $\omega$ ,  $\rho$ ,  $\phi$  and photon fields, respectively.

The anomalous magnetic moments (AMMs) of the baryons interact with the external magnetic field via terms of the form of  $\kappa_b \sigma_{\mu\nu} F^{\mu\nu}$  where  $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$  and  $\kappa_b$  is the strength of the AMM for a baryon, *i.e.*  $\kappa_p = 1.7928 \mu_N$  for a proton with  $\mu_N$  the nuclear magneton.

In general, the AMMs could depend upon the matter density. Therefore, one can take account of the medium effect through density dependent AMMs which can be evaluated within the quark meson coupling (QMC) model [9]. In this report, however, we did not take account of the effect of density dependent AMMs because the calculation here is performed within the QHD model.

Energy spectra for the baryons and leptons are given by

$$\begin{aligned}
E_b^C &= \sqrt{k_z^2 + \left( \sqrt{M_b^{*2} + 2\nu|q_b|B} - s\kappa_b B \right)^2} \\
&\quad + g_{\omega b} \omega_0 + g_{\phi b} \phi_0 + g_{\rho b} I_z^b \rho_{30}, \\
E_b^N &= \sqrt{k_z^2 + \left( \sqrt{M_b^{*2} + k_x^2 + k_y^2} - s\kappa_b B \right)^2} \\
&\quad + g_{\omega b} \omega_0 + g_{\phi b} \phi_0 + g_{\rho b} I_z^b \rho_{30}, \\
E_l &= \sqrt{k_z^2 + m_l^2 + 2\nu|q_l|B}, \quad (2)
\end{aligned}$$

where  $E_b^C$  and  $E_b^N$  denote the energies of charged and neutral baryons, respectively. The Landau quantization for charged particles in a magnetic field is denoted as  $\nu = n + 1/2 - \text{sgn}(q)s/2 = 0, 1, 2, \dots$ , where the sign of the electric charge is denoted as  $q \text{sgn}(q)$  and  $s = 1(-1)$  is for spin up (down).

Chemical potentials for the baryons and leptons are given by

$$\mu_b = E_f^b + g_{\omega b} \omega_0 + g_{\phi b} \phi_0 + g_{\rho b} I_z^b \rho_{30}, \quad (3)$$

$$\mu_l = \sqrt{k_f^2 + m_l^2 + 2\nu|q_l|B}, \quad (4)$$

where  $E_f^b$  is the baryon Fermi energy and  $k_f$  is the lepton Fermi momentum. For charged particles,  $E_f^b$  is written as

$$E_f^b = k_f^{b2} + \left( \sqrt{M_b^{*2} + 2\nu|q_b|B} - s\kappa_b B \right)^2, \quad (5)$$

where  $k_f^b$  is the baryon Fermi momentum and  $\nu = 0$  for neutral baryons.

We apply three constraints for calculating the properties of a neutron stars: 1) baryon number conservation; 2) charge neutrality; and 3) chemical equilibrium. The meson field equations are solved along with the chemical potentials for baryons and leptons subject to these three constraints. The total energy density is then given by

$\varepsilon_{tot} = \varepsilon_m + \varepsilon_f$ , where the energy density for the matter fields is given as

$$\varepsilon_m = \sum_b \varepsilon_b + \sum_l \varepsilon_l + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\rho^2 \rho^2 + U(\sigma), \quad (6)$$

and the energy density due to the magnetic field is given by  $\varepsilon_f = B^2/2$ . The total pressure is then

$$P_{tot} = P_m + \frac{1}{2} B^2, \quad (7)$$

where the pressure due to the matter fields is obtained from  $P_m = \sum_i \mu_i \rho_v^i - \varepsilon_m$ . The relation between the mass and radius for a static and spherical symmetric neutron star is generated from a solution to the Tolman-Oppenheimer-Volkoff (TOV) equations using the equation of state (EoS) described above.

