

# Plasma damping effects on the radiative energy loss of relativistic particles

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The energy loss of a relativistic charge undergoing multiple scatterings while traversing an infinite, polarizable and absorptive plasma is investigated. Polarization and damping mechanisms in the medium are phenomenologically modelled by a complex index of refraction. Apart from the known Ter-Mikaelian effect related to the dielectric polarization of matter, we find an additional, substantial reduction of the energy loss due to damping of radiation. The observed effect is more prominent for larger damping and/or larger energy of the charge. A conceivable analog of this phenomenon in QCD could influence the study of jet quenching phenomena in ultra-relativistic heavy-ion collisions at RHIC and LHC.

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The observed strong suppression of high transverse momentum hadron-yields in relativistic nuclear collisions [1, 2] has been interpreted as a signature for the formation of a strongly interacting, deconfined and dense quark-gluon plasma (QGP) [3, 4], in which energetic partons suffer radiative and collisional energy loss [5–10]. Important in the context of radiative energy loss of relativistic particles, as realized by Landau, Pomeranchuk [11] and Migdal [12] for QED, and later on generalized to QCD [6, 13], is the possibility of a destructive interference between radiation amplitudes when the charged particle undergoes multiple scatterings within the formation time of radiation, resulting in a suppression of the radiation spectrum compared to the sum of incoherent emissions at successive scatterings (LPM effect).

As pointed out by Ter-Mikaelian [14], the radiation spectrum, and hence the energy loss, is also modified by the dielectric polarization of the medium, which gives rise to medium-modifications in the dispersion relation of radiated quanta (TM effect). The QCD analog of the TM effect was studied in [15, 16] by investigating the gluon radiation spectrum in the QGP. These approaches made use of dielectric functions either including a constant thermal gluon mass [15] or being related to the hard thermal loop (HTL) gluon self-energy [16]. In none of these studies, however, the possible additional influence of the damping of radiation in an absorptive medium was taken into account. The intent of our Letter is to investigate this influence.

We study in linear response theory the energy loss of an energetic point-charge in an absorptive, dielectric medium. Polarization and radiation damping effects are both taken into account by employing a complex medium index of refraction. Following the original approach in [11], the charge's velocity vector  $\vec{v}(t)$  is modelled to change with time due to successive scatterings in an infinite medium. Our studies, however, also qualita-

tively apply in the case, in which the size of the medium is large compared to the formation length of radiated quanta. A detailed derivation and discussion of the results will be reported elsewhere. Here, we want to focus on two essential findings: (i) Damping of radiation in an absorptive medium can lead to a substantial *reduction* of the radiative energy loss and (ii) the observed effect intensifies with increasing medium damping and/or increasing energy  $E$  of the charge. Our investigations, being strictly valid for electro-magnetic plasmas, represent the classical, abelian approximation for the dynamics of a color charge in the QGP. It may, thus, be conceivable that radiation damping is also of some impact in parton energy loss studies. Throughout this work natural units are used, i. e.  $\hbar = c = 1$ .

In line with [10], we determine the energy loss from the negative mechanical work  $W$  performed on the charge by its electric field. As  $W$  accounts for the total energy loss of the charge, it incorporates in an absorptive medium in particular both, the energy radiated out of and the amount of energy dissipated inside the medium. Thus, a study of  $W$  is particularly suitable for our purposes. It is comfortably evaluated in the mixed spatial coordinate and frequency representation of its integrand via

$$W = 2 \operatorname{Re} \left( \int d^3 \vec{r}' \int_0^\infty d\omega \vec{E}(\vec{r}', \omega) \vec{j}(\vec{r}', \omega)^* \right), \quad (1)$$

where  $\vec{j}(\vec{r}', t) = q \vec{v}(t) \delta^{(3)}(\vec{r}' - \vec{r}(t))$  is the classical current of charge  $q$  in space-time coordinates and  $\vec{E}$  is the total electric field of the charge inside the medium.

