

Non-Perturbative Prediction of the Ferromagnetic Transition in Repulsive Fermi Gases

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It is generally believed that a dilute spin-1/2 Fermi gas with repulsive interactions can undergo a ferromagnetic phase transition to a spin-polarized state at a critical gas parameter $(k_F a)_c$. The perturbation theory fails to predict quantitatively the ferromagnetic transition since $(k_F a)_c$ is not small. In this Letter we study the non-perturbative effects on the ferromagnetic transition by summing the particle-particle ladder diagrams to all orders in the gas parameter. To the leading order of the effective range expansion, such a resummation predicts a second order ferromagnetic phase transition. The predicted critical gas parameter $(k_F a)_c = 0.858$ is in good agreement with recent Quantum Monte Carlo result $(k_F a)_c = 0.86$ for a nearly zero-range potential [S. Pilati, *et al.*, Phys. Rev. Lett. **105**, 030405 (2010)].

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Itinerant ferromagnetism is a fundamental problem in condensed matter physics, which can be dated back to the basic model proposed by Stoner [1]. While the problem of itinerant ferromagnetism in electronic systems is quite complicated and the phase transition theory is still qualitative, a dilute spin-1/2 Fermi gas with repulsive interactions may serve as a clean system to simulate the Stoner model. It is generally thought that the repulsive Fermi gas could undergo a ferromagnetic phase transition (FMPT) to a spin-polarized state with increased interaction strength [2]. Recently, the experimentalists realized a two-component “repulsive” Fermi gas of ⁶Li atoms in a harmonic trap by using a nonadiabatic field switch to the upper branch of a Feshbach resonance with a positive s-wave scattering length [3]. Therefore, it is possible to investigate itinerant ferromagnetism in cold Fermi gases.

The physical picture of the ferromagnetism in repulsive Fermi gases can be understood as a result of the competition between the repulsive interaction and the Pauli exclusion principle. The former tends to induce polarization and reduce the interaction energy, while the latter prefers balanced spin populations and hence a reduced kinetic energy. With increasing repulsion, the reduced interaction energy for a polarized state will overcome the gain in kinetic energy, and a FMPT should occur when the minimum of the energy landscape shifts to nonzero polarization or magnetization.

Quantitatively, to study the FMPT in dilute Fermi gases at zero temperature, we should calculate the energy density \mathcal{E} as a function of the spin polarization or magnetization $x = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ at given dimensionless gas parameter $k_F a$ which represents the interaction strength [2]. Here, k_F is the Fermi momentum related to the total density $n = n_\uparrow + n_\downarrow$ by $n = k_F^3/(3\pi^2)$ and $a > 0$ is the s-wave scattering length. Generally, the energy density can be expressed as $\mathcal{E}(x) = \frac{3}{5}nE_F f(x)$, where $E_F = k_F^2/(2M)$ is the Fermi energy with M being the fermion mass. The dimensionless function $f(x)$, which depends on the gas parameter $k_F a$, represents the energy landscape with respect to the magnetization x .

Known results for $f(x)$ are based on the perturbation theory (PTh) which treats the gas parameter $k_F a$ as a small number.

Up to order $O((k_F a)^2)$, the result is universal, i.e., independent of the details of the short range interaction,

$$f(x) = \frac{1}{2}(\eta_\uparrow^5 + \eta_\downarrow^5) + \frac{10k_F a}{9\pi} \eta_\uparrow^3 \eta_\downarrow^3 + \frac{(k_F a)^2}{21\pi^2} \xi(\eta_\uparrow, \eta_\downarrow), \quad (1)$$

where $\eta_\uparrow = (1+x)^{1/3}$ and $\eta_\downarrow = (1-x)^{1/3}$. The 0th-order term corresponds to the kinetic energy, and the 1st-order term coincides with the Hartree-Fock mean-field theory [2]. The coefficient $\xi(\eta_\uparrow, \eta_\downarrow)$ in the 2nd-order term was first evaluated by Kanno [4]. Its explicit form is