### B. Slowly rotating neutron stars

To calculate the moment of inertia for a slowly rotating neutron star, we follow the methods detailed in Refs. [14, 15] which are briefly summarized here. The metric of an axially symmetric neutron star can be written in generalized rotating Schwarzschild coordinates as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - 2\omega(r) r^2 \sin^2 \theta dt d\phi. \quad (8)$$

If the neutron star is rotating uniformly with a stellar frequency  $\Omega$  far below the Kepler frequency

$$\Omega \ll \Omega_{\max} \approx \sqrt{\frac{M}{R^3}}, \quad (9)$$

the slow-motion approximation is valid and the moment of inertia can be written:

$$I \equiv \frac{J}{\Omega} = \frac{8\pi}{3} \int_0^R r^4 e^{-\nu(r)} \frac{\tilde{\omega}(r)}{\Omega} \frac{(\varepsilon(r) + P(r))}{\sqrt{1 - 2M(r)/r}} dr, \quad (10)$$

where  $J$  is the angular momentum, while  $\nu(r)$  and  $\tilde{\omega}(r)$  are radially-dependent metric functions.  $M(r)$ ,  $\varepsilon(r)$ , and  $P(r)$  are the mass of the star, energy density, and pressure, respectively, derived from a solution to the TOV equation.

The metric functions  $\nu(r)$  and  $\lambda(r)$  in Eq. (8) are unchanged from the values for a spherically symmetric neutron star. Thus,  $\lambda(r)$  is obtained from the mass  $M(r)$  by the usual Schwarzschild solution:

$$g_{11}(r) = e^{2\lambda(r)} = \left(1 - 2M(r)/r\right)^{-1} \quad (11)$$

and  $\nu(r)$  can be calculated from

$$\nu(r) = \frac{1}{2} \ln \left(1 - \frac{2M}{R}\right) - \int_r^R \frac{(M(x) + 4\pi x^3 P(x))}{x^2 (1 - 2M(x)/x)} dx. \quad (12)$$

The metric function  $\omega(r)$  denotes a frequency corresponding to the dragging of local inertial frames by the rotating star. The relative frequency  $\tilde{\omega}(r) \equiv \Omega - \omega(r)$  appearing in Eq. (10) represents the angular velocity of the fluid as measured in a local inertial reference frame. In particular, the dimensionless relative frequency  $\tilde{\omega}(r) \equiv \tilde{\omega}(r)/\Omega$  satisfies the following second-order differential equation

$$\frac{d}{dr} \left( r^4 j(r) \frac{d\tilde{\omega}(r)}{dr} \right) + 4r^3 \frac{dj(r)}{dr} \tilde{\omega}(r) = 0, \quad (13)$$

where

$$j(r) = e^{-\nu(r) - \lambda(r)} = e^{-\nu(r)} \sqrt{1 - 2M(r)/r} \quad \text{if } r \leq R, \\ 1 \quad \text{if } r > R. \quad (14)$$

Note that  $\tilde{\omega}(r)$  is subject to the following two boundary conditions:

$$\tilde{\omega}'(0) = 0, \quad (15)$$

$$\tilde{\omega}(R) + \frac{R}{3} \tilde{\omega}'(R) = 1. \quad (16)$$

In numerical calculations, one can integrate Eq. (13) from the center of the star to its surface for various central frequencies. When the boundary conditions at the surface, Eq. (16) are not satisfied for an arbitrary choice of  $\tilde{\omega}_c$ , one must rescale the function and its derivative by an appropriate constant to correct for the mismatch. After solving both  $\tilde{\omega}(r)$  and  $I$ , one can check the consistency of the results by testing the accuracy to which the following condition is satisfied:

$$\tilde{\omega}'(R) = \frac{6I}{R^4}. \quad (17)$$

### III. RESULTS AND DISCUSSION

We use the parameter set given in Ref. [16] for the coupling constants,  $g_{\sigma N}$ ,  $g_{\omega N}$  and  $g_{\rho N}$ , where  $N$  denotes the nucleon. The coupling constants for hyperons in a nuclear medium,  $g_{\omega Y}$  and  $g_{\sigma Y}$ , are determined by the quark counting rule and the relevant hyperon potentials at the saturation density. The strength of these potentials is fixed at  $U_\Lambda = -30$  MeV,  $U_\Sigma = 30$  MeV and  $U_\Xi = -15$  MeV. Since the magnetic fields may also depend upon the density, we utilize the form for the density-dependent magnetic fields suggested in Refs. [2, 8, 9]

