$Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ as the $B^*\bar{B}$ and $B^*\bar{B}^*$ molecular states

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In the framework of the one-boson-exchange model, we have studied the interaction of the $B^*\bar{B}$ and $B^*\bar{B}^*$ system. After considering the S-wave and D-wave mixing, we notice that both $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ can be interpreted as the $B^*\bar{B}$ and $B^*\bar{B}^*$ molecular states quite naturally. Within the same framework, there also exist several molecular charmonia including $X(3872)$ and several other molecular bottomonia, which are the partners of $Z_b(10610)$ and $Z_b(10650)$. The long-range one-pion-exchange force alone is strong enough to form these loosely bound molecular states, which ensures the numerical results quite model-independent and robust.

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I. INTRODUCTION

Very recently, the Belle Collaboration announced two charged bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$. These two states were observed in the invariant mass spectra of $h_b(nP)\pi^{\pm}$ $(n = 1, 2)$ and $\Upsilon(mS)\pi^{\pm}$ $(m = 1, 2, 3)$ of the corresponding $\Upsilon(5S) \to h_b(nP)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(mS)\pi^+\pi^-$ hidden-bottom decays [1]. With the above five hidden-bottom decay channels, Belle extracted the $Z_b(10610)$ and $Z_b(10650)$ parameters. The obtained averages over all five channels are $M_{Z_b(10610)} = 10608.4 \pm 2.0$ MeV/c^2, $\Gamma_{Z_b(10610)} = 15.6 \pm 2.5$ MeV/c^2, $M_{Z_b(10650)} = 10653.2 \pm 1.5$ MeV/c^2, $\Gamma_{Z_b(10650)} = 14.4 \pm 3.2$ MeV/c^2 [1]. In addition, the analysis of the angular distribution indicates both $Z_b(10610)$ and $Z_b(10650)$ favor $I^G(J^P) = 1^+(1^+)$.

If $Z_b(10610)$ and $Z_b(10650)$ arise from the resonance structures, they are good candidates of non-conventional bottomonium-like states. The masses of the $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ $b\bar{b}q\bar{q}$ tetraquark states were found to be around $10.1 \sim 10.2$ GeV in the framework of QCD sum rule formalism [2], which are significantly lower than these two charged Z_b states. Therefore, it's hard to accommodate them as tetraquarks. If comparing the experimental measurement with the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, one notices that $Z_b(10610)$ and $Z_b(10650)$ are close to thresholds of $B\bar{B}^*$ and $B^*\bar{B}^*$, respectively. One plausible explanation is that both $Z_b(10610)$ and $Z_b(10650)$ are either $B^*\bar{B}^*$ or $B^*\bar{B}^*$ molecular states respectively.

Before the observations of two charged $Z_b(10610)$ and $Z_b(10650)$ states, there have been many theoretical works which focused on the molecular systems composed of $B^{(*)}$ and $\bar{B}^{(*)}$ meson pair and indicated that there probably exist loosely bound S-wave $B^*\bar{B}^*$ or $B^*\bar{B}^*$ molecular states [3, 4]. To some extent, such studies were stimulated by a series of near-threshold charomonium-like X, Y, Z states in the past eight years.

Molecular states involving charmed quarks were first proposed by Voloshin and Okun more than thirty years ago [5]. Later, De Rujula, Georgi and Glashow speculated $\psi(4040)$ as a $D^*\bar{D}^*$ molecular charmonium [6]. Tönqvist calculated the possible deuteron-like two-meson bound states such as $D\bar{D}^*$ and $D^*\bar{D}^*$ using the quark-pion interaction model [7, 8]. The observations of $X(3872)$, three charged charomonium-like states $Z^+(4350)$, $Z_1^+(4050)$, $Z^+(4250)$ and $Y(4140)$, $Y(4274)$ etc. again inspired theorists' interest in the molecular system composed of charmed meson pair (see Refs. [3, 4, 9–37] for details).

As the first observed charged bottomonium-like states, $Z_b(10610)$ and $Z_b(10650)$ have attracted the attention of many theoretical groups. The authors discussed the special decay behavior of the $J = 1$ S-wave $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states based on the heavy quark symmetry in Ref. [38]. Chen, Liu and Zhu [39] found that the intermediate $Z_b(10610)$ and $Z_b(10650)$ contribution to $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ naturally explains Belle's previous observation of the anomalous $\Upsilon(2S)\pi^+\pi^-$ production near the peak of $\Upsilon(5S)$ at $\sqrt{s} = 10.87$ GeV [40], where the resulting $d\Gamma(\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-)/dm_{\pi^+\pi^-}$ and $d\Gamma(\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-)/d\cos\theta$ distributions agree with Belle's measurement after inclusion of these Z_b states [39]. The authors of Ref. [41] tried to reproduce the masses of $Z_b(10610)$ and $Z_b(10650)$ using a molecular bottomonium-like current in the QCD sum rule calculation. Yang *et al.* studied the mass spectra of the S-wave $[\bar{b}q][b\bar{q}]$, $[\bar{b}q]^*[b\bar{q}]$, $[\bar{b}q]^*[b\bar{q}]^*$ in the chiral quark model and indicated that $Z_b(10610)$ and $Z_b(10650)$ are good candidates of the S-wave $B\bar{B}^*$ and $B^*\bar{B}^*$ bound states [42]. Bugg proposed a non-exotic explanation of $Z_b(10610)$ and $Z_b(10650)$, which are interpreted as the orthogonal linear combinations of the $q\bar{q}$ and meson-meson states, namely $b\bar{b}+B\bar{B}^*$ and $b\bar{b}+B^*\bar{B}^*$

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[43], respectively. Nieves and Valderrama suggested the possible existence of two positive C-parity isoscalar states: a $^3S_1 - ^3D_1$ state with a binding energy of 90-100 MeV and a $^3P_0$ state located about 20-30 MeV below the $B\bar{B}^*$ threshold [44]. Unfortunately, the quantum number of the above states does not match those of these two charged Z_b states. Danilkin, Orlovsky and Simonov studied the interaction between a light hadron and heavy quarkonium through the transition to a pair of intermediate heavy mesons. Based on the above coupled-channel effect, the authors discussed the resonance structures close to the $B^{(*)}\bar{B}^*$ threshold [45]. Using the chromomagnetic interaction, the authors of Ref. [46] discussed the possibility of $Z_b(10610)$ and $Z_b(10650)$ being tetraquark states. In contrast, the $b\bar{b}q\bar{q}$ tetraquark states were predicted to be around 10.2 ∼ 10.3 GeV using the color-magnetic interaction with the flavor symmetry breaking corrections [47], consistent with the values extracted from the QCD sum rule approach [2].

As emphasized in Ref. [39], future dynamical study of the mass and decay pattern of the S-wave $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states are very desirable. In this work, we perform more thorough study of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems using the One-Boson-Exchange (OBE) model. Different from our former work in Refs. [3, 4], we not only consider S-wave interaction but also include D-wave contribution between $B^{(*)}$ and $\bar{B}^{(*)}$. Such a study will be helpful to answer whether the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular bottomonia exist or not.

This paper is organized as follows. After the introduction, we present the formalism of the study of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems, which includes the relevant effective Lagrangian and coupling constants, the derivation of the effective potential of the $B\bar{B}^*$ and $B^*\bar{B}^*$ system, the numerical results etc.. Finally, the paper ends with the discussion and conclusion.