From Maxwell's equations for a linear dispersive medium, one obtains in Fourier-space

$$\vec{E}_{\vec{k}}(\omega) = \frac{iq}{(2\pi)^2} \int dt' \frac{e^{i\omega t' - i\vec{k}\vec{r}(t')}}{\omega \epsilon(\omega)} \times \left\{ \frac{\vec{v}_L(t') k^2}{(\omega^2 \mu(\omega) \epsilon(\omega) - k^2)} - \frac{\vec{v}(t') \omega^2 \mu(\omega) \epsilon(\omega)}{(\omega^2 \mu(\omega) \epsilon(\omega) - k^2)} \right\}, \quad (2)$$

where  $\vec{v}_L = (\vec{k}\vec{v})\vec{k}/k^2$  is the longitudinal component of  $\vec{v}$  with respect to the wave vector  $\vec{k}$ , and  $\epsilon(\omega)$  and  $\mu(\omega)$  denote permittivity and permeability of the matter, respectively, which are defined to be complex in order to account for the damping of (time-like) excitations in the plasma [17]. For positive  $\omega$ , one finds from (1) in differential form

$$\frac{dW}{d\omega} = \text{Re} \left( -\frac{iq^2}{8\pi^4} \int dt \int dt' \int d^3\vec{k} \frac{e^{-i\omega(t-t') + i\vec{k}\vec{\Delta}r}}{\omega\epsilon(\omega)} \times \left\{ \frac{(\vec{k}\vec{v}(t))(\vec{k}\vec{v}(t'))}{(k^2 - \omega^2 n^2(\omega))} - \frac{\vec{v}(t)\vec{v}(t')\omega^2 n^2(\omega)}{(k^2 - \omega^2 n^2(\omega))} \right\} \right), \quad (3)$$

where  $\vec{\Delta}r = \vec{r}(t) - \vec{r}(t')$  and  $n^2(\omega) = \mu(\omega)\epsilon(\omega)$  is the complex squared index of refraction.

In the following, we assume for simplicity  $\epsilon(\omega)$  and  $\mu(\omega)$  to depend on  $\omega$  only, i. e. spatial distortions in the plasma are not considered. Then,  $dW/d\omega$  in (3) is sensitive to simple poles in the complex momentum-plane and the momentum-integrals in (3) can easily be evaluated analytically by contour integration. The inclusion of an explicit  $\vec{k}$ -dependence in  $\epsilon$  and  $\mu$  is left for future studies.

By decomposing the index of refraction into real and imaginary parts,  $n(\omega) = n_r(\omega) + in_i(\omega)$ , and defining  $\vec{g} = \omega n(\omega)\vec{\Delta}r$  one obtains from (3)

$$\frac{dW}{d\omega} = \text{Re} \left( \frac{iq^2}{4\pi^2} \int dt \int dt' \frac{\omega^2 n^3(\omega)}{\epsilon(\omega)} e^{-i\omega(t-t')} \mathcal{A}(t, t') \right) \quad (4)$$

with

$$\mathcal{A}(t, t') = (\vec{v}(t)\vec{v}(t') + (\nabla_{\vec{g}}\vec{v}(t))(\nabla_{\vec{g}}\vec{v}(t'))) \frac{e^{i \text{sgn}(n_i)g}}{g}. \quad (5)$$

This is the main result of our work. Essential here is the exponential factor

$$e^{i \text{sgn}(n_i)g} = e^{i \text{sgn}(n_i)\omega n_r \Delta r} e^{-\omega |n_i| \Delta r}, \quad (6)$$

which implies that irrespective of the sign of  $n_i(\omega)$ ,  $\text{sgn}(n_i)$ , the mechanical work is exponentially damped for  $|n_i| \neq 0$ . This is a direct consequence of the fact that, depending on  $\text{sgn}(n_i)$ , only one of the two simple poles in (3) contributes to the energy loss. Only the phase factor in (6), associated with  $n_r(\omega)$ , is affected by  $\text{sgn}(n_i)$ .