$$B(\rho/\rho_0) = B^{surf} + B_0 [1 - \exp\{-\beta(\rho/\rho_0)^\gamma\}], \quad (18)$$

where  $B^{surf}$  is the magnetic field at the neutron-star surface, taken from observations to be  $\sim 10^{15}$  G, and  $B_0$  represents the magnetic field saturation strength in the high density region.

In this work, we use  $\beta = 0.02$  and  $\gamma = 3$ . Since the magnetic field is usually written in units of the critical field for the electron  $B_e^c = m_e^2/e = 4.414 \times 10^{13}$  G, the  $B$  and the  $B_0$  in Eq. (18) can be expressed as dimensionless quantities  $B^* = B/B_e^c$  and  $B_0^* = B_0/B_e^c$ . Based upon this magnetic field representation, we investigate the structure of neutron stars as a function of magnetic field strength both with and without hyperonic matter.

### A. Particle populations, adiabatic index and equation of state

The populations of particles as a function of density in our model are shown in Fig. 1, for four cases. Shown are particle fractions  $Y_i$  for both pure nucleonic and hyperonic equations of state, and both with and without magnetic fields. Since the details about the effects of magnetic fields in neutron stars have been discussed in our previous paper [9], we briefly explain the magnetic field effects on populations of particles. In Fig. 1, one can notice the effects of a magnetic field from the difference between the (no  $B$ ) and ( $B_0^* = 5 \times 10^4$ ) figures for both nucleonic (upper) and hyperonic phases (lower).

In general, charged particles are strongly affected by the EM interaction term ( $eB$ ), which gives rise to an increase in the electron fraction. Therefore the condition of charge neutrality and the EM interaction enhances the proton fraction. This suppresses hyperons by baryon number conservation. This phenomenon explicitly appears if we utilize  $B_0^* = 10^5$  as in our previous paper [9]. But its effect becomes indiscernible in the present case with a weaker field strength,  $B_0^* = 5 \times 10^4$ .

The most prominent feature caused by the strong magnetic field in Fig. 1 is the kink pattern in the populations of electrons and muons, which is clearly shown for the hyperonic phase. Of course, that pattern is caused by the change of Landau levels owing to the existence of a magnetic field.

In Fig. 2, we show the adiabatic index  $\Gamma \equiv \frac{\partial \ln P}{\partial \ln \rho} = (1 + \frac{\epsilon}{P}) \frac{dP}{d\epsilon}$  for both the nucleonic (upper) and the hyperonic (lower) equation of state. This is of particular interest because the adiabatic index may be used as a critical factor for understanding the radial stability of a star [17, 18].

In the nucleonic phase, one can see multiple kinks similar to the results shown in Ref. [4] based upon a cold Fermi gas model. This oscillatory pattern takes place as charged particles begin to rapidly fill the next unoccupied Landau level when the density increases. In the hyperonic phase, one can see a sudden large increase in the adiabatic index at  $\approx 3\rho_0$ . This is caused by the production of  $\Xi^-$  hyperons around that density as shown in Fig. 1.