II. DEDUCTION OF EFFECTIVE POTENTIAL

A. Flavor wave function

We list the flavor wave functions of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems constructed in Refs. [3, 4]. The $B\bar{B}^*$ systems can be categorized as the isovector and isoscalar states with the corresponding flavor wave functions

$$\begin{cases} |Z^{(T)}_{B\bar{B}^*}{}^{+}\rangle = \frac{1}{\sqrt{2}}\Big(|B^{*+}\bar{B}^0\rangle + cB^+\bar{B}^{*0}\Big), \\ |Z^{(T)}_{B\bar{B}^*}{}^{-}\rangle = \frac{1}{\sqrt{2}}\Big(|B^{*-}\bar{B}^0\rangle + cB^-\bar{B}^{*0}\Big), \\ |Z^{(T)}_{B\bar{B}^*}{}^{0}\rangle = \frac{1}{2}\Big[\Big(|B^{*+}B^-\rangle - B^{*0}\bar{B}^0\Big) + c\Big(B^+B^{*-} - B^0\bar{B}^{*0}\Big)\Big], \end{cases} \quad (1)$$

$$|Z^{(S)}_{B\bar{B}^*}{}^{0}\rangle = \frac{1}{2}\Big[\Big(|B^{*+}B^-\rangle + B^{*0}\bar{B}^0\Big) + c\Big(B^+B^{*-} + B^0\bar{B}^{*0}\Big)\Big], \quad (2)$$

where $c = \pm$ corresponds to C-parity $C = \mp$ respectively [3, 4]. The flavor wave functions of the $B^*\bar{B}^*$ systems can be constructed as

$$\begin{cases} |Z^{(T)}_{B^*\bar{B}^*}[\mathrm{J}]^+\rangle = |B^{*+}\bar{B}^{*0}\rangle \\ |Z^{(T)}_{B^*\bar{B}^*}[\mathrm{J}]^-\rangle = |B^{*-}\bar{B}^{*0}\rangle \\ |Z^{(T)}_{B^*\bar{B}^*}[\mathrm{J}]^0\rangle = \frac{1}{\sqrt{2}}\Big(|B^{*+}B^{*-}\rangle - |B^{*0}\bar{B}^{*0}\rangle\Big) \end{cases} \quad (3)$$

for the isovector states, and

$$|Z^{(S)}_{B^*\bar{B}^*}[\mathrm{J}]^0\rangle = \frac{1}{\sqrt{2}}\Big(|B^{*+}B^{*-}\rangle + |B^{*0}\bar{B}^{*0}\rangle\Big) \quad (4)$$

for the isoscalar state. In the above expressions, the superscripts T and S in Eqs. (1)-(4) are applied to distinguish the isovector and isoscalar states, respectively. The total angular momentum of the S-wave $B^*\bar{B}^*$ systems is $J = 0, 1, 2$. Thus, we use the extra notation [J] in Eqs. (3)-(4) to distinguish the $B^*\bar{B}^*$ systems with different total angular momentum J.

Belle indicated that both $Z_b(10610)$ and $Z_b(10650)$ belong to the isotriplet states. If $Z_b(10610)$ and $Z_b(10650)$ are the $B\bar{B}^*$ or $B^*\bar{B}^*$ molecular states respectively, they should correspond to $Z^{(T)}_{B\bar{B}^*}$ and $Z^{(T)}_{B^*\bar{B}^*}[1]$ in Eqs. (1) and (3), respectively. Since $Z_b(10610)^0$ is of C-odd parity, i.e., $C = -1$, thus the coefficient $c = +1$ is taken in Eq. (1). The choice of the coefficient $c = -1$ and $C = +1$ leads to $X(3872)$ and its partners, where $X(3872)$ corresponds to $Z^{(S)}_{D\bar{D}^*}{}'$ listed in Table. I.

In Table I, we summarize the quantum numbers of the states when we discuss whether there exist the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states. Moreover, we extend the same formalism to study the $D\bar{D}^*$ and $D^*\bar{D}^*$ systems, where the flavor wave function of the $D\bar{D}^*$ and $D^*\bar{D}^*$ systems can be obtained with replacement $B^{(*)} \to \bar{D}^{(*)}$ and $\bar{B}^{(*)} \to D^{(*)}$.

TABLE I: A summary of the $B\bar{B}^*$, $B^*\bar{B}^*$, $D\bar{D}^*$, $D^*\bar{D}^*$ systems. If taking $c = -1$ in Eqs. (1) and (2), we obtain the flavor wave functions of $Z^{(T)}_{B\bar{B}^*}{}'$ and $Z^{(S)}_{B\bar{B}^*}{}'$, which are the partners of $Z^{(T)}_{B\bar{B}^*}$ and $Z^{(S)}_{B\bar{B}^*}$ respectively.

$B\bar{B}^*/B^*\bar{B}^*$ systems	$D\bar{D}^*/D^*\bar{D}^*$ systems	$I^G(J^{PC})$
$Z^{(T)}_{B\bar{B}^*}$	$Z^{(T)}_{D\bar{D}^*}$	$1^+(1^+)$
$Z^{(S)}_{B\bar{B}^*}$	$Z^{(S)}_{D\bar{D}^*}$	$0^-(1^{+-})$
$Z^{(T)}_{B^*\bar{B}^*}[\mathrm{J}]$	$Z^{(T)}_{D^*\bar{D}^*}[\mathrm{J}]$	$1^-(0^+), 1^-(2^+), 1^+(1^+)$
$Z^{(S)}_{B^*\bar{B}^*}[\mathrm{J}]$	$Z^{(S)}_{D^*\bar{D}^*}[\mathrm{J}]$	$0^+(0^{++}), 0^+(2^{++}), 0^-(1^{+-})$
$Z^{(T)}_{B\bar{B}^*}{}'$	$Z^{(T)}_{D\bar{D}^*}{}'$	$1^-(1^+)$
$Z^{(S)}_{B\bar{B}^*}{}'$	$Z^{(S)}_{D\bar{D}^*}{}'$	$0^+(1^{++})$

B. Effective Lagrangian and coupling constant

In order to obtain the effective potential of the $B\bar{B}^*$ and $B^*\bar{B}^*$ system, we employ the OBE model, which is an effective framework to describe the $B\bar{B}^*$ or $B^*\bar{B}^*$ interaction by exchanging the light pseudoscalar, scalar and vector mesons. In terms of heavy quark limit and chiral symmetry, the interactions of light pesudoscalar, vector and scalar mesons interacting with S-wave heavy flavor mesons were constructed in Refs. [28, 48–53]

$$\begin{aligned} \mathcal{L}_{HH\mathbb{P}} &= ig\langle H^{(Q)}_b \gamma_\mu A^\mu_{ba} \gamma_5 \bar{H}^{(Q)}_a \rangle \\ &\quad + ig\langle \bar{H}^{(\bar{Q})}_a \gamma_\mu A^\mu_{ab} \gamma_5 H^{(\bar{Q})}_b \rangle, \end{aligned} \quad (5)$$

$$\mathcal{L}_{HH\mathbb{V}} = i\beta\langle H_b^{(Q)} v_\mu(\mathcal{V}_{ba}^\mu - \rho_{ba}^\mu)\bar{H}_a^{(Q)}\rangle + i\lambda\langle H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)\bar{H}_a^{(Q)}\rangle - i\beta\langle \bar{H}_a^{(\bar{Q})} v_\mu(\mathcal{V}_{ab}^\mu - \rho_{ab}^\mu)H_b^{(\bar{Q})}\rangle + i\lambda\langle H_b^{(\bar{Q})}\sigma_{\mu\nu}F'^{\mu\nu}(\rho)\bar{H}_a^{(\bar{Q})}\rangle, \quad (6)$$

$$\mathcal{L}_{HH\sigma} = g_s\langle H_a^{(Q)}\sigma\bar{H}_a^{(Q)}\rangle + g_s\langle \bar{H}_a^{(\bar{Q})}\sigma H_a^{(\bar{Q})}\rangle, \quad (7)$$

where the multiplet field $H^{(Q)}$ is composed of the pseudoscalar $\mathcal{P}$ and vector $\mathcal{P}^*$ with $\mathcal{P}^{(*)T} = (D^{(*)+}, D^{(*)0})$ or $(\bar{B}^{(*)0}, B^{(*)-})$. And $H^{(Q)}$ and $\bar{H}^{(Q)}$ are defined by

$$H_a^{(Q)} = \frac{1+\not{v}}{2}[\mathcal{P}_{a\mu}^*\gamma^\mu - \mathcal{P}_a\gamma_5], \quad (8)$$

$$\bar{H}_a^{(Q)} = [\mathcal{P}_{a\mu}^{*\dagger}\gamma^\mu + \mathcal{P}_a^\dagger\gamma_5]\frac{1+\not{v}}{2}. \quad (9)$$

Here, $\bar{H} = \gamma_0 H^\dagger\gamma_0$ and $v = (1, \mathbf{0})$.