Due to the symmetry property of  $\mathcal{A}(t, t')$  under variable exchange, (4) can be written in a form such that only  $t > t'$  has to be considered in the time-integration. Omitting as in [11] those terms stemming from the action of  $\nabla_{\vec{g}}$  on  $1/g$  in (5),  $\mathcal{A}(t, t')$  entering (4) reduces to

$$\mathcal{A}_{(0)}(t, t') = \frac{1}{g} \left( \vec{v}(t)\vec{v}(t') - \frac{(\vec{v}(t)\vec{g})(\vec{v}(t')\vec{g})}{g^2} \right) e^{i \text{sgn}(n_i)g}. \quad (7)$$

For constant  $\vec{v}$ ,  $\mathcal{A}_{(0)}(t, t') = 0$  and the corresponding energy loss determined via (4) vanishes.

Going beyond constant  $\vec{v}$ , we study, as in [11], the case, where the velocity  $\vec{v}(t') = v'\hat{z}$  is changed due to multiple scatterings according to  $\vec{v}(t' + \bar{t}) = v'\hat{z} \cos \theta_{\bar{t}} + v'\vec{e}_{\perp} \sin \theta_{\bar{t}}$  for  $\bar{t} > 0$ , i. e. where  $v^2$  remains constant. Assuming the relative deflection angle,  $\theta_{\bar{t}}$ , to be small within  $\bar{t} = t - t'$ , one obtains  $\vec{v}(t)\vec{v}(t') \simeq v'^2(1 - \theta_{\bar{t}}^2/2)$  in (7). Likewise, by omitting terms of order  $\mathcal{O}(\theta^4)$ , one finds  $\vec{\Delta}r \simeq v'\hat{z}\bar{t} - \frac{1}{2}v'\hat{z}\mathcal{I}_2 + v'\vec{e}_{\perp}\mathcal{I}_1$  with  $\mathcal{I}_2 = \int_0^{\bar{t}} \theta_{\tau}^2 d\tau$  and  $\mathcal{I}_1 = \int_0^{\bar{t}} \theta_{\tau} d\tau$ . Averaging over the deflection angles, one finds  $\langle \Delta r \rangle \simeq v'\bar{t}\sqrt{1 - \hat{q}\bar{t}/(3E^2)}$  with  $\hat{q} = E^2\langle \theta_{\bar{t}}^2 \rangle/\bar{t}$  in units of GeV<sup>2</sup>/fm. The parameter  $\hat{q}$  is the mean accumulated transverse momentum squared of the deflected charge per unit time. Approximating  $dz = v'dt'$  and using (7), one obtains from (4)

$$\begin{aligned} \frac{d^2W}{dzd\omega} &\simeq -\text{Re} \left( \frac{2i\alpha}{3\pi} \frac{\hat{q}}{E^2} \int_0^{\infty} d\bar{t} \frac{\omega n^2}{\epsilon} \cos(\omega\bar{t}) \right. \\ &\quad \times \exp \left[ i \text{sgn}(n_i)\omega n_r \beta \bar{t} \left( 1 - \frac{\hat{q}}{6E^2}\bar{t} \right) \right] \\ &\quad \left. \times \exp \left[ -\omega |n_i| \beta \bar{t} \left( 1 - \frac{\hat{q}}{6E^2}\bar{t} \right) \right] \right) \quad (8) \end{aligned}$$

with  $\beta = v'$  and coupling  $\alpha = q^2/(4\pi)$ .

For  $\hat{q} = 0$ , i. e. when the charge suffers no deflections,  $d^2W/(dzd\omega)$  vanishes. Furthermore, in the vacuum limit, i. e. when setting  $\epsilon(\omega) = \mu(\omega) = 1$ , (8) becomes the negative of the radiation intensity determined in [11]. Thus, we interpret the negative of expression (8) as radiative energy loss spectrum per unit length. We restrict ourselves to the case  $\text{sgn}(n_i) = \text{sgn}(n_r)$  in order to account for an actual loss of energy. Moreover, we do not distinct in the following between longitudinal and transverse excitations by setting  $\mu(\omega) = 1$ , which implies  $\epsilon_L = \epsilon_T = \epsilon$  for isotropic and homogeneous media [17]. A differentiation between longitudinal and transverse excitations is left for future investigations.

