Three triton states in ^{9}Li

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We focus on a characteristic non-α cluster structure in light neutron-rich nuclei; three triton structure in ^{9}Li. This is an analogy to the case of three α state in ^{12}C (Hoyle state). The α clusters behave as bosons, however tritons have Fermionic nature, and how the three cluster structure is different from ^{12}C is an intriguing problem. For this purpose, we introduce three triton wave functions. In addition, $\alpha+t+n+n$ wave functions are prepared to describe other low-lying states of ^{9}Li, and the coupling effect between them is taken into account. The states with dominantly the three triton components appear below the three triton threshold energy, where three triton correlation is important, however the root mean square radius is not enhanced contrary to the α gas states in ^{12}C and ^{16}O.

I. INTRODUCTION

Recently, gas-like states comprised of α clusters in atomic nuclei have attracted increased interest [1–6]. One candidate is the second 0^+ state of ^{12}C* (3α) at $E_x = 7.65$ MeV. The state is well expressed by THSR (Tohsaki Horiuchi Schuck Röpke) wave function, where all of the α-clusters occupy the same $0s$-orbital with a spatially extended distribution [2]. In addition, a candidate for the gas-like state of 4α clusters around the threshold energy in ^{16}O has been studied both by theoretical and experimental approaches [1, 3, 7].

Furthermore, not only α clusters, but also various other types of clusters are expected to appear in the excited states of neutron-rich nuclei around the corresponding threshold energies. One recent example is $\alpha+\alpha+t$ structure; the third $3/2^-$ state of ^{11}B at $E_x = 8.56$ MeV has been discussed to have the $\alpha+\alpha+t$ structure from both theoretical and experimental sides [8]. Another example is ^{10}Be; although all the low-lying states of ^{10}Be have been found to be well described by the combination of three molecular-orbits for the two valence neutrons around two α-clusters[9], the $\alpha+t+t$ cluster structure is found to coexist with the $\alpha+\alpha+n+n$ structure around $E_x = 15$ MeV, close to the corresponding threshold[10]. The existence of ^{7}Li$+t$ cluster configuration of ^{10}Be has been reported in the recent experiment with ^{8}Li$(d, t)^7$Li reaction [11, 12], and since the ^{7}Li nucleus is well-known to be described as $\alpha+t$ cluster structure, the resonance state is considered to have the $\alpha+t+t$ cluster configuration. This is one of the first example that completely different cluster configurations of triton-type ($\alpha + t + t$) and α type ($\alpha+\alpha+n+n$) coexist in a single nucleus around the same energy region. A well known other example is the appearance (mixing) of ^{4}He$+2n$ and $t+t$ structures in ^{6}He [13–15].

In the present study, we focus on more drastic non-α cluster structure in light neutron-rich nuclei; three tri-ton structure in ^{9}Li, where all the clusters are tritons. This is an analogy to the case of three α state in ^{12}C (Hoyle state). In the case of α cluster, it consists of four nucleons and the spin of the system is zero, thus it behaves as a boson. On the other hand, a triton consists of three nucleons and has Fermionic nature, and how the three cluster structure is different from the Hoyle state in ^{12}C is an intriguing problem. Proposing an effective wave function for multi-triton (Fermion) systems, just like THSR wave function in the α cluster case, is the final goal. In this paper, we focus on whether three triton states really appear or not in the excited states of ^{9}Li as the first step. For this purpose, we introduce three triton wave functions. Also $\alpha+t+n+n$ wave functions are introduced to describe other low-lying states of ^{9}Li. It has been well known that ^{7}Li is described by a $\alpha+t$ model, thus $\alpha+t+n+n$ model is considered to work to large extent for the low-lying states of ^{9}Li. The validity of $\alpha+t+n+n$ model for ^{9}Li has been shown in Ref.[16], and here, three triton states are coupled.