As given in Refs. [28, 54], the anti-charmed or bottom meson fields $\widetilde{\mathcal{P}}^{(*)T} = (D^{(*)-}, \bar{D}^{(*)0})$ or $(B^{(*)0}, B^{(*)+})$ satisfy

$$\widetilde{P}_{a\mu}^* = -CP_{a\mu}^*C^{-1},\ \widetilde{P}_a = CP_aC^{-1}. \quad (10)$$

The multiplet field $H^{(\bar{Q})}$ with the heavy antiquark can be defined as

$$H_a^{(\bar{Q})} = C(CH_a^{(Q)}C^{-1})^TC^{-1} = [\widetilde{P}_a^{*\mu}\gamma_\mu - \widetilde{P}_a\gamma_5]\ \frac{1-\not{v}}{2}, \quad (11)$$

$$\bar{H}_a^{(\bar{Q})} = \frac{1-\not{v}}{2}[\widetilde{P}_a^{*\mu}\gamma_\mu + \widetilde{P}_a\gamma_5]. \quad (12)$$

If considering the following charge conjugation transformation,

$$C\xi C^{-1} = \xi^T,\ \ C\mathcal{V}_\mu C^{-1} = -\mathcal{V}_\mu^T,\ \ C\mathcal{A}_\mu C^{-1} = \mathcal{A}_\mu^T,\ \ C\rho^\mu C^{-1} = -\rho^{\mu T}, \quad (13)$$

one obtains the Lagrangian relevant to the mesons with heavy antiquark $\bar{Q}$ which is converted from the one related to the meson with heavy quark Q, where the Lagrangians are given in Eqs. (5)-(7) [28, 54]. In the above expressions, the $\mathcal{P}(\widetilde{\mathcal{P}})$ and $\mathcal{P}^*(\widetilde{\mathcal{P}}^*)$ satisfy the normalization relations $\langle 0|\mathcal{P}|Q\bar{q}(0^-)\rangle = \langle 0|\widetilde{\mathcal{P}}|\bar{Q}q(0^-)\rangle = \sqrt{M_\mathcal{P}}$ and $\langle 0|\mathcal{P}_\mu^*|Q\bar{q}(1^-)\rangle = \langle 0|\widetilde{\mathcal{P}}_\mu^*|\bar{Q}q(1^-)\rangle = \epsilon_\mu\sqrt{M_{\mathcal{P}^*}}$. The axial current is $A^\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) = \frac{i}{f_\pi}\partial_\mu\mathbb{P} + \cdots$ with $\xi = \exp(i\mathbb{P}/f_\pi)$ and $f_\pi = 132$ MeV. $\rho_{ba}^\mu = ig_V\mathbb{V}_{ba}^\mu/\sqrt{2}$, $F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + [\rho_\mu,\ \rho_\nu]$, $F'_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - [\rho_\mu,\ \rho_\nu]$ and $g_V = m_\rho/f_\pi$, with $g_V = 5.8$. Here, $\mathbb{P}$ and $\mathbb{V}$ are two by two pseudoscalar and vector matrices

$$\mathbb{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{\eta}{\sqrt{6}} & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{\eta}{\sqrt{6}} \end{pmatrix}, \quad (14)$$

$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}. \quad (15)$$

By expanding Eqs. (5)-(7), one further obtains the effective Lagrangian of the light pseudoscalar mesons $\mathbb{P}$ with the heavy flavor mesons

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{P}} = -i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\lambda\dagger}\partial^\nu\mathbb{P}_{ba} + i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\lambda}\partial^\nu\mathbb{P}_{ab}, \quad (16)$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{P}} = -\frac{2g}{f_\pi}(\mathcal{P}_b\mathcal{P}_{a\lambda}^{*\dagger} + \mathcal{P}_{b\lambda}^*\mathcal{P}_a^\dagger)\partial^\lambda\mathbb{P}_{ba} + \frac{2g}{f_\pi}(\widetilde{\mathcal{P}}_{a\lambda}^{*\dagger}\widetilde{\mathcal{P}}_b + \widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_{b\lambda}^*)\partial^\lambda\mathbb{P}_{ab}. \quad (17)$$

The effective Lagrangian depicting the coupling of the light vector mesons $\mathbb{V}$ and heavy flavor mesons reads as

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}} = -\sqrt{2}\beta g_V\mathcal{P}_b\mathcal{P}_a^\dagger v\cdot\mathbb{V}_{ba} + \sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b v\cdot\mathbb{V}_{ab}, \quad (18)$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}} = -2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}(\mathcal{P}_b\mathcal{P}_a^{*\mu\dagger} + \mathcal{P}_b^{*\mu}\mathcal{P}_a^\dagger)(\partial^\alpha\mathbb{V}^\beta)_{ba} - 2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}(\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b + \widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b^{*\mu})(\partial^\alpha\mathbb{V}^\beta)_{ab}, \quad (19)$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{V}} = \sqrt{2}\beta g_V\mathcal{P}_b^*\cdot\mathcal{P}_a^{*\dagger}v\cdot\mathbb{V}_{ba} - i2\sqrt{2}\lambda g_V\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\nu\dagger}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ba} - \sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^{*\dagger}\widetilde{\mathcal{P}}_b^* v\cdot\mathbb{V}_{ab} - i2\sqrt{2}\lambda g_V\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\nu}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ab}. \quad (20)$$

The effective Lagrangian of the scalar meson σ interacting with the heavy flavor mesons can be expressed as

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma} = -2g_s\mathcal{P}_b\mathcal{P}_b^\dagger\sigma - 2g_s\widetilde{\mathcal{P}}_b\widetilde{\mathcal{P}}_b^\dagger\sigma, \quad (21)$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\sigma} = 2g_s\mathcal{P}_b^*\cdot\mathcal{P}_b^{*\dagger}\sigma + 2g_s\widetilde{\mathcal{P}}_b^*\cdot\widetilde{\mathcal{P}}_b^{*\dagger}\sigma. \quad (22)$$

As shown in Eqs. (16)-(20), the terms for the interactions between the anti-heavy flavor mesons and light mesons can be obtained by taking the following replacements in the corresponding terms for the interactions between the heavy flavor mesons and light mesons:

$$v \to -v,\ a \to b,\ b \to a,\ \mathcal{P}_\mu^* \to \widetilde{\mathcal{P}}_\mu^{*\dagger},\ \mathcal{P} \to -\widetilde{\mathcal{P}}^\dagger,\ \mathcal{P}_\mu^{*\dagger} \to \widetilde{\mathcal{P}}_\mu^*,\ \mathcal{P}^\dagger \to -\widetilde{\mathcal{P}}.$$

$g = 0.59$ is extracted from the experimental width of D^{*+} [55]. The parameter β relevant to the vector meson can be fixed as $\beta = 0.9$ by the vector meson dominance mechanism while $\lambda = 0.56$ GeV^{-1} was obtained by comparing the form factor calculated by light cone sum rule with the one obtained by lattice QCD. As the coupling constant related to the scalar meson σ, $g_s = g_\pi/(2\sqrt{6})$ with $g_\pi = 3.73$ was given in Refs. [4, 53].

C. Effective potential

With the above preparation, we deduce the effective potentials of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems in the following . Generally, the scattering amplitude $i\mathcal{M}(J,J_Z)$ is related to the interaction potential in the momentum space in terms of the Breit approximation

$$\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\mathbf{q}) \;=\; -\frac{\mathcal{M}(B^{(*)}\bar{B}^{(*)}\to B^{(*)}\bar{B}^{(*)})}{\sqrt{\prod_i 2M_i \prod_f 2M_f}},$$

where M_i and M_j denote the masses of the initial and final states respectively. The potential in the coordinate space $\mathcal{V}(\mathbf{r})$ is obtained after performing the Fourier transformation

$$\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\mathbf{r}) = \int \frac{d\mathbf{p}}{(2\pi)^3}\, e^{i\mathbf{p}\cdot\mathbf{r}}\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\mathbf{q})\mathcal{F}^2(q^2,m_E^2), \tag{23}$$

where we need to introduce the monopole form factor (FF) $\mathcal{F}(q^2,m_E^2) = (\Lambda^2-m_E^2)/(\Lambda^2-q^2)$ to reflect the structure effect of the vertex of the heavy mesons interacting with the light mesons. m_E denotes the exchange meson mass. For $q^2\to 0$ we can treat FF as a constant while for $\Lambda \gg m$ FF approaches unity. The behavior of FF indicates [3] (1) when the distance becomes infinitely large, the interaction vertex looks like a perfect point corresponding of the constant FF; (2) when the distance is very small, the inner structure would manifest itself. In reality, the phenomenological cutoff Λ is around one to several GeV, which also plays the role of regulating the effective potential.