As the approach assumes small deflection angles for arbitrary values of  $\bar{t}$ , it is necessary to impose as physical constraint a natural upper boundary in the  $\bar{t}$ -integral. We restrict (8) to  $\bar{t} \leq 2E^2/\hat{q}$ , i. e. exactly where  $\langle \Delta r \rangle/v'$  reaches its maximum. Increasing the cut-off value reasonably does not influence our numerical results presented below.

We use for the complex squared index of refraction  $n^2(\omega) = \epsilon(\omega)$  the following formal ansatz

$$n^2(\omega) = 1 - \frac{m^2}{\omega^2} + 2i\frac{\Gamma}{\omega}. \quad (9)$$

This structure is based on the assumption that radiated quanta formed inside a medium follow medium-modified dispersion relations of plasma modes, cf. [15, 16], acquiring a finite in-medium mass  $m$  and being damped in the

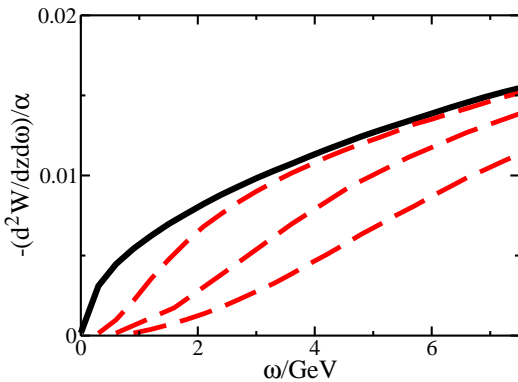


FIG. 1: (Color online) Radiative energy loss spectrum as a function of  $\omega$  for  $\Gamma = 0$ . The kinematic parameters are chosen as  $E = 20$  GeV, charge mass  $M = 1$  GeV and  $\hat{q} = 2.5$  GeV<sup>2</sup>/fm. Black solid curve shows the vacuum limit (setting  $m = 0$ ), while the red long-dashed curves depict the corresponding results for a medium with real  $n^2(\omega)$  ( $m = 0.3, 0.6, 0.9$  GeV from top to bottom, respectively).

absorptive medium with a rate related to  $\Gamma$  [18, 19]. Expression (9) is connected with a Lorentz-type spectral function for intermediate hard quanta [18], where  $m$  and  $\Gamma$  are in general free parameters, which may depend on plasma temperature, coupling and/or frequency. The corresponding dispersion relation of plasma modes follows from  $Re(k^2 - \omega^2 \epsilon(\omega)) = 0$ . This implies (i) the absence of radiation for  $\omega < m$  (for  $\omega \rightarrow m^+$  the phase velocity of the plasma modes vanishes), (ii) time-like plasma modes and (iii)  $Im(\epsilon) = 2\Gamma/\omega \neq 0$  for  $\Gamma \neq 0$  with support in the  $\omega$ -region, where the plasma modes exist. Our ansatz for the dielectric function, therefore, differs from an  $\epsilon(\omega)$  deduced from a leading-order HTL self-energy expression, cf. [10, 16]. In this case, plasma modes are also time-like [20–22], while the support of  $Im(\epsilon)$  is restricted to the space-like region. Only at next-to-leading-order, the damping of plasma modes emerges in HTL-based approaches.

We now want to quantify (8), simplifying however the considerations by being restricted to constant  $\Gamma$  and  $m$  values inspired by [23]. We discuss first the case of a polarizable, non-damping medium employing  $n^2(\omega)$  from (9) with  $\Gamma = 0$ . Fig. 1 exhibits the radiative energy loss spectrum for  $\Gamma = 0$  as a function of  $\omega$  for  $m = 0.3, 0.6, 0.9$  GeV (red long-dashed curves from top to bottom, respectively). For comparison also the vacuum limit ( $m = 0$ ) is shown (black solid curve). With increasing in-medium mass an increasing reduction of the spectrum is observed (TM effect, cf. also [15, 16]). This reduction is more pronounced at small  $\omega$ , while the generic approach of the vacuum result with increasing  $\omega$  is hampered for increasing  $m$ .