II. METHOD

We introduce two kind of the basis states; the basis states with various $\alpha+t+n+n$ configurations, $\{\Psi(1)_i^{J^\pi KM}\}$, and those with various $t+t+t$ configurations, $\{\Psi(2)_j^{J^\pi KM}\}$. It has been known that ^{7}Li is well described by $\alpha+t$, and $\alpha+t+n+n$ model is the first order approximation for the low-lying states of ^{9}Li. In addition, we introduce three triton wave functions $\{\Psi(2)_j^{J^\pi KM}\}$. The total wave function $\Phi^{J^\pi MK}$ is therefore

$$\Phi^{J^\pi KM} = \sum_i c_i \Psi(1)_i^{J^\pi KM} + \sum_j d_j \Psi(2)_j^{J^\pi KM}. \quad (1)$$

The eigen states of Hamiltonian are obtained by diagonalizing the Hamiltonian matrix, and the coefficients

$\{c_i\}$ and $\{d_j\}$ for the linear combination of each Slater determinant are obtained.

The i-th basis state of $\{\Psi(1)_i^{J^\pi KM}\}$ with the $\alpha+t+n+n$ configuration has the following form,

$$\Psi_i(1)^{J^\pi KM} = P^\pi P^{JMK}$$
$$[\mathcal{A}\, \phi(\alpha, \vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4, \vec{R}_1)$$
$$\phi(t, \vec{r}_5\vec{r}_6\vec{r}_7, \vec{R}_2)$$
$$\phi(n^{(1)}, \vec{r}_8, \vec{R}_3)\phi(n^{(2)}, \vec{r}_9, \vec{R}_4)]_i, \quad (2)$$

where $\mathcal{A}$ is the antisymmetrizer, and $\phi(\alpha, \vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4, \vec{R}_1)$, $\phi(t, \vec{r}_5\vec{r}_6\vec{r}_7, \vec{R}_2)$, $\phi(n^{(1)}, \vec{r}_8, \vec{R}_3)$, $\phi(n^{(2)}, \vec{r}_9, \vec{R}_4)$ are wave functions of α, triton, the first valence neutron, and the second valence neutron, respectively. Here, $\{\vec{r}_i\}$ represents spatial coordinates of nucleons, and each nucleon is described as locally shifted Gaussian centered at $\vec{R}$ with the size parameter of $\nu = 1/2b^2$, $b = 1.46$ fm. The α cluster consists of four nucleons (spin-up proton, spin-down proton, spin-up neutron, and spin-down neutron), which share a common Gaussian center parameter $\vec{R}_1$, although the spin and isospin of each nucleon are not explicitly described in this formula for simplicity. The triton consists of three nucleons (proton, spin-up neutron, and spin-down neutron) which are centered at $\vec{R}_2$. The Gaussian center parameters of two valence neutrons are $\vec{R}_3$ and $\vec{R}_4$. The z components of the spins of the two valence neutrons are introduced to be anti-parallel for simplicity. The index i in Eq. (2) specifies a set of values of Gaussian center parameters for $\vec{R}_1$, $\vec{R}_2$, $\vec{R}_3$, and $\vec{R}_4$. The projection onto good parity and angular momentum (projection operators P^π and P^{JMK}) is performed numerically. The number of mesh points for the Euler angle integral is $16^3 = 4096$.

For the three triton basis sates, the j-th basis state of $\{\Psi(2)_j^{J^\pi KM}\}$ has the following form,

$$\Psi_j(2)^{J^\pi KM} = P^\pi P^{JMK}$$
$$[\mathcal{A}\, \phi(t^{(1)}, \vec{r}_1\vec{r}_2\vec{r}_3, \vec{R}_1)$$
$$\phi(t^{(2)}, \vec{r}_4\vec{r}_5\vec{r}_6, \vec{R}_2)$$
$$\phi(t^{(3)}, \vec{r}_7\vec{r}_8\vec{r}_9, \vec{R}_3)]_j, \quad (3)$$

where $\phi(t^{(1)}, \vec{r}_1\vec{r}_2\vec{r}_3, \vec{R}_1)$, $\phi(t^{(2)}, \vec{r}_4\vec{r}_5\vec{r}_6, \vec{R}_2)$, $\phi(t^{(3)}, \vec{r}_7\vec{r}_8\vec{r}_9, \vec{R}_3)$ are wave functions of three tritons. In each triton cluster, three nucleons (proton, spin-up neutron, spin-down neutron) share a common Gaussian center parameter $\{\vec{R}_1, \vec{R}_2,$ or $\vec{R}_3\}$. The z components of the spins of two of the protons are parallel, and the remaining one is anti-parallel, for simplicity.