In this work, we consider both S-wave and D-wave interactions between $B^{(*)}$ and $\bar{B}^{(*)}$ mesons. In general, the $B\bar{B}^*$ and $B^*\bar{B}^*$ states can be expressed as

$$\left|Z_{B\bar{B}^*}^{(\alpha)\ (\prime)}\right\rangle \;=\; \begin{pmatrix}\left|BB^*({}^3S_1)\right\rangle\\ \left|BB^*({}^3D_1)\right\rangle\end{pmatrix},\ \left|Z_{B^*\bar{B}^*}^{(\alpha)}[0]\right\rangle = \begin{pmatrix}\left|B^*\bar{B}^*({}^1S_0)\right\rangle\\ \left|B^*\bar{B}^*({}^5D_0)\right\rangle\end{pmatrix},$$

$$\left|Z_{B^*\bar{B}^*}^{(\alpha)}[1]\right\rangle \;=\; \begin{pmatrix}\left|B^*\bar{B}^*({}^3S_1)\right\rangle\\ \left|B^*\bar{B}^*({}^3D_1)\right\rangle\\ \left|B^*\bar{B}^*({}^5D_1)\right\rangle\end{pmatrix},\ \left|Z_{B^*\bar{B}^*}^{(\alpha)}[2]\right\rangle = \begin{pmatrix}\left|B^*\bar{B}^*({}^5S_2)\right\rangle\\ \left|B^*\bar{B}^*({}^1D_2)\right\rangle\\ \left|B^*\bar{B}^*({}^3D_2)\right\rangle\\ \left|B^*\bar{B}^*({}^5D_2)\right\rangle\end{pmatrix} \tag{24}$$

with $\alpha = S,T$, where we use the notation ${}^{2S+1}L_J$ to denote the total spin S, angular momentum L, total angular momentum J of the $B\bar{B}^*$ or $B^*\bar{B}^*$ system. Indices S and D indicate that the couplings between B^* and $\bar{B}^*$ occur via the S-wave and D-wave interactions, respectively.

Thus, the total effective potentials of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems are

$$V_{\text{Total}}^{Z_{B\bar{B}^*}^{(\alpha)\ (\prime)}} \;=\; \left\langle Z_{B\bar{B}^*}^{(\alpha)\ (\prime)}\right|\sum_{E=\pi,\eta,\sigma,\rho,\omega}\mathcal{V}_E^{B\bar{B}^*}(r)\left|Z_{B\bar{B}^*}^{(\alpha)\ (\prime)}\right\rangle, \tag{25}$$

$$V_{\text{Total}}^{Z_{B^*\bar{B}^*}^{(\alpha)}[\text{J}]} \;=\; \left\langle Z_{B^*\bar{B}^*}^{(\alpha)}[\text{J}]\right|\sum_{E=\pi,\eta,\sigma,\rho,\omega}\mathcal{V}_E^{B^*\bar{B}^*}(r)\left|Z_{B^*\bar{B}^*}^{(\alpha)}[\text{J}]\right\rangle, \tag{26}$$

which are 2×2 and $(J+2)\times(J+2)$ matrices respectively. We impose the following constraint

$$\left|B\bar{B}^*\left({}^{2S+1}L_J\right)\right\rangle \;=\; \sum_{m,m_L,m_S} C_{1m,Lm_L}^{JM}\epsilon_n^m Y_{Lm_L}, \tag{27}$$

$$\left|B^*\bar{B}^*\left({}^{2S+1}L_J\right)\right\rangle \;=\; \sum_{m,m',m_L,m_S} C_{Sm_S,Lm_L}^{JM}C_{1m,1m'}^{Sm_S}\epsilon_{n'}^{m'}\epsilon_n^m Y_{Lm_L}, \tag{28}$$

to the effective potential obtained from the scattering amplitude. C_{1m,Lm_L}^{JM}, C_{Sm_S,Lm_L}^{JM} and $C_{1m,1m'}^{Sm_S}$ are the Clebsch-Gordan coefficients. Y_{Lm_L} is the spherical harmonics function. The polarization vector for the vector heavy flavor meson is defined as $\epsilon_\pm^m = \mp\frac{1}{\sqrt{2}}(\epsilon_x^m \pm i\epsilon_y^m)$ and $\epsilon_0^m = \epsilon_z^m$. Here, the polarization vector in Eqs. (27)-(28) is just the one appearing in the effective potentials which will be presented later.

1. The $B\bar{B}^*$ system

The general expressions of the total effective potentials of the isoscalar and isovector $B\bar{B}^*$ systems are

$$\mathcal{V}^{Z_{B\bar{B}^*}^{(T)\ (\prime)}} \;=\; V_\sigma^{\text{Direct}} - \frac{1}{2}V_\rho^{\text{Direct}} + \frac{1}{2}V_\omega^{\text{Direct}} + \frac{c}{4}\Big(-2V_\pi^{\text{Cross}} + \frac{2}{3}V_\eta^{\text{Cross}} - 2V_\rho^{\text{Corss}} + 2V_\omega^{\text{Corss}}\Big), \tag{29}$$

$$\mathcal{V}^{Z_{B\bar{B}^*}^{(S)\ (\prime)}} \;=\; V_\sigma^{\text{Direct}} + \frac{3}{2}V_\rho^{\text{Direct}} + \frac{1}{2}V_\omega^{\text{Direct}} + \frac{c}{4}\Big(6V_\pi^{\text{Cross}} + \frac{2}{3}V_\eta^{\text{Cross}} + 6V_\rho^{\text{Corss}} + 2V_\omega^{\text{Corss}}\Big), \tag{30}$$

where the subpotentials from the π, η, σ, ρ and ω meson exchanges are written as

$$V_\pi^{\text{Cross}} \;=\; -\frac{g^2}{f_\pi^2}\Big[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_2,m_2,r) + \frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_2,m_2,r)\Big], \tag{31}$$

$$V_\eta^{\text{Cross}} \;=\; -\frac{g^2}{f_\pi^2}\Big[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_3,m_3,r) + \frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_3,m_3,r)\Big], \tag{32}$$

$$V_\sigma^{\text{Direct}} \;=\; -g_s^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda,m_\sigma,r), \tag{33}$$

$$V_\rho^{\text{Direct}} \;=\; -\frac{1}{2}\beta^2 g_V^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda,m_\rho,r), \tag{34}$$

$$V_\rho^{\text{Cross}} \;=\; 2\lambda^2 g_V^2\Big[\frac{2}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_0,m_0,r) - \frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_0,m_0,r)\Big], \tag{35}$$

$$V_\omega^{\text{Direct}} \;=\; -\frac{1}{2}\beta^2 g_V^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda,m_\omega,r), \tag{36}$$

$$V_\omega^{\text{Cross}} \;=\; 2\lambda^2 g_V^2\Big[\frac{2}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_1,m_1,r) - \frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_1,m_1,r)\Big]. \tag{37}$$