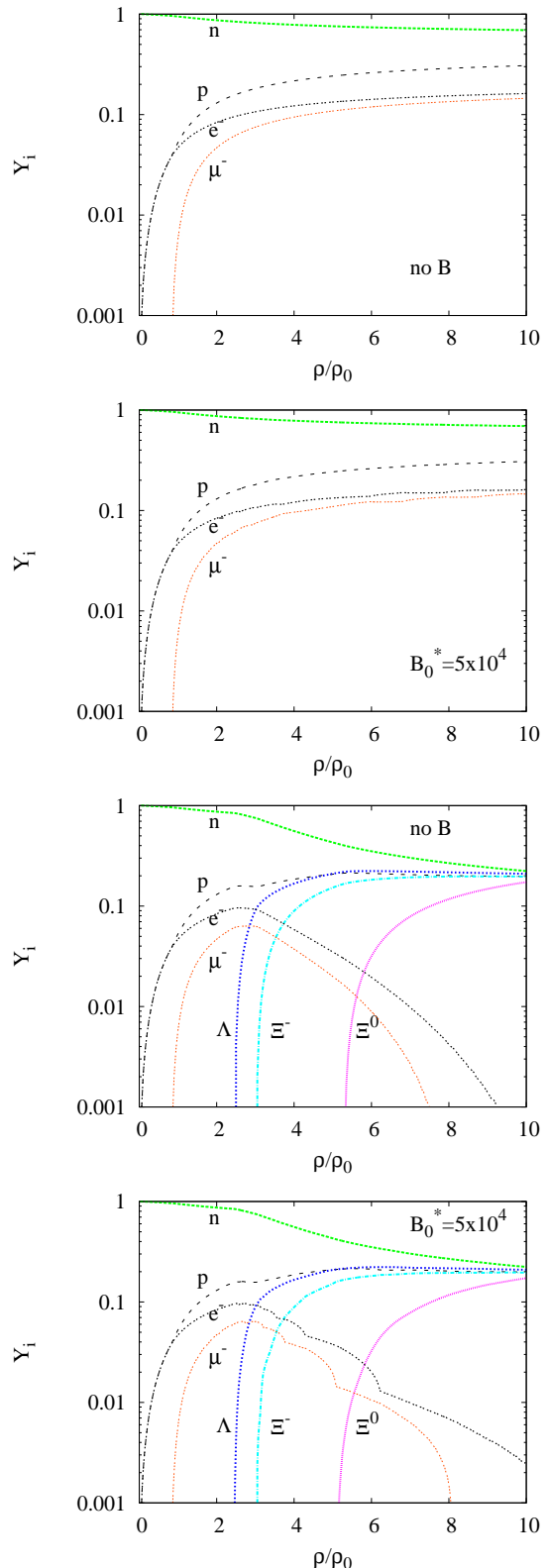


FIG. 1: Populations of particles in a neutron star. The upper two panels are for the nucleonic phase and the lower two panels are for the hyperonic phase with and without a magnetic field, respectively, where we used  $B_0^* = 5 \times 10^4$  as explained in the text. Note, this field strength is a bit different value from the value used in our previous paper [9].

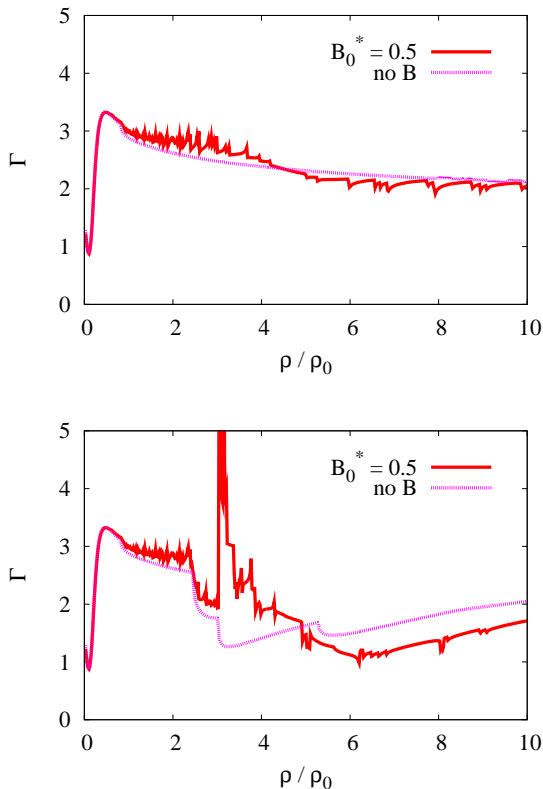


FIG. 2: Adiabatic index for the nucleonic (upper) and hyperonic (lower) equations of state. The adiabatic index is defined as  $\Gamma \equiv \frac{\partial \ln P}{\partial \ln \rho} = (1 + \frac{\epsilon}{P}) \frac{dP}{d\epsilon}$ . Here  $B_0^*$  is given in units of  $10^4$ .