The Hamiltonian operator ($\hat{H}$) has the following form:

$$\hat{H} = \sum_{i=1}^{A} \hat{t}_i - \hat{T}_{c.m.} + \sum_{i>j}^{A} \hat{v}_{ij}. \quad (4)$$

where the two-body interaction ($\hat{v}_{ij}$) includes the central, spin-orbit, and Coulomb parts. As the $N-N$ interaction,

for the central part, we use the Volkov No.2 effective potential [17]:

$$V(r) = (W - MP^\sigma P^\tau + BP^\sigma - HP^\tau)$$
$$\times (V_1 \exp(-r^2/c_1^2) + V_2 \exp(-r^2/c_2^2)), \quad (5)$$

where $c_1 = 1.01$ fm, $c_2 = 1.8$ fm, $V_1 = 61.14$ MeV, $V_2 = -60.65$ MeV, $W = 1 - M$ and $M = 0.60$. The singlet-even channel of the original Volkov interaction without the Bartlet (B) and Heisenberg (H) parameters has been known to be too strong, thus $B = H = 0.08$ is introduced to remove the bound state of two neutrons. For the spin-orbit term, we introduce the G3RS potential [18]:

$$V_{ls} = V_0(e^{-d_1 r^2} - e^{-d_2 r^2})P(^3O)\vec{L} \cdot \vec{S}, \quad (6)$$

where $d_1 = 5.0$ fm^{-2}, $d_2 = 2.778$ fm^{-2}, $V_0 = 2000$ MeV, and $P(^3O)$ is a projection operator onto a triplet odd state. The operator $\vec{L}$ stands for the relative angular momentum and $\vec{S}$ is the spin ($\vec{S}_1 + \vec{S}_2$). All of the parameters of this interaction were determined from the $\alpha + n$ and $\alpha + \alpha$ scattering phase shifts [19].