In the above expressions, we define

$$\Lambda_2^2 \;=\; \Lambda^2-(m_{B^*}-m_B)^2,\quad m_2^2 = m_\pi^2-(m_{B^*}-m_B)^2,$$

$$\Lambda_3^2 = \Lambda^2 - (m_{B^*} - m_B)^2, \quad m_3^2 = m_\eta^2 - (m_{B^*} - m_B)^2,$$
$$\Lambda_0^2 = \Lambda^2 - (m_{B^*} - m_B)^2, \quad m_0^2 = m_\rho^2 - (m_{B^*} - m_B)^2,$$
$$\Lambda_1^2 = \Lambda^2 - (m_{B^*} - m_B)^2, \quad m_1^2 = m_\omega^2 - (m_{B^*} - m_B)^2,$$

and $S(\hat{\mathbf{r}}, \mathbf{a}, \mathbf{b}) = 3(\hat{\mathbf{r}} \cdot \mathbf{a})(\hat{\mathbf{r}} \cdot \mathbf{b}) - \mathbf{a} \cdot \mathbf{b}$. Additionally, functions $Y(\Lambda, m, r)$, $Z(\Lambda, m, r)$ and $T(\Lambda, m, r)$ are defined as

$$Y(\Lambda, m_E, r) = \frac{1}{4\pi r}(e^{-m_E r} - e^{-\Lambda r}) - \frac{\Lambda^2 - m_E^2}{8\pi\Lambda} e^{-\Lambda r}, \quad (38)$$
$$Z(\Lambda, m_E, r) = \nabla^2 Y(\Lambda, m_E, r) = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} Y(\Lambda, m_E, r), \quad (39)$$
$$T(\Lambda, m_E, r) = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} Y(\Lambda, m_E, r). \quad (40)$$

In Eqs. (29)-(30), $c = +1$ corresponds to the $Z^{(T)}_{B\bar{B}^*}$ and $Z^{(S)}_{B\bar{B}^*}$ states including these two charged Z_b states observed by Belle collaboration while taking $c = -1$ corresponds to the $Z^{(T)}_{B\bar{B}^*}{}'$ and $Z^{(S)}_{B\bar{B}^*}{}'$ states which are partner states of X(3872).

As indicated in Eq. (24), we consider both S-wave and D-wave interactions between the B and $\bar{B}^*$ mesons. Finally the total effective potential can be obtained by making the replacement in the subpotentials

$$\left.\begin{matrix}(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_3^\dagger) \\ (\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)\end{matrix}\right\} \rightarrowtail \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3^\dagger) \rightarrowtail \begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix},$$

which results in the total effective potential of the $B\bar{B}^*$ system, i.e, a two by two matrix.

The effective potential of the $D\bar{D}^*$ system is similar to that of $B\bar{B}^*$ system. The η, σ, ρ and ω meson exchange potentials of $D\bar{D}^*$ system can be easily obtained by replacing the parameters for the $B\bar{B}^*$ system with the ones for $D\bar{D}^*$ system. Since the mass gap of m_D^* and m_D is larger than the mass of π, which is different from the case of the $B\bar{B}^*$ system, the π exchange potential of the $D\bar{D}^*$ system is [3, 4]

$$V_\pi^{\rm Cross} = -\frac{g^2}{f_\pi^2}\Bigg[\frac{1}{3}(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_3^\dagger) Z_\pi^{DD^*}(\Lambda_4, m_4, r) + \frac{1}{3} S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3^\dagger) T_\pi^{DD^*}(\Lambda_4, m_4, r)\Bigg], \quad (41)$$

where

$$Y_\pi^{DD^*}(\Lambda_4, m_4, r) = \frac{1}{4\pi r}\Bigg(-e^{-\Lambda_4 r} - \frac{r(\Lambda_4^2 + m_4^2)}{2\Lambda_4} e^{-\Lambda_4 r} + \cos(m_4 r)\Bigg), \quad (42)$$
$$Z_\pi^{DD^*}(\Lambda_4, m_4, r) = \nabla^2 Y_\pi^{DD^*}(\Lambda_4, m_4, r) = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \times Y_\pi^{DD^*}(\Lambda_4, m_4, r), \quad (43)$$
$$T_\pi^{DD^*}(\Lambda_4, m_4, r) = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} Y_\pi^{DD^*}(\Lambda_4, m_4, r). \quad (44)$$

In the present case, the parameters Λ_4 and m_4 are defined as

$$\Lambda_4 = \sqrt{\Lambda^2 - (m_{D^*} - m_D)^2}, \quad (45)$$
$$m_4 = \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2}. \quad (46)$$

2. The $B^*\bar{B}^*$ system

For the isoscalar and isovector $B^*\bar{B}^*$ systems, the general expressions of the total effective potentials are

$$\mathcal{V}^{Z^{(T)}_{B^*\bar{B}^*}{}^{(\prime)}} = W_\sigma - \frac{1}{2}W_\rho + \frac{1}{2}W_\omega - \frac{1}{2}W_\pi + \frac{1}{6}W_\eta, \quad (47)$$
$$\mathcal{V}^{Z^{(S)}_{B^*\bar{B}^*}{}^{(\prime)}} = W_\sigma + \frac{3}{2}W_\rho + \frac{1}{2}V_\omega + \frac{3}{2}W_\pi + \frac{1}{6}W_\eta, \quad (48)$$

respectively, where the π, η, σ, ρ and ω meson exchanges can contribute to the effective potentials. The corresponding subpotentials are expressed as

$$W_\pi = -\frac{g^2}{f_\pi^2}\Bigg[\frac{1}{3}(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) Z(\Lambda, m_\pi, r) + \frac{1}{3} S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) T(\Lambda, m_\pi, r)\Bigg], \quad (49)$$
$$W_\eta = -\frac{g^2}{f_\pi^2}\Bigg[\frac{1}{3}(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) Z(\Lambda, m_\eta, r) + \frac{1}{3} S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) T(\Lambda, m_\eta, r)\Bigg], \quad (50)$$
$$W_\sigma = -g_s^2 (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) Y(\Lambda, m_\sigma, r), \quad (51)$$
$$W_\rho = -\frac{1}{4}\Bigg\{2\beta^2 g_V^2 (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) Y(\Lambda, m_\rho, r) - 8\lambda^2 g_V^2 \Bigg[\frac{2}{3}(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) Z(\Lambda, m_\rho, r) - \frac{1}{3} S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) T(\Lambda, m_\rho, r)\Bigg]\Bigg\}, \quad (52)$$
$$W_\omega = -\frac{1}{4}\Bigg\{2\beta^2 g_V^2 (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) Y(\Lambda, m_\omega, r) - 8\lambda^2 g_V^2 \Bigg[\frac{2}{3}(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) Z(\Lambda, m_\omega, r) - \frac{1}{3} S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) T(\Lambda, m_\omega, r)\Bigg]\Bigg\}. \quad (53)$$

Here, the definitions of $Y(\Lambda, m, r)$, $Z(\Lambda, m, r)$, $T(\Lambda, m, r)$ and $S(\hat{\mathbf{r}}, \mathbf{a}, \mathbf{b})$ are given in Sec. II C 1.

In this work, we consider both S-wave and D-wave interactions between the B^* and $\bar{B}^*$ mesons, which are illustrated in Eq. (24). Thus, the total effective potential of the $B^*\bar{B}^*$ with $J = 0, 1, 2$ is 2×2, 3×3, 4×4 matrices, which can be obtained by replacing the corresponding terms in the subpotentials, i.e.,

$$(\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (54)$$
$$(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \quad (55)$$
$$S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \quad (56)$$

for the $B^*\bar{B}^*$ states with $J=0$,

$$(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\tag{57}$$

$$(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix},\tag{58}$$

$$S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger,\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}0&-\sqrt{2}&0\\-\sqrt{2}&1&0\\0&0&1\end{pmatrix}\tag{59}$$

for the $B^*\bar{B}^*$ states with $J=1$, and

$$(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\tag{60}$$

$$(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}-1&0&0&0\\0&2&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix},\tag{61}$$

$$S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger,\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}0&\sqrt{\frac{2}{5}}&0&-\sqrt{\frac{14}{5}}\\\sqrt{\frac{2}{5}}&0&0&-\frac{2}{\sqrt{7}}\\0&0&-1&0\\-\sqrt{\frac{14}{5}}&-\frac{2}{\sqrt{7}}&0&-\frac{3}{7}\end{pmatrix}\tag{62}$$

for the $B^*\bar{B}^*$ states with $J=2$.

The potentials of the $D^*\bar{D}^*$ system and $B^*\bar{B}^*$ system have the same form. We only need to replace the parameters for the $B^*\bar{B}^*$ system with the ones for the $D^*\bar{D}^*$ system.