$$\begin{aligned} \xi = & 22\eta_\uparrow^3 \eta_\downarrow^3 (\eta_\uparrow + \eta_\downarrow) - 4\eta_\uparrow^7 \ln \frac{\eta_\uparrow + \eta_\downarrow}{\eta_\uparrow} - 4\eta_\downarrow^7 \ln \frac{\eta_\uparrow + \eta_\downarrow}{\eta_\downarrow} \\ & + \frac{1}{2}(\eta_\uparrow - \eta_\downarrow)^2 \eta_\uparrow \eta_\downarrow (\eta_\uparrow + \eta_\downarrow) [15(\eta_\uparrow^2 + \eta_\downarrow^2) + 11\eta_\uparrow \eta_\downarrow] \\ & + \frac{7}{4}(\eta_\uparrow - \eta_\downarrow)^4 (\eta_\uparrow + \eta_\downarrow) (\eta_\uparrow^2 + \eta_\downarrow^2 + 3\eta_\uparrow \eta_\downarrow) \ln \left| \frac{\eta_\uparrow - \eta_\downarrow}{\eta_\uparrow + \eta_\downarrow} \right|. \quad (2) \end{aligned}$$

Setting $x = 0$, we recover the well-known equation of state for hard sphere Fermi gases, which was first obtained by Huang, Yang, and Lee [5] and recovered by Hammer and Furnstahl [6] in recent years using effective field theory.

In the 1st-order PTh, the FMPT is of second order and occurs at $k_F a = \pi/2$ [2]. However, taking into account the 2nd-order corrections, one finds a first order FMPT at $k_F a = 1.054$ [7]. This can be understood by noticing the non-analytical term $\propto x^4 \ln|x|$ with positive coefficient in the small- x expansion of the coefficient $\xi(\eta_\uparrow, \eta_\downarrow)$. In fact, Belitz *et al.* [8] have argued that the correlation effects or the coupling of the order parameter to gapless modes generally leads to non-analytical terms in the free energy. The general form of the Ginzburg-Landau free energy then takes the form $f_{GL}(x) = tx^2 + vx^4 \ln|x| + \alpha x^4 + O(x^6)$, where we can keep $\alpha > 0$. If the coefficient v is positive, the phase transition is always of first order. On the other hand, for negative v , we always have a second order phase transition. Up to the order $O((k_F a)^2)$, we find that the Fermi gas problem corresponds to the case $v > 0$.

However, since the critical gas parameter of FMPT is not small, there naturally arises a serious problem: Does the

Fermi gas problem really correspond to the case $\nu > 0$ if the non-perturbative effects at $k_{\text{F}a} \sim 1$ are taken into account? For the two-body problem in the vacuum, it is well known that an infinite set of bubble diagrams with the leading-order contact interaction must be resummed if the two-body scattering length is large [9]. Therefore it is natural to extend the resummation method to finite density so that the predicted equation of state works well even at $k_{\text{F}a} \sim 1$. The results from recent Quantum Monte Carlo (QMC) simulations [10, 11] enable us to judge how good the resummation theory is.

The main purpose of this Letter is to calculate the function $f(x)$ by resumming certain class of ladder diagrams. We require that the resummation theory (RTh) satisfies the following two criteria: (i) The physical result does not depend on the renormalization scale; (ii) The function $f(x)$ recovers Eq. (1) when we expand $f(x)$ to the order $O((k_{\text{F}a})^2)$.

For a short range interaction characterized by a momentum scale Λ , one can construct the effective field theory [9] describing scattering at momenta $k \ll \Lambda$ according to the effective range expansion $k \cot \delta = -1/a + \frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n (k^2/\Lambda^2)^{n+1}$ for the s -wave scattering phase shift δ . We will keep the first term in this expansion and neglect the effective range effect. In this universal case, it is possible to obtain a non-perturbative result of $f(x)$ satisfying the criterions (i) and (ii).