III. RESULTS

We diagonalize the Hamiltonian comprised of the basis states and obtain the eigen states and coefficients for the linear combination of the basis states. The energy convergence of $3/2^-$ states of ^{9}Li is shown in Fig. 1. From 1 to 150 on the horizontal axis are the wave functions with various $\alpha+t+n+n$ configurations, $\{\Psi(1)_i^{J^\pi KM}\}$, and from 151 to 300 are those with various $t+t+t$ configurations, $\{\Psi(2)_j^{J^\pi KM}\}$. The ground state of ^{9}Li converges at -40.59 MeV within the present framework. This is 6.13 MeV below the $\alpha+t+n+n$ threshold (calculated at -34.46 MeV) compared with the experimental value of 8.22 MeV. We can see that the energies of some of the states drastically decrease after superposing three triton configurations, and eventually those states converge at -30.66 MeV as the third $3/2^-$ state, -29.25 MeV as the 5th $3/2^-$ state, and -26.18 MeV as the 7th $3/2^-$ state. In this model, the three triton threshold is calculated at -20.63 MeV, corresponding to $E_x = 19.95$ MeV, compared with the experimental value of 19.55 MeV. These states are considered to have predominantly the three triton configuration, which converges at the energy much above the $\alpha+t+n+n$ threshold, however they are below the three triton threshold. Therefore, the states are bound states with respect to the three triton threshold. The energy and root mean square (rms) radius are listed in Table I. Although the third $3/2^-$ state has large excitation energy of $E_x = 9.93$ MeV, it is much below the three triton threshold, and the calculated rms radius of 2.39 fm is even smaller than that of the ground state (2.43 fm). Therefore, this nature should be called "three triton correlation" rather than "three triton cluster"". The 5th and 7th $3/2^-$ are closer to the threshold
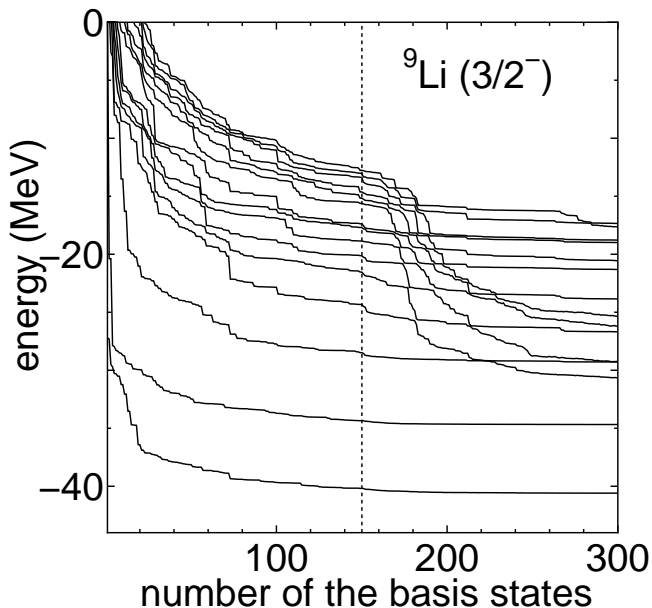
FIG. 1: The energy convergence of $3/2^-$ states of ^{9}Li. From 1 to 150 on the horizontal axis are the wave functions with various $\alpha+t+n+n$ configurations, $\{\Psi(1)_i^{J^\pi KM}\}$, and from 151 to 300 are those with various $t+t+t$ configurations, $\{\Psi(2)_j^{J^\pi KM}\}$.
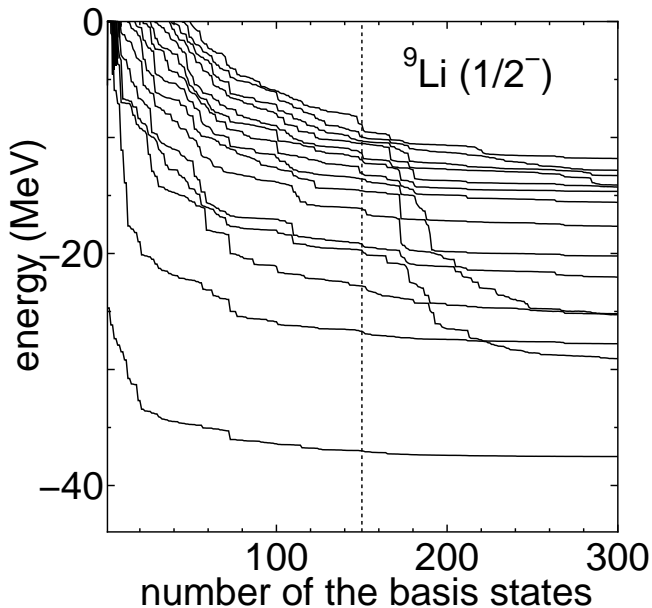

FIG. 2: The energy convergence of $1/2^-$ states of ^{9}Li. The definition of the horizontal axis is the same as Fig. 1.

FIG. 3: The energy convergence of $1/2^+$ states of ^{9}Li. The definition of the horizontal axis is the same as Fig. 1.

and -25.27 MeV as the 4th $1/2^-$ state. The second $1/2^-$ state is below the three triton threshold by 8.43 MeV, and the rms radius is also small (2.37 fm) similarly to the $3/2^-$ case. The 4th $1/2^-$ state is closer to the threshold, however the calculated rms radius is 2.38 fm. The lowest $1/2^-$ state is obtained at -37.50 MeV, 3.09 MeV above the ground state (experimental value is E_x = 2.691 MeV).