III. NUMERICAL RESULTS

With the obtained effective potentials, we can find the bound state solution by solving the coupled-channel Schrödinger equation. Corresponding to the systems in Eqs. (29)-(30), the kinetic terms for the $Z^{(\alpha)}_{B\bar{B}^*}{}'$ and $Z^{(\alpha)}_{B^*\bar{B}^*}[\mathrm{J}]$ (J $=0,1,2$) systems are

$$K_{Z^{(\alpha)}_{B\bar{B}^*}{}'}=\mathrm{diag}\left(-\frac{\triangle}{2\tilde{m}_1},\ -\frac{\triangle_2}{2\tilde{m}_1}\right),\tag{63}$$

$$K_{Z^{(\alpha)}_{B^*\bar{B}^*}[0]}=\mathrm{diag}\left(-\frac{\triangle}{2\tilde{m}_2},\ -\frac{\triangle_2}{2\tilde{m}_2}\right),\tag{64}$$

$$K_{Z^{(\alpha)}_{B^*\bar{B}^*}[1]}=\mathrm{diag}\left(-\frac{\triangle}{2\tilde{m}_2},\ -\frac{\triangle_2}{2\tilde{m}_2},\ -\frac{\triangle_2}{2\tilde{m}_2}\right),\tag{65}$$

$$K_{Z^{(\alpha)}_{B^*\bar{B}^*}[2]}=\mathrm{diag}\left(-\frac{\triangle}{2\tilde{m}_2},\ -\frac{\triangle_2}{2\tilde{m}_2},\ -\frac{\triangle_2}{2\tilde{m}_2},\ -\frac{\triangle_2}{2\tilde{m}_2}\right),\tag{66}$$

respectively. Here, $\triangle=\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, $\triangle_2=\triangle-\frac{6}{r^2}$. $\tilde{m}_1=m_Bm_{B^*}/(m_B+m_{B^*})$ and $\tilde{m}_2=m_{B^*}/2$ are the reduced masses of the $Z^{(i)}_{b1}$ and $Z^{(i)}_{b2}$ systems, where m_B and m_{B^*} denote the masses of the pseudoscalar and vector bottom mesons [59], respectively. Of course, the kinematic terms for the $D\bar{D}^*$ and $D^*\bar{D}^*$ systems are of the same forms as those for the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems, where we replace the mass of $D^{(*)}$ with that of $B^{(*)}$.

In this work, the FESSDE program [56, 57] is adopted to produce the numerical values for the binding energy and the relevant root-mean-square r with the variation of the cutoff in the region of $0.8\le\Lambda\le5$ GeV. Moreover, we also use MATSCE [58], a MATLAB package for solving coupled-channel Schrödinger equation, to perform an independent cross-check.

Throughout this work, we will first present the numerical results of the obtained bound state solutions when all types of the one-meson-exchange (OME) potentials are included. The one-pion-exchange force contributes to the long range interaction between the heavy meson pair, which is clear and well-known. In contrast, the scalar and vector meson exchanges are used to mimic the intermediate and short range interaction between the heavy mesons, which are not determined very precisely. In order to find out whether the existence of the possible bound molecular states is sensitive to the details of the short-range interaction, we will also study the case when only the one-pion-exchange (OPE) contribution is considered. In the following illustration, we use OME and OPE to distinguish such two cases. If the OPE force alone is strong enough to form a loosely bound state, such a case is particularly interesting phenomenologically.

A. The $B\bar{B}^*$ and $D\bar{D}^*$ systems

In the following, we first present the numerical results for the $Z^{(T)}_{B\bar{B}^*}$ and $Z^{(S)}_{B\bar{B}^*}$ states where $Z^{(T)}_{B\bar{B}^*}$ corresponds to $Z_b(10610)$ observed by Belle [1]. As shown in Table I, there exist two systems with $c=-1$ and $C=+1$ in the flavor wave functions, where are marked as $Z^{(T)}_{B\bar{B}^*}{}'$ and $Z^{(S)}_{B\bar{B}^*}{}'$.

1. In Table II, we present the numerical results of the obtained bound state solutions in both OME and OPE cases. We find the bound state solutions for the two isoscalar $Z^{(S)}_{B\bar{B}^*}$ and $Z^{(S)}_{B\bar{B}^*}{}'$ with reasonable Λ values ($\Lambda\sim$ 1 GeV), which indicates the existence of the $Z^{(S)}_{B\bar{B}^*}$ and $Z^{(S)}_{B\bar{B}^*}{}'$ molecular states.

2. For the $Z^{(T)}_{B\bar{B}^*}$ state, we also find the bound state solution with Λ around 2.2 GeV. Our result shows that

$Z^{(T)}_{B\bar{B}^*}$ could be as a molecular state with a very shallow binding energy. In addition, its binding energy is not strongly dependent on Λ. Thus, it is quite natural to interpret $Z_b(10610)$ as a $B\bar{B}^*$ molecular state with isospin $I = 1$.

3. For the $Z^{(T)\prime}_{B\bar{B}^*}$ system, the bound state solution can be found in the region $\Lambda > 4.7$ GeV. To some extent, the value of Λ for the $Z^{(T)\prime}_{B\bar{B}^*}$ seems a little large compared to 1 GeV.

4. We also discuss the case when we only consider the OPE potential. For the $\{Z^{(T)}_{B\bar{B}^*}, Z^{(T)\prime}_{B\bar{B}^*}, Z^{(S)\prime}_{B\bar{B}^*}\}$ or $Z^{(S)}_{B\bar{B}^*}$, we need to decrease or increase the Λ value to obtain the same binding energy as that from OME. The one pion meson exchange potential indeed plays the crucial role in the formation of the BB^* bound states.

TABLE II: The obtained bound state solutions (binding energy E and root-mean-square radius $r_{\rm RMS}$) for the $B\bar{B}^*$ systems. Here, we discuss two situations, i.e., including all one meson exchange (OME) contribution and only considering one pion exchange (OPE) potential.

		OME			OPE		
$I^G(J^{PC})$	State	Λ	E (MeV)	$r_{\rm RMS}$ (fm)	Λ	E (MeV)	$r_{\rm RMS}$ (fm)
$1^+(1^+)$	$Z^{(T)}_{B\bar{B}^*}$	2.1	-0.22	3.05	2.2	-8.69	0.62
		2.3	-1.64	1.31	2.4	-20.29	0.47
		2.5	-4.74	0.84	2.6	-38.54	0.36
$1^-(1^+)$	$Z^{(T)\prime}_{B\bar{B}^*}$	4.9	-0.14	3.64	4.5	-17.79	0.56
		5.0	-0.41	2.45	4.6	-22.65	0.52
		5.1	-0.85	1.80	4.7	-28.29	0.48
$0^-(1^{+-})$	$Z^{(S)}_{B\bar{B}^*}$	1.0	-0.28	3.35	1.8	-10.09	0.96
		1.05	-1.81	1.71	1.9	-15.11	0.84
		1.1	-5.36	1.18	2.0	-21.53	0.76
$0^+(1^{++})$	$Z^{(S)\prime}_{B\bar{B}^*}$	0.8	-0.95	1.84	1.0	-7.68	0.82
		0.9	-6.81	0.91	1.1	-15.30	0.65
		1.0	-19.92	0.65	1.2	-26.53	0.53

We extend the formalism in Sec. II to study the $D\bar{D}^*$ systems. As shown in Table III, we can exclude the existence of the $Z^{(T)}_{D\bar{D}^*}$ and $Z^{(T)\prime}_{D\bar{D}^*}$ since we do not find any bound state solution for the $Z^{(T)}_{D\bar{D}^*}$ and $Z^{(T)\prime}_{D\bar{D}^*}$ states. For the two isoscalar $Z^{(S)}_{D\bar{D}^*}$ and $Z^{(S)\prime}_{D\bar{D}^*}$, there exist loosely bound states with reasonable Λ values. If only considering the OPE exchange potential, we notice: (1) the bound state solution of the $Z^{(T)}_{D\bar{D}^*}$ appears when $\Lambda \sim 4.6$ GeV, which largely deviates from 1 GeV; (2) there still does not exist any bound state solution for $Z^{(T)\prime}_{D\bar{D}^*}$; (3) for $Z^{(S)}_{D\bar{D}^*}$ and $Z^{(S)\prime}_{D\bar{D}^*}$, Λ becomes larger in order to find the bound state solution. The comparison between the OME and OPE results also reflects the importance of one pion exchange in the $D\bar{D}^*$ systems. We need to specify that $Z^{(S)\prime}_{D\bar{D}^*}$ with $0^+(1^{++})$ directly corresponds to the observed $X(3872)$ [60].