At finite density, the free propagators for the two spin components are given by [12]

$$\mathcal{G}_{\sigma}(k_0, \mathbf{k}) = \frac{\Theta(|\mathbf{k}| - k_{\text{F}}^{\sigma})}{k_0 - \omega_{\mathbf{k}} + i\epsilon} + \frac{\Theta(k_{\text{F}}^{\sigma} - |\mathbf{k}|)}{k_0 - \omega_{\mathbf{k}} - i\epsilon}, \quad (3)$$

where $\sigma = \uparrow, \downarrow$, $k_{\text{F}}^{\uparrow, \downarrow} = k_{\text{F}}\eta_{\uparrow, \downarrow}$ are the Fermi momenta of the two spin components, $\omega_{\mathbf{k}} = \mathbf{k}^2/(2M)$ is the free dispersion, and $\Theta(z)$ is the Heaviside step function. For each spin component, the propagator describes two types of excitations, particles with momentum $|\mathbf{k}| > k_{\text{F}}^{\sigma}$ and holes with $|\mathbf{k}| < k_{\text{F}}^{\sigma}$. The simplest resummation scheme which satisfies the criterions (i) and (ii) is to sum the particle-particle (pp) ladders [9, 13, 14]. The elementary pp bubble shown in Fig. 1(a) is given by

$$B(\mathbf{p}_1, \mathbf{p}_2) = M \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\Theta(|\mathbf{k}_1| - k_{\text{F}}^{\uparrow})\Theta(|\mathbf{k}_2| - k_{\text{F}}^{\downarrow})}{\mathbf{q}^2 - \mathbf{k}^2 + i\epsilon} \quad (4)$$

where $\mathbf{k}_{1,2} = \mathbf{P} \pm \mathbf{k}$. Here \mathbf{p}_1 and \mathbf{p}_2 are external momenta of the two spin components, and we have defined their half sum $\mathbf{P} = (\mathbf{p}_1 + \mathbf{p}_2)/2$ and half difference $\mathbf{q} = (\mathbf{p}_1 - \mathbf{p}_2)/2$. Notice that the imaginary part of B vanishes automatically.

We can separate B into a vacuum part and a medium part using the identity $\Theta(-z) = 1 - \Theta(z)$. The vacuum part is linearly divergent and we choose the dimensional regularization with power divergence subtraction (PDS) [9]. For convenience, we define two dimensionless quantities: $s = |\mathbf{P}|/k_{\text{F}}$ and $\kappa = |\mathbf{q}|/k_{\text{F}}$. The elementary pp bubble can be evaluated as

$$B(\kappa, s) = -\frac{M\mu}{4\pi} + \frac{Mk_{\text{F}}}{4\pi^2} R_{\text{pp}}(s, \kappa), \quad (5)$$

where μ is the renormalization scale introduced in the PDS scheme [9]. The function $R_{\text{pp}}(s, \kappa)$ is defined as $R_{\text{pp}}(s, \kappa) =$

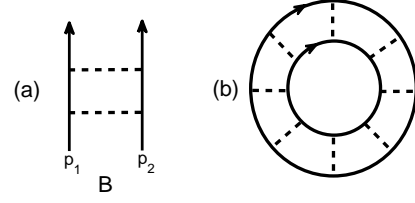


FIG. 1: (a) The elementary particle-particle bubble $B(\mathbf{p}_1, \mathbf{p}_2)$ with external momenta \mathbf{p}_1 and \mathbf{p}_2 for the two spin components. The solid line with arrow corresponds to the particle term of the propagator (3). The dashed line represents the interaction vertex g . (b) A typical particle-particle ladder diagram contributing to the interaction energy.