The situation is slightly different in the $1/2^+$ case. Figure 3 shows that the state largely affected by the three triton configurations converges as the 4th $1/2^+$ state at -22.25 MeV. This is below the three triton threshold by 1.62 MeV, and the calculated radius is rather large, 2.62 fm, larger than that of the ground state by 0.19 fm. In the ^{9}Li case, positive parity states correspond to higher nordal states, and this could be one of the reasons for the (slightly) large radius. However, it is not enhanced compared with the neutron-halo structure in ^{11}Li or Hoyle state in ^{12}C, and it is smaller than the yrast $1/2^+$ state (2.71 fm). It is also shown that the second $1/2^+$ state is affected when three triton states are coupled to the $\alpha+t+n+n$ basis states. Therefore, the mixing (or coupling) effect of two components, $\alpha+t+n+n$ and $t+t+t$ would be important in the second $1/2^+$ state.

The result for the $3/2^+$ states is shown in Fig. 4, and the 7th $3/2^+$ state is largely affected by the three triton configurations and converges at -22.07 MeV. This state is also close to the threshold, however the calculated rms radius of 2.60 fm is not so strongly enhanced.

The component of three triton clusters for each state is listed in Table II. Here the triton cluster component

energy, however the calculated rms radii are almost the same.

Similar situation can be seen also for the $1/2^-$ states in Fig. 2. The energies of two of the states which drastically decrease after superposing three triton configurations converge at -29.06 MeV as the second $1/2^-$ state
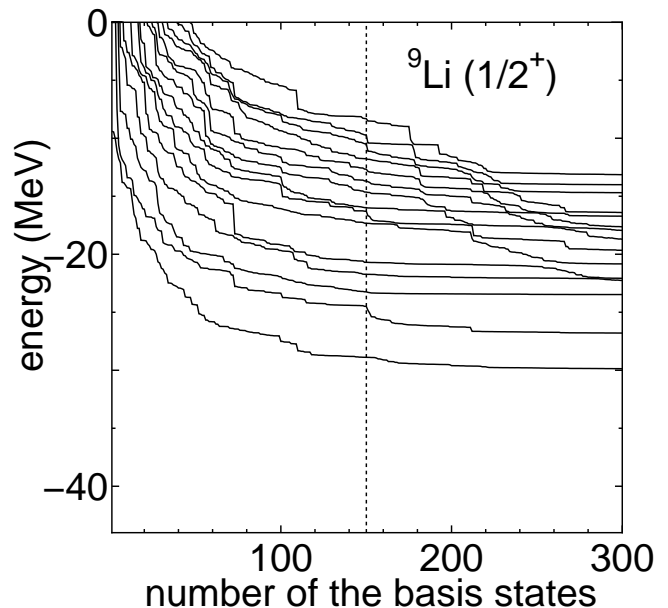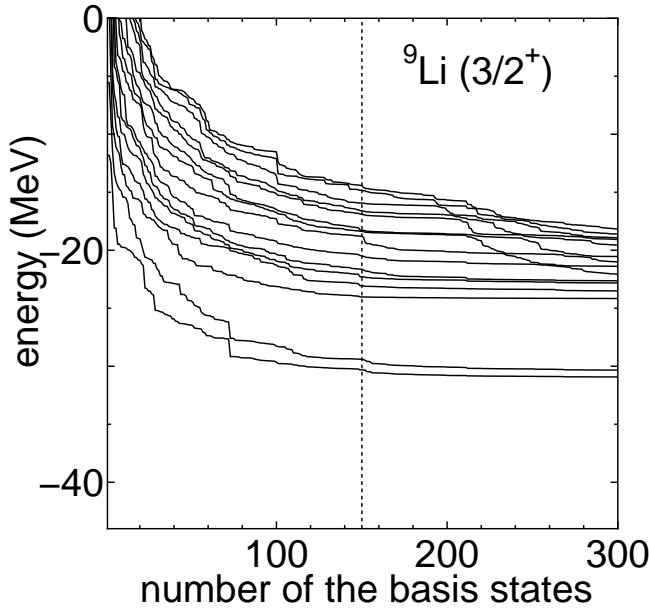
FIG. 4: The energy convergence of $3/2^+$ states of ^{9}Li. The definition of the horizontal axis is the same as Fig. 1.