The BaBar Collaboration measured the radiative decay of $X(3872)$ and found a ratio of $B(X(3872) \to \psi(2S)\gamma)/B(X(3872) \to J/\psi\gamma) = 3.4 \pm 1.4$ [61], which contradicts the prediction with a purely $D\bar{D}^*$ molecular assignment to $X(3872)$ [13]. However, very recently Belle reported a new measurement of the radiative decay of $X(3872)$, where only the decay mode $X(3872) \to J/\psi\gamma$ was observed and the upper limit $B(X(3872) \to \psi(2S)\gamma)/B(X(3872) \to J/\psi\gamma) < 2.1$ was given [62]. The inconsistence between the Belle and BaBar results indicate that the study of $X(3872)$ is still an important research topic. Our numerical results suggest that the mass of the loosely bound molecular state $Z^{(S)\prime}_{D\bar{D}^*}$ is consistent with that of $X(3872)$. The assignment of $X(3872)$ as a molecular candidate is still very attractive.

TABLE III: The obtained bound state solutions (binding energy E and root-mean-square radius $r_{\rm RMS}$) for $D\bar{D}^*$ systems. Here, we discuss two situations, i.e., including all one meson exchange (OME) contribution and only considering one pion exchange (OPE) potential to $B\bar{B}^*$ systems.

		OME			OPE		
$I^G(J^{PC})$	State	Λ	E (MeV)	$r_{\rm RMS}$ (fm)	Λ	E (MeV)	$r_{\rm RMS}$ (fm)
$1^+(1^{+-})$	$Z^{(T)}_{D\bar{D}^*}$	-	-	-	4.6	-0.85	1.46
					4.7	-3.42	1.17
					4.8	-7.18	0.93
					4.9	-12.40	0.75
$1^-(1^{++})$	$Z^{(T)\prime}_{D\bar{D}^*}$	-	-	-	-	-	-
$0^-(1^{+-})$	$Z^{(S)}_{D\bar{D}^*}$	1.3	-	-	3.4	-0.11	1.74
		1.4	-1.56	1.61	3.5	-2.03	1.50
		1.5	-12.95	0.98	3.6	-4.79	1.26
		1.6	-35.73	0.69	3.7	-9.62	1.06
$0^+(1^{++})$	$Z^{(S)\prime}_{D\bar{D}^*}$	1.1	-0.61		1.7	-3.01	1.37
		1.2	-4.42	1.38	1.8	-7.41	1.06
		1.3	-11.78	1.05	1.9	-14.15	0.84
		1.4	-21.88	0.86	2	-23.82	0.68

B. The $B^*\bar{B}^*$ and $D^*\bar{D}^*$ systems

The numerical results of the $B^*\bar{B}^*$ systems are presented in Table IV, which include the obtained binding energy and the corresponding root-mean-square radius. We find the bound state solution for all the $B^*\bar{B}^*$ states with reasonable Λ values:

1. A loosely bound state exists for $Z^{(T)}_{B^*\bar{B}^*}[1]$ corresponding to the observed $Z_b(10650)$ with Λ slightly above 2 GeV. Only considering the OPE potential, the obtained binding energy becomes deeper with the same Λ value.

2. In addition, the $B^*\bar{B}^*$ can form loosely bound molecular states $Z^{(S)}_{B^*\bar{B}^*}[0]$, $Z^{(T)}_{B^*\bar{B}^*}[0]$, $Z^{(S)}_{B^*\bar{B}^*}[1]$ and $Z^{(S)}_{B^*\bar{B}^*}[2]$ with very reasonable Λ values. Comparing the results between OME and OPE cases, one notices again that the one pion exchange indeed is very important to form the $B^*\bar{B}^*$ bound state.

3. For the $Z^{(T)}_{B^*\bar{B}^*}[2]$ state, the existence of the loosely bound state requires the value of Λ around 4.4 GeV.

TABLE IV: The obtained bound state solutions (binding energy E and root-mean-square radius $r_{\rm RMS}$) for $B^*\bar{B}^*$ systems. Here, we discuss two situations, i.e., including all one meson exchange (OME) contribution and only considering one pion exchange (OPE) potential to $B\bar{B}^*$systems.

		OME			OPE		
$I^G(J^{PC})$	State	Λ	E (MeV)	$r_{\rm RMS}$ (fm)	Λ	E (MeV)	$r_{\rm RMS}$ (fm)
$1^+(0^+)$	$Z^{(T)}_{B^*B^*}[0]$	1.2	-	-	1	-	-
		1.4	-1.44	1.24	1.2	-0.32	1.53
		1.6	-6.16	0.77	1.4	-5.69	0.78
		1.8	-15.15	0.54	1.6	-18.82	0.50
$0^-(0^{+-})$	$Z^{(S)}_{B^*B^*}[0]$	0.9	-	-	1	-	-
		1	-0.81	2.11	1.2	-0.52	2.76
		1.1	-9.98	1.02	1.4	-5.74	1.12
		1.2	-35.16	0.70	1.6	-20.92	0.77
$1^+(1^+)$	$Z^{(T)}_{B^*B^*}[1]$	2.2	-0.81	1.38	2	-2.17	1.15
		2.4	-3.31	0.95	2.2	-8.01	0.68
		2.6	-7.80	0.68	2.4	-19.00	0.48
		2.8	-14.94	0.52	2.6	-36.36	0.38
$0^-(1^{+-})$	$Z^{(S)}_{B^*B^*}[1]$	1.	-0.01	2.07	1.4	-0.51	1.90
		1.1	-5.50	1.17	1.6	3.65	-1.32
		1.2	-21.76	-0.75	1.8	-10.26	0.96
		1.3	-53.68	0.55	2.0	-21.81	0.75
$1^+(2^+)$	$Z^{(T)}_{B^*B^*}[2]$	4.4	-0.44	1.59	3.6	-2.82	1.12
		4.6	-1.59	1.28	3.8	-6.21	0.85
		4.8	-3.42	1.01	4.0	-11.41	0.68
		5.	-6.16	0.81	4.2	-18.77	0.57
$0^-(2^{+-})$	$Z^{(S)}_{B^*B^*}[2]$	0.8	-2.33	1.32	0.8	-1.81	1.48
		0.9	-10.45	0.84	0.9	-5.64	1.01
		1.0	-27.14	0.63	1.0	-12.28	0.76

In the following, we also present the numerical results for the $D^*\bar{D}^*$ systems in Table V. Our calculation indicates:

1. We find the bound state solutions for the three isoscalar states $Z^{(S)}_{D^*\bar{D}^*}[0]$, $Z^{(S)}_{D^*\bar{D}^*}[1]$ and $Z^{(S)}_{D^*\bar{D}^*}[2]$, where the corresponding Λ is around 1 GeV. If only considering the OPE contribution for the $Z^{(S)}_{D^*\bar{D}^*}[0]$, $Z^{(S)}_{D^*\bar{D}^*}[1]$ states, we need to largely increase Λ value in order to obtain a loosely bound state. Here, either $Z^{(S)}_{D^*\bar{D}^*}[0]$ or $Z^{(S)}_{D^*\bar{D}^*}[2]$ could correspond to the observed $Y(3930)$ by Belle [63] and BaBar [64], which is consistent with the conclusion in Ref. [35].

2. There does not exist the bound state $Z^{(T)}_{D^*\bar{D}^*}[2]$. The value of Λ is about 3.6 GeV in order to form a bound state $Z^{(T)}_{D^*\bar{D}^*}[0]$. In the range $0.8 < \Lambda < 5$ GeV, we cannot find the bound state solution for $Z^{(T)}_{D^*\bar{D}^*}[1]$ in the OME case. Thus, we exclude the existence of the $Z^{(T)}_{D^*\bar{D}^*}[1]$ molecular state.

TABLE V: The obtained bound state solutions (binding energy E and root-mean-square radius $r_{\rm RMS}$) for $D^*\bar{D}^*$ systems. Here, we discuss two situations, i.e., including all one meson exchange (OME) contribution and only considering one pion exchange (OPE) potential to $D^*\bar{D}^*$ systems.