$R_{\uparrow}(s, \kappa) + R_{\downarrow}(s, \kappa) + R_{\uparrow\downarrow}(s, \kappa)$, where $R_{\sigma}(s, \kappa)$ reads

$$R_{\sigma}(s, \kappa) = \frac{\eta_{\sigma}^2 - (s + \kappa)^2}{4s} \ln \left| \frac{\eta_{\sigma} + s + \kappa}{\eta_{\sigma} - s - \kappa} \right| + (\kappa \rightarrow -\kappa) + \eta_{\sigma}, \quad (6)$$

and the function $R_{\uparrow\downarrow}(s, \kappa)$ is

$$R_{\uparrow\downarrow}(s, \kappa) = \begin{cases} -\Theta(x)R_{\downarrow} - \Theta(-x)R_{\uparrow} & , \quad 0 < s < \frac{|\eta_{\uparrow} - \eta_{\downarrow}|}{2} \\ K_{\uparrow}(s, \kappa) + K_{\downarrow}(s, \kappa) & , \quad \frac{|\eta_{\uparrow} - \eta_{\downarrow}|}{2} < s < \frac{\eta_{\uparrow} + \eta_{\downarrow}}{2} \\ 0 & , \quad \text{elsewhere.} \end{cases} \quad (7)$$

Here $K_{\sigma}(s, \kappa)$ is defined as

$$K_{\sigma}(s, \kappa) = \frac{\eta_{\sigma}^2 - s^2 - \kappa^2}{4s} \ln \left| \frac{(\eta_{\sigma} - s)^2 - \kappa^2}{r^2 - s^2 - \kappa^2} \right| + \frac{\kappa}{2} \ln \left| \frac{\eta_{\sigma} - s + \kappa}{\eta_{\sigma} - s - \kappa} \right| + \frac{s - \eta_{\sigma}}{2}, \quad (8)$$

where $r^2 = (\eta_{\uparrow}^2 + \eta_{\downarrow}^2)/2$.

Particle-particle ladder diagrams which contribute to the interaction energy can be built from the elementary bubble, see Fig. 1(b) for a typical example. All contributions form a geometric series and the interaction energy is given by

$$\mathcal{E}_{\text{int}} = g \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \int \frac{d^3\mathbf{p}_2}{(2\pi)^3} \frac{\Theta(k_{\text{F}}^{\uparrow} - |\mathbf{p}_1|)\Theta(k_{\text{F}}^{\downarrow} - |\mathbf{p}_2|)}{1 - gB(\mathbf{p}_1, \mathbf{p}_2)}, \quad (9)$$

where the running coupling constant is given by $g(\mu) = (-\mu + 1/a)^{-1}4\pi/M$ from the renormalization group equation in the vacuum [9]. Finally, \mathcal{E}_{int} is independent of the renormalization scale μ , and the function $f(x)$ can be expressed as

$$f(x) = \frac{1}{2}(\eta_{\uparrow}^5 + \eta_{\downarrow}^5) + \frac{80}{\pi} \int_0^{\infty} s^2 ds \int_0^{\infty} \kappa d\kappa I(s, \kappa) F(s, \kappa), \quad (10)$$

where $F(s, \kappa)$ is given by $F(s, \kappa) = k_{\text{F}a}[1 - k_{\text{F}a}R_{\text{pp}}(s, \kappa)/\pi]^{-1}$. The function $I(s, \kappa)$ appears due to the angular integration. Its explicit form is

$$I(s, \kappa) = \Theta(r^2 - s^2 - \kappa^2)\Theta(\eta_{\uparrow} - |s - \kappa|)\Theta(\eta_{\downarrow} - |s - \kappa|) \times \left[\frac{\eta_{\uparrow}^2 - (s + \kappa)^2}{4s} \Theta(s + \kappa - \eta_{\uparrow}) + (\eta_{\uparrow} \rightarrow \eta_{\downarrow}) + \kappa \right]. \quad (11)$$