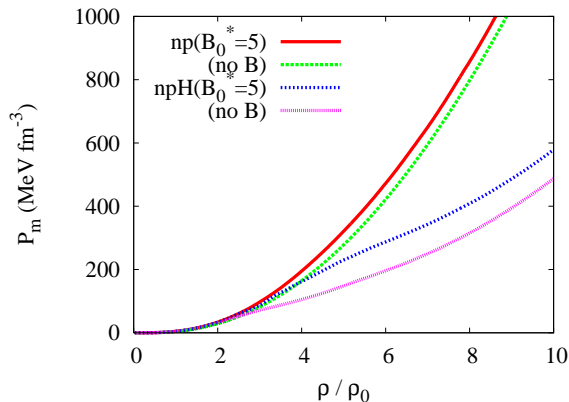


FIG. 3: Equation of state pressure as a function of density in units of the nuclear saturation density.

In the high density region beyond  $3\rho_0$ , similar kink patterns appear in both the nucleonic and hyperonic phases. At high density, however, the kinks are relatively smaller than the kinks in the lower density region. This is because the high density increases the occupied levels but does not cause a rapid shift in the Landau levels. This means that there should exist a critical density for such

discontinuous pressure response, at least in the hyperonic phase.

Similarly to the Fermi gas model [4], the magnetic fields in the QHD model also give rise to sudden jumps from the occupied Landau levels to the unoccupied levels. This leads to kinks in the adiabatic index because of the unstable structure in the magnetic pressure. These oscillatory kinks can be conjectured to cause star-quakes. Specifically, in the hyperonic phase, the instability caused by the magnetic field becomes significantly larger than that in the nucleonic phase.

In Fig. 3, the EOSs are shown for both nucleonic and hyperonic matter. Magnetic fields can make the EOS stiff in both phases. In particular, since the magnetic fields suppress hyperons, the effect of stiffness owing to the magnetic field in the hyperonic phase is far larger than in the nucleonic phase. Thus, the stiff EOS gives a large mass and radius for a neutron star. That is shown in Fig. 4.

## B. Mass-radius relation and the moment of inertia

The mass-radius relations for both the nucleonic and the hyperonic phases are compared with observational limits in Fig. 4. In this context, it is particularly noteworthy that larger maximum masses and radii are possible when a strong interior magnetic field is present. Neutron stars and heavy ion collision data provide valuable constraints on the EOS for dense matter [19]. There are indeed recent reports of evidence for higher maximum masses for neutron stars. For instance,  $M = 2.0 \pm 0.1 M_\odot$  for 4U 1636-536 was reported in Ref. [20]. Very recently authors in Ref. [21] investigated seven neutron stars, six binaries and an isolated neutron star (RX J1865-3754) and showed that  $M \approx 1.9 - 2.3 M_\odot$  and  $R \approx 11 - 13$  km. That is labeled as the SLB data in Fig. 4, which also shows the  $2\sigma$  lower and upper limits.

Pulsar I of the globular cluster Terzan 5 (Ter 5 I) shows a lower mass limit  $M_{N,Star} \geq 1.68 M_\odot$  at the 95 % confidence level [22]. Another constraint deduced independently of given models is obtained from XTE J1739-285 [23]. That work provides a constrained curve for the ratio between mass and radius. Although we do not indicate it here, another recent paper [24] has also reported a mass of  $M \approx 1.97 M_\odot$  for PSR J1614-2230 from a detection of the Shapiro time delay.