		OME			OPE		
$I^G(J^{PC})$	State	Λ	E (MeV)	$r_{\rm RMS}$ (fm)	Λ	E (MeV)	$r_{\rm RMS}$ (fm)
$1^+(0^+)$	$Z^{(T)}_{D^*D^*}[0]$	3.6	-0.94	1.74	2.8	-2.03	1.47
		3.8	-6.16	1.00	2.9	-6.10	1.00
		4	-16.44	0.66	3	-12.51	0.74
		4.2	-33.23	0.49	3.1	-21.56	0.59
$0^-(0^{+-})$	$Z^{(S)}_{D^*D^*}[0]$	1.4	-1.72	1.62	3	-5.70	1.24
		1.5	-17.98	0.88	3.1	-12.15	0.96
		1.6	-54.60	0.47	3.2	-21.83	0.78
$1^+(1^+)$	$Z^{(T)}_{D^*D^*}[1]$	-	-	-	4.7	-6.96	0.94
					4.8	-12.29	0.73
					4.9	-19.36	0.60
					5	-28.31	0.51
$0^-(1^{+-})$	$Z^{(S)}_{D^*D^*}[1]$	1.3	-		3.6	-9.91	1.01
		1.4	-3.44	1.44	3.7	-15.25	0.87
		1.5	-16.57	0.90	3.8	-22.07	0.76
		1.6	-41.25	0.66	3.9	-30.53	0.68
$1^+(2^+)$	$Z^{(T)}_{D^*D^*}[2]$	-	-	-	-	-	
$0^-(2^{+-})$	$Z^{(S)}_{D^*D^*}[2]$	1.1	-0.61	1.72	1.6	-3.89	1.28
		1.2	-7.50	1.19	1.7	-9.64	0.98
		1.3	-19.22	0.89	1.8	-18.38	0.77
		1.4	-35.93	0.73	1.9	-30.71	0.64

IV. SUMMARY

Stimulated by the newly observed bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$, we have carried out a systematical study of the $B\bar{B}^*$ and $B^*\bar{B}^*$ system using the one boson exchange model in our work. We have considered both the S-wave and D-wave interaction between the $B^{(*)}$ and $\bar{B}^*$ mesons, which results in the mixing of the S-wave and D-wave contribution as discussed in Sec. II. Our numerical results indicate that the $Z_b(10610)$ and $Z_b(10650)$ signals can be interpreted as the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states with $I^G(J^P) = 1^+(1^+)$ respectively.

As a byproduct, we also predict the existences of six other $B\bar{B}^*$ and $B^*\bar{B}^*$ bound states (see Table VI) within the same framework. We want to stress that the long-range interaction between the heavy meson pair arises from the one-pion-exchange force, which is clearly known. This OPE force alone is strong enough to form the above loosely bound molecular states, which makes the present results quite model-independent and robust.

The observation of these $Z_b(10610)$ and $Z_b(10650)$ states shows that the hidden-bottom decay are very important decay channels, which is characteristic and helpful to the search of the molecular bottomonium. After taking into account of the phase space [59, 65–67] and the conservation of quantum number, the $Z^{(S)}_{B\bar{B}^*}$, $Z^{(S)\prime}_{B\bar{B}^*}$, $Z^{(T)}_{B^*\bar{B}^*}[0]$, $Z^{(S)}_{B^*\bar{B}^*}[0]$, $Z^{(S)}_{B^*\bar{B}^*}[1]$ and $Z^{(S)}_{B^*\bar{B}^*}[2]$ molecular states can decay into

$$\Big\{\Upsilon(1S)\eta, \Upsilon(2S)\eta, h_b(1P)\eta, \eta_b(1S)\omega\Big\},$$
$$\Big\{\Upsilon(1S)\omega, \chi_{b0}(1P)\eta, \chi_{b1}(1P)\eta, \chi_{b2}(1P)\eta\Big\},$$
$$\Big\{\chi_{b1}(1P)\pi, \chi_{b1}(2P)\pi, \Upsilon(1S)\rho, \eta_b(1S)\pi\Big\},$$
$$\Big\{\Upsilon(1S)\omega, \chi_{b1}(1P)\eta, \eta_b(1S)\eta\Big\},$$
$$\Big\{\chi_{b0}(1P)\omega, \Upsilon(1S)\eta, \Upsilon(2S)\eta, \eta_b(1S)\omega, h_b(1P)\eta\Big\},$$
$$\Big\{\Upsilon(1S)\omega, \chi_{b1}(1P)\eta, \chi_{b2}(1P)\eta, \eta_b(1S)\eta\Big\},$$

respectively. The above modes can be used in the future experimental search of the partner states of $Z_b(10610)$ and $Z_b(10650)$.

We also extend our formalism to study the molecular charmonia. The observed possible molecular charmonia are listed in Table VI. The possible hidden-charm decay channels of the molecular states $Z^{(S)}_{D\bar{D}^*}$, $Z^{(S)}_{D^*\bar{D}^*}[0]$, $Z^{(S)}_{D^*\bar{D}^*}[1]$ and $Z^{(S)}_{D^*\bar{D}^*}[2]$ are

$$\Big\{\eta_c(1S)\omega, J/\psi(1S)\eta\Big\},$$
$$\Big\{J/\psi\omega, \eta_c(1S)\eta\Big\},$$
$$\Big\{\eta_c(1S)\omega, J/\psi(1S)\eta\Big\},$$
$$\Big\{J/\psi(1S)\omega, \eta_c(1S)\eta\Big\},$$

respectively. Due to the limit of phase space, the hidden-charm decays for the other one $Z^{(S)\prime}_{D\bar{D}^*}$ molecular state are $J/\psi(1S)$ or $\eta_c(1S)$ plus multi-pions.

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TABLE VI: A summary of the $B\bar{B}^*$, $B^*\bar{B}^*$, $D\bar{D}^*$, $D^*\bar{D}^*$ systems. Here, we use ✓ and × to mark the corresponding systems with and without the bound states solution when taking a reasonable Λ value, respectively. The criteria of the choice of the reasonable Λ may be strongly biased.

$I^G(J^P)$	System	Remark	Experiment [1]	System	Remark	Experiment
$1^+(1^+)$	$Z_{B\bar{B}^*}^{(T)}$	✓	$Z_b(10610)$	$Z_{D\bar{D}^*}^{(T)}$	×	
$0^-(1^{+-})$	$Z_{B\bar{B}^*}^{(S)}$	✓		$Z_{D\bar{D}^*}^{(S)}$	✓	
$1^-(1^+)$	$Z_{B\bar{B}^*}^{(T)\prime}$	×		$Z_{D\bar{D}^*}^{(T)\prime}$	×	
$0^+(1^{++})$	$Z_{B\bar{B}^*}^{(S)\prime}$	✓		$Z_{D\bar{D}^*}^{(S)\prime}$	✓	$X(3872)$ [60]
$1^-(0^+)$	$Z_{B^*\bar{B}^*}^{(T)}[0]$	✓		$Z_{D^*\bar{D}^*}^{(T)}[0]$	×	
$0^+(0^{++})$	$Z_{B^*\bar{B}^*}^{(S)}[0]$	✓		$Z_{D^*\bar{D}^*}^{(S)}[0]$	✓	$Y(3930)$ [65–67]
$1^+(1^+)$	$Z_{B^*\bar{B}^*}^{(T)}[1]$	✓	$Z_b(10650)$	$Z_{D^*\bar{D}^*}^{(T)}[1]$	×	
$0^-(1^{+-})$	$Z_{B^*\bar{B}^*}^{(S)}[1]$	✓		$Z_{D^*\bar{D}^*}^{(S)}[1]$	✓	
$1^-(2^+)$	$Z_{B^*\bar{B}^*}^{(T)}[2]$	×		$Z_{D^*\bar{D}^*}^{(T)}[2]$	×	
$0^+(2^{++})$	$Z_{B^*\bar{B}^*}^{(S)}[2]$	✓		$Z_{D^*\bar{D}^*}^{(S)}[2]$	✓	$Y(3940)$ [65–67]

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