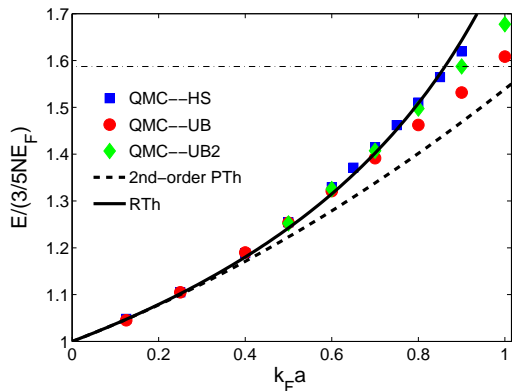


FIG. 2: (Color-online) The equation of state for the balanced case $x = 0$. The blue squares are the QMC data for the hard sphere (HS) potential [10], the red circles for the upper branch (UB) of a square well potential [10], and the green diamonds for the upper branch (UB2) of an attractive short range potential [11]. For UB and UB2 cases, the effective range is much smaller than the s -wave scattering length a . The solid line is the result calculated from RTh. The dashed line is the 2nd-order perturbative result [5]. The dash-dotted horizontal line corresponds to the energy of the fully polarized state, i.e., $f(1) = 2^{2/3}$.

To check the consistency with Eq. (1), we expand the function F as $F = k_F a + (k_F a)^2 R_{pp}/\pi + O((k_F a)^3)$. One can check that $\int_0^\infty s^2 ds \int_0^\infty \kappa d\kappa I = \eta_\uparrow^3 \eta_\downarrow^3 / 72$ and $\int_0^\infty s^2 ds \int_0^\infty \kappa d\kappa I R_{pp} = \xi(\eta_\uparrow, \eta_\downarrow) / 1680$. Therefore we can compare the results from our RTh and the 2nd-order PTh on the same footing and study the non-perturbative effects on the FMPT.

To show the advantage of our RTh, we first compare the equation of state for the balanced case ($x = 0$) with that obtained from the QMC simulations [10, 11]. The results are shown in Fig. 2. While the 2nd-order perturbative result can fit the QMC data only for $k_F a \lesssim 0.4$, our RTh can fit well the data up to $k_F a \sim 0.8$ where the FMPT is estimated to occur [10]. With increasing $k_F a$, the result of the 2nd-order PTh becomes lower and lower than the QMC data. Therefore, the 2nd-order PTh overestimates the critical gas parameter, and our RTh may predict the FMPT more accurately.

The order of the ferromagnetic phase transition and the critical gas parameter $(k_F a)_c$ can be obtained by studying carefully the behavior of the energy landscape, i.e., the function $f(x)$. To very high numerical accuracy, we haven't found any maximum at $x \neq 0$ in the energy landscape. Instead, we find a second order phase transition at $k_F a = 0.858$ where the function $f(x)$ starts to develop a minimum at $x \neq 0$. This is in contrast to the 2nd-order PTh which predicts a strong first order phase transition at $k_F a = 1.054$ [7] where the magnetization jumps from zero to $x_c = 0.573$.

Since an analytical result for the function $f(x)$ can not be achieved in our RTh, we cannot understand clearly how the non-perturbative effects modify the order of the phase transition. In fact, analytical results cannot be obtained from the or-

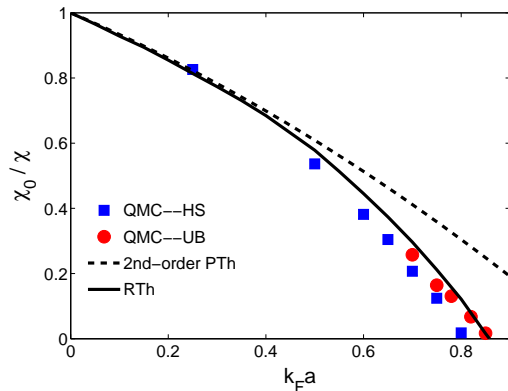


FIG. 3: (Color-online) The dimensionless inverse spin susceptibility χ_0/χ . The blue squares and red circles are the QMC data [10] for the HS and UB cases, respectively. The solid line is the result calculated from RTh. The dashed line is the 2nd-order perturbative result.