For the case of a nucleonic EOS, the mass and radius relations satisfy all constraints. The hyperonic phase, however, shows some discrepancies. Although a hyperonic phase cannot satisfy all of the constraints, magnetized hyperonic neutron stars can explain large neutron-star masses within the constraints from the observational data.

In Fig. 5, we show the moment of inertia in units of  $I_{45} = 10^{45} \text{ g cm}^2$  in order to investigate the spin up of the star. One can easily notice the large difference in the moment of inertia between stars with and without

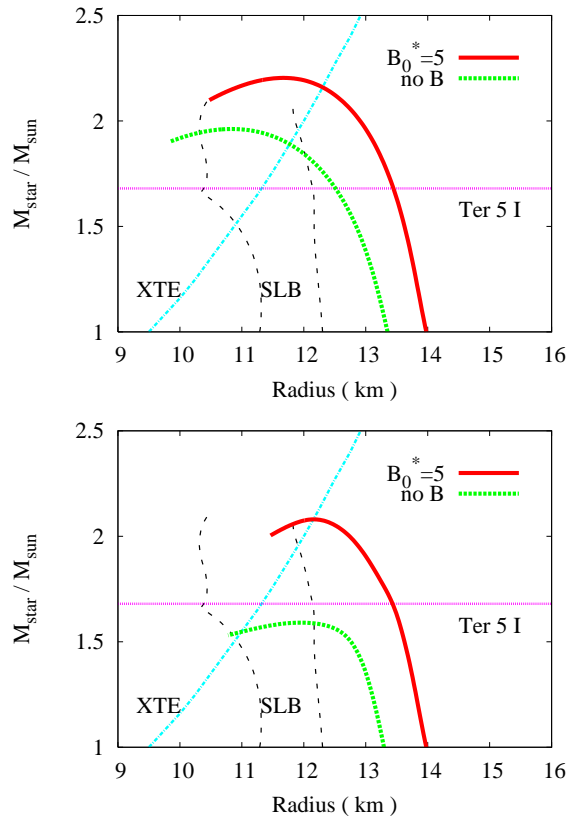


FIG. 4: Mass-radius relations for nucleonic (upper) and hyperonic (lower) equations of state. Three constraints are shown in the figure. XTE is a short expression of XTE J1739-285 [23], Ter 5 I is Pulsar I of the globular cluster Terzan 5 ( $M_{\text{star}} = 1.68M_{\odot}$ ) [22], and SLB is taken from the data ( $2\sigma$  limits) on  $r_{\text{ph}} \gg R$  in Ref. [21] where  $r_{\text{ph}}$  is the photospheric radius.  $B_0^*$  values are given in units of  $10^4$ . Detailed explanations are given in text.

magnetic fields. This implies that reducing the magnetic field strength could cause a decrease in the moment of inertia. If there is a mechanism for this reduction, one might expect an decrease in the spin period by angular momentum conservation.

For illustration, we plot  $I_{45}$  as a function of  $B_0^*$  on Fig. 5 for models with a central density of  $\rho = 4\rho_0$ . For a can field strength around  $B_0^* = 10^4$ , relatively small changes of the magnetic field give rise to a significant change in the moment of inertia leading to a spin change of the neutron stars.

To examine the effect of SGR flares on the moment of inertia, consider the released magnetic energy is  $E_{\text{flare}} \sim 2 \times 10^{46}$  erg for the largest observed magnetar flare yet observed [10]. If we assume that this magnetic energy released from a region below the surface of thickness of thickness  $\Delta r$ , than the released energy is related to the change in the magnetic energy density  $(\Delta B)^2/2$  times a volume factor,  $E_{\text{flare}} = (4\pi R^2 \Delta r) \times (\Delta B)^2/2$  where  $R$  is the radius of the emission region taken to be  $\sim 10$  km. An emission  $\sim 10^{46}$  erg corresponds to a change in the