The additional effect of radiation damping in an absorptive medium is shown in Fig. 2 for different values of

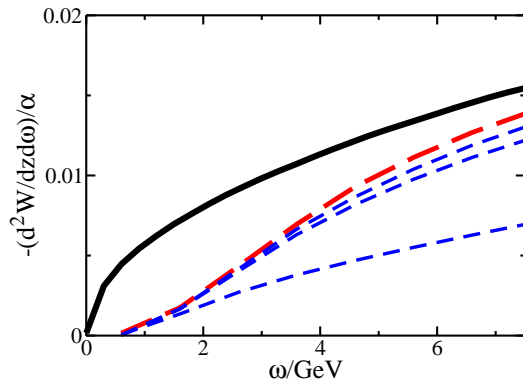


FIG. 2: (Color online) Radiative energy loss spectrum as a function of  $\omega$  with the same kinematic parameters as in Fig. 1. The black solid curve shows the vacuum limit and the red long-dashed curve depicts the result for a medium with real  $n^2(\omega)$  using  $m = 0.6$  GeV. The blue short-dashed curves exhibit the additional influence of radiation damping, employing  $n^2(\omega)$  from (9) with  $\Gamma = 5, 10, 50$  MeV (from top to bottom, respectively).

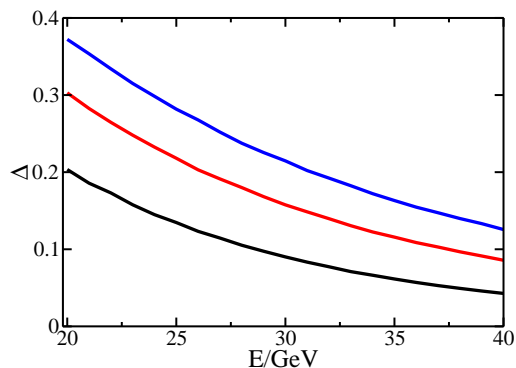


FIG. 3: (Color online) Ratio  $\Delta$  as a function of  $E$  for constant  $\omega = 3, 5, 7$  GeV (solid curves from bottom to top, respectively), and fixed  $\Gamma = 50$  MeV and  $m = 0$ . Charge mass  $M$  and parameter  $\hat{q}$  as in Figs. 1 and 2.  $\Delta \rightarrow 1$  for negligible damping effects.

$\Gamma = 5, 10, 50$  MeV (blue short-dashed curves from top to bottom, respectively) and fixed  $m = 0.6$  GeV. Even for small values of  $\Gamma$  (the maximal ratio considered in Fig. 2 is  $\Gamma/m = 0.083$ ), radiation damping leads to a significant reduction of the radiative energy loss spectrum. This is a natural consequence of the sensitivity of (8) on the poles in the complex  $k$ -plane determined from  $\omega^2 - k^2 - m^2 + 2i\Gamma\omega = 0$ , which implies a non-negligible effect of  $\Gamma$  even for small  $\Gamma/m$ . In the limit  $\omega \rightarrow m^+$ , the behavior for a polarizable, non-damping medium is recovered.

It is interesting to investigate the dependence of (8) on the energy of the charge for a given  $\Gamma$ . In Fig. 3, the ratio of the radiative energy loss spectrum between an absorptive ( $\Gamma \neq 0$ ) and a non-absorptive ( $\Gamma = 0$ )

medium,  $\Delta \equiv d^2W_{\Gamma \neq 0}/d^2W_{\Gamma=0}$ , is exhibited as a function of  $E$  for constant  $\omega$ ,  $\Gamma = 50$  MeV and  $m = 0$ . This allows for studying the relative importance of radiation damping effects in an absorptive medium compared to the vacuum. We observe a sensitive energy dependence with increasing importance of radiation damping effects for increasing  $E$ .