der $O((k_F a)^3)$ even for the balanced case $x = 0$ in the PTh [6]. However, some definite conclusions can be drawn from our numerical results: (1) Higher-order terms in the gas parameter can also generate non-analytical terms $\propto x^4 \ln|x|$ and may generate other important non-analytical terms which are not known due to the mathematical limitation; (2) The coefficients of the non-analytical terms generated by the higher-order contributions are certainly not always positive, and they are generally proportional to $(k_F a)^n$ for the n -th-order contributions. Since the phase transition occurs at a gas parameter $k_F a \sim 1$, the non-perturbative effects from the sum of the higher order contributions are very important. As we have shown numerically, their effects are not only reducing the critical value of the gas parameter but also changing the order of the phase transition.

A second order FMPT is precisely controlled by the spin (or magnetic) susceptibility χ . To show this, we expand the function $f(x)$ near $x = 0$ as $f(x) = f(0) + tx^2 + \dots$. The coefficient t is related to the spin susceptibility by $t = \frac{5}{9} \chi_0/\chi$ [10] where $\chi_0 = 3n/(2E_F)$ is the spin susceptibility of a non-interacting Fermi gas. Therefore, the second order FMPT occurs exactly when the inverse of the spin susceptibility vanishes. In the 2nd-order PTh, the inverse spin susceptibility can be analytically evaluated as $\frac{\chi_0}{\chi} = 1 - \frac{2}{\pi} k_F a - \frac{16(2+\ln 2)}{15\pi^2} (k_F a)^2$ [4], which vanishes at $k_F a = 1.058$. This differs from the critical gas parameter $(k_F a)_c = 1.054$ due to the fact that the FMPT is of first order in the 2nd-order PTh.

The quantity χ_0/χ can be calculated numerically from our RTh. It is shown and compared with the QMC data in Fig. 3. We find that the inverse spin susceptibility predicted by our RTh deviates from the 2nd-order PTh result for $k_F a \gtrsim 0.4$. Our RTh result agrees well with the QMC data for the upper branch (UB) of the square well potential where the effective range is much smaller than the scattering length. Nevertheless, the RTh result is also not bad for the hard sphere (HS) case. The predicted critical gas parameter $(k_F a)_c = 0.858$ is

in good agreement with the results $(k_{Fa})_c = 0.86$ for the UB case and $(k_{Fa})_c = 0.82$ for the HS case [10]. Two remarks may help us understand the good agreements: (1) In a very interesting paper [13], Steele suggested that the pp ladder sum is the leading-order contribution of the $1/D$ expansion, where D is the number of space-time dimensions. All other contributions like hole-hole ladder sum and effective range corrections are suppressed by a factor $1/D$; (2) In a recent work [15], Liu *et al.* found that for sufficiently small $a > 0$, the energy spectrum of three interacting fermions in the upper branch of the Feshbach resonance can be interpreted as that of three “repulsively” interacting fermions. Therefore, for many-body problem the upper branch may serve as a universal “repulsive” Fermi gas for sufficiently small k_{Fa} .

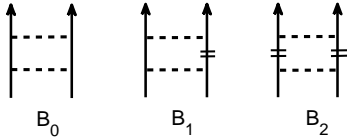


FIG. 4: The elementary bubbles organized in the number of the MI. The solid line with a cut represents the MI part of the propagator, and the pure solid line corresponds to the vacuum part.

In the final part we check whether our conclusion that the FMPT is of second order is changed by other contributions. We consider the contribution of the hole-hole (hh) ladder diagrams by summing the combined pp and hh ladders to all orders in k_{Fa} while keeping the criteria (i) and (ii) satisfied.