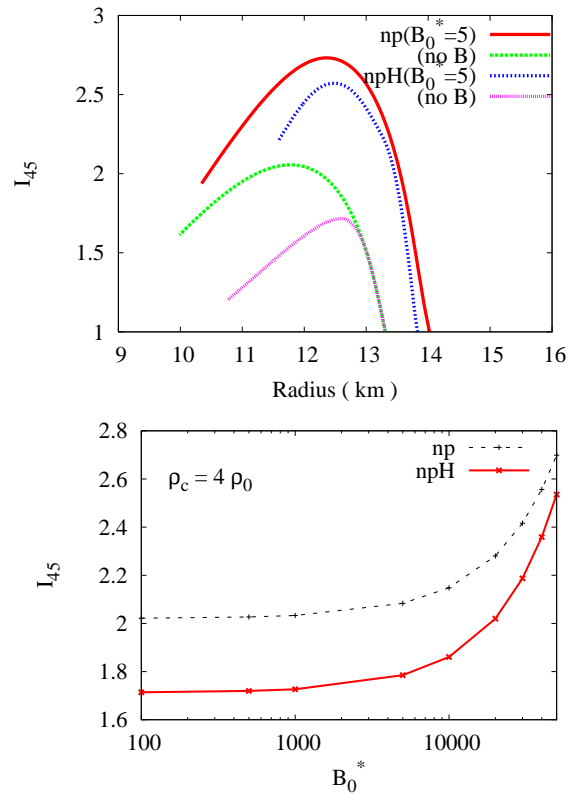


FIG. 5: The moment of inertia ( $I_{45} = 10^{45} \text{ g} \cdot \text{cm}^2$ ) for nucleonic and hyperonic equations of state with and without magnetic fields as a function of of radius (upper) and magnetic field strength (lower) figure.

magnetic field of only  $\Delta B \sim 10^{16}$  G from a thickness of  $\Delta r = 1$  m ( or  $\sim 10^{15}$  G for  $\Delta r = 100$  m ). From Fig. 5 it is apparent that a change of  $\sim 10^{16}$  G corresponds to about a 1% decrease in the moment of inertia and the rotation period. For a typical flare associated with AXPs, however, the energy released is only  $\sim 10^{40}$  erg [25], corresponding to  $\Delta B \sim 10^{13}$  G, implying a decrease of the rotation period by a factor of  $\sim 10^{-5}$ . Interestingly, this is comparable to the glitches observed [26] in some AXPs in association with a radiative event. Thus, an association of AXP emission and glitches is possible in this mechanism.

#### IV. SUMMARY

We have investigated neutron stars which contain strong interior magnetic fields. The populations of particles, the adiabatic index, the EOS, the mass-radius relation, and the moment of inertia have been calculated by using the QHD Lagrangian modified to include strong magnetic fields. We discuss the effects of a strong interior magnetic field on the physical quantities characterizing neutron stars or magnetars.

Possible changes in the mass-radius relation for neutron stars with strong magnetic fields are compared with the constraints deduced from observational data. The mass-radius relations for both nucleonic and hyperonic phases with strong magnetic fields satisfy the maximum mass suggested by Ter 5 I and XTE J1739-285, but the constraint in Ref. [21] turns out to be inconsistent with the results for the hyperonic phase unless a strong interior magnetic field is present.

The possibility of star-quakes, magnetar flares and changes in the spin rate of AXPs associated with flares has also been addressed by determining the adiabatic index and the moment of inertia with a magnetic QHD equation of state. Our results suggest that changes in the pressure response of the star with changes in density could lead to star quakes and magnetic flare energy release consistent with magnetar flares if the interior mag-

netic field strength is large enough. The implied change in the moment of inertia associated with magnetic flares is also consistent with glitches in the AXP period occasionally associated with radiative events.

### Acknowledgments

This work was supported by the National Research Foundation of Korea (Grant No. 2011-0077273). Work at NAOJ was supported by Grants-in-Aid for Scientific Research of the JSPS (20244035), and for Scientific Research on Innovative Area of MEXT (20105004). Work at the University of Notre Dame was supported by the U.S. Department of Energy under Nuclear Theory Grant DE-FG02-95-ER40934.

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