The observations made in Figs. 1-3 can be understood by qualitatively analyzing (8). Here, we focus on the discussion of observations made in the regime  $\omega \gg m \gg \Gamma$ . In the case of a non-damping but polarizable medium, the vacuum result will be approached with increasing  $\omega$  as the exponential damping factor in (8) is 1 and  $n_r \rightarrow 1$  with increasing  $\omega$ .

Analyzing (8) for  $\Gamma \neq 0$ , one finds from (9) that  $\omega|n_i| \rightarrow \Gamma$  and  $\text{sgn}(n_i)\omega n_r \rightarrow \omega$  in the considered  $\omega$ -regime. In this limit, (8) gives

$$\begin{aligned} \frac{d^2W}{dzd\omega} \simeq & \frac{\alpha\omega\hat{q}}{3\pi E^2} \int_0^\infty d\bar{t} \exp \left[ -\Gamma\beta\bar{t} \left( 1 - \frac{\hat{q}}{6E^2}\bar{t} \right) \right] \\ & \times \left\{ \sin \left[ \omega\bar{t}(\beta - 1) - \omega\beta\frac{\hat{q}}{6E^2}\bar{t}^2 \right] \right. \\ & \left. + \sin \left[ \omega\bar{t}(\beta + 1) - \omega\beta\frac{\hat{q}}{6E^2}\bar{t}^2 \right] \right\}. \end{aligned} \quad (10)$$

The exponential damping factor in (10) is responsible for the suppression of the spectrum compared to the  $\Gamma = 0$  case. It is, unless  $\Gamma$  depends on  $\omega$ , formally frequency independent.

In order to elucidate the impact of the exponential damping factor, it is necessary to study the formation time  $t_f$  of radiation from a relativistic charge, which in general depends on  $\omega$ . A detailed discussion of  $t_f$  in a polarizable and absorptive medium is presented in [24]. We determine  $t_f$  from the phase factor  $\Phi(t)$  of the dominant sine-function in (10). As is well known, radiation can be considered as decoupled from its emitter, once a phase  $\Phi(t_f) \sim 1$  has been accumulated. This leads to the condition  $1 \sim \omega t_f(1 - \beta) + \omega\beta\hat{q}t_f^2/(6E^2)$ . Accordingly,  $t_f$  is roughly given by the minimum of the two limiting solutions,  $2E^2/(\omega M^2)$  and  $E\sqrt{6/(\omega\hat{q})}$ . The increase of  $t_f$  with  $E$  explains why a rather small  $\Gamma$  can lead to a large suppression of radiation. The exponential damping factor in (10) reduces from 1 at  $\bar{t} = 0$  to approximately  $\exp[-\Gamma t_f]$  within the formation time interval. This gives rise to the observed sensitive  $\Gamma$ - and  $E$ -dependence of the spectrum, unless  $t_f$  is comparable to or larger than another competing time scale  $t_d \sim 1/\Gamma$ , cf. [24], where the exponential damping factor becomes of order  $\mathcal{O}(1/e)$ , such that the amplitude in (10) is damped away before radiation could be formed.

In summary, we have shown that the radiative energy loss of an energetic charge can be substantially reduced in an absorptive medium (modelled by  $n_i \neq 0$ ). This effect occurs in addition to the known TM effect, which leads to

a reduction of the radiative energy loss in a polarizable, non-absorptive medium (described by  $n_r \neq 0$  and  $n_i = 0$ ). The observed effect increases with increasing medium damping and/or increasing energy of the charge. Our investigations, being restricted here to the regime  $\omega \ll E$ , represent the classical, non-abelian approximation to the dynamics of a color charge in the QGP. However, still important effects specific to QCD are missing such as gluon rescatterings. This will be explored in future work.

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