Following a recent work by Kaiser [16], we rewrite the propagator (3) in an alternative form $\mathcal{G}_\sigma(k_0, \mathbf{k}) = \mathcal{G}^0(k_0, \mathbf{k}) + 2\pi i \delta(k_0 - \omega_{\mathbf{k}}) \Theta(k_F^\sigma - |\mathbf{k}|)$, where the first term corresponds to the vacuum propagator $\mathcal{G}^0(k_0, \mathbf{k}) = (k_0 - \omega_{\mathbf{k}} + i\epsilon)^{-1}$ and the second term is a so-called “medium-insertion” (MI) [16]. The elementary bubbles in this treatment are shown in Fig. 4. The first diagram B_0 is purely a vacuum part and we use the same PDS scheme. For our purpose of resummation, we are interested in the following two quantities, $B_0 + B_1 + B_2$ and $B_0 + B_1$, which are mutually complex conjugate. We have

$$B_0 + B_1 + B_2 = -\frac{M\mu}{4\pi} + \frac{Mk_F}{4\pi^2} [R(s, \kappa) - i\pi I(s, \kappa)], \quad (12)$$

where $R(s, \kappa) = R_\uparrow(s, \kappa) + R_\downarrow(s, \kappa)$.

To sum all ladder diagrams built from the elementary bubbles, we first notice that the non-vanishing contributions to the interaction energy come from diagrams with at least two adjacent MIs [16]. Then a typical n -th-order contribution would look like the ring diagram of Fig. 1 (b) with n vertices and at least two adjacent MIs. Naively, all these n -th-order diagrams are summed to give $g^n[(B_0 + B_1 + B_2)^n - (B_0 + B_1)^n]$ where the subtraction gets rid of those diagrams which have no adjacent MI pairs. However, this expression is complex and therefore cannot be the correct one. The crucial observations are that: (1) Each n -th-order ring diagram has a n -rotational symmetry. Therefore, we should introduce an additional factor

$1/n$; (2) An n -th-order ring diagram comes from closing two open MI-lines of an n -th-order ladder diagram, which introduces an integration over the allowed phase space $|\mathbf{p}_1| < k_F^\uparrow$ and $|\mathbf{p}_2| < k_F^\downarrow$, but does not contribute a factor B_2 to the energy as the naive expression does. These amendments lead to the correct n -th-order contribution to the interaction energy [16]: $g^n[(B_0 + B_1 + B_2)^n - (B_0 + B_1)^n]/(2iIn)$. The summation over n leads to two complex-conjugated logarithms and the final result is real.

Therefore, the function $f(x)$ in this theory also takes the form (10), while the function $F(s, \kappa)$ becomes $F(s, \kappa) = [\ln(1 - \frac{k_{Fa}}{\pi} R + ik_{Fa}I) - c.c.]/(2iI)$. We can also check that $\int_0^\infty s^2 ds \int_0^\pi \kappa d\kappa IR = \xi(\eta_\uparrow, \eta_\downarrow)/1680$, which reflects the fact that the hh ladders start to contribute at order $O((k_{Fa})^3)$ [6, 13, 14]. Numerically, we also find a second order phase transition, which occurs at a smaller gas parameter $k_{Fa} = 0.786$. We notice that the inclusion of hh ladders may not improve the quantitative result.

In summary, we have studied the non-perturbative effects on the ferromagnetic phase transition in repulsive Fermi gases by summing the ladder diagrams to all orders in the gas parameter k_{Fa} . The non-perturbative effects not only reduce the critical gas parameter but also change the order of the phase transition. The resummation of particle-particle ladders predicts a second order phase transition occurring at $k_{Fa} = 0.858$, in good agreement with the QMC result [10]. The equation of state and the spin susceptibility calculated from our resummation theory are also in good agreement with the QMC results. Therefore, the resummation theory provides a more quantitative way to study the ferromagnetic transition in repulsive Fermi gases.

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