TABLE I: The calculated root mean square radius (fm) of the $1/2^+$, $3/2^+$, $1/2^-$, and $3/2^-$ states. The energies (MeV) are shown in the parentheses.

	$1/2^+$	$3/2^+$	$1/2^-$	$3/2^-$
1	2.71 (-29.86)	2.72 (-30.92)	2.53 (-37.50)	2.43 (-40.59)
2	2.92 (-26.79)	2.75 (-30.33)	2.37 (-29.06)	2.59 (-34.68)
3	2.62 (-23.47)	2.66 (-24.16)	2.78 (-27.78)	2.39 (-30.66)
4	2.62 (-22.25)	2.77 (-23.50)	2.38 (-25.27)	2.69 (-29.29)
5	2.71 (-22.07)	2.85 (-22.82)	2.84 (-25.19)	2.38 (-29.25)
6	2.66 (-20.83)	2.74 (-22.64)	2.97 (-22.04)	2.86 (-26.67)
7	3.11 (-19.66)	2.60 (-22.07)	2.88 (-20.21)	2.34 (-26.18)

(C_i^{3t}) of the state ϕ_i is defined as

$$C_i^{3t} = \sum_k \langle \phi_i | \psi_k^{3t} \rangle \langle \psi_k^{3t} | \phi_i \rangle. \qquad (7)$$

Here ψ_k^{3t} is the k-th orthonormal state obtained by diagonalizing the Hamiltonian matrix only within the basis states of three triton clusters (from 151 to 300 on the horizontal axis of Figs. 1-4). It can be confirmed that states discussed to be affected by three triton basis states really have large three triton components; the second $1/2^-$ and third $3/2^-$ states have the amount of $\sim$1.00, and the 4th $1/2^-$ and 5th $3/2^-$ states have the amount of 0.96$\sim$0.99. The second $1/2^+$ has mixed structure; C_i^{3t} is around 0.76.

In a similar way, the component of $\alpha+t+n+n$ for each state is listed in Table III. Here the $\alpha+t+n+n$ component $(C_i^{\alpha tnn})$ of the state ϕ_i is defined as

$$C_i^{\alpha tnn} = \sum_k \langle \phi_i | \psi_k^{\alpha tnn} \rangle \langle \psi_k^{\alpha tnn} | \phi_i \rangle, \qquad (8)$$

TABLE II: The components of three triton clusters C_i^{3t} defined in Eq. (7). The values in the parentheses show the energies of the states.

	$1/2^+$	$3/2^+$	$1/2^-$	$3/2^-$
1	0.66 (-29.86)	0.67 (-30.92)	0.65 (-37.50)	0.67 (-40.59)
2	0.76 (-26.79)	0.75 (-30.33)	1.00 (-29.06)	0.77 (-34.68)
3	0.01 (-23.47)	0.06 (-24.16)	0.44 (-27.78)	1.00 (-30.66)
4	0.88 (-22.25)	0.30 (-23.50)	0.99 (-25.27)	0.40 (-29.29)
5	0.23 (-22.07)	0.51 (-22.82)	0.66 (-25.19)	0.96 (-29.25)
6	0.08 (-20.83)	0.23 (-22.64)	0.69 (-22.04)	0.68 (-26.67)
7	0.71 (-19.66)	0.99 (-22.07)	0.34 (-20.21)	1.00 (-26.18)

TABLE III: The components of $\alpha+t+n+n$ configurations $C_i^{\alpha tnn}$ defined in Eq. (8). The values in the parentheses show the energies of the states.

	$1/2^+$	$3/2^+$	$1/2^-$	$3/2^-$
1	0.97 (-29.86)	0.98 (-30.92)	0.99 (-37.50)	0.99 (-40.59)
2	0.86 (-26.79)	0.96 (-30.33)	0.00 (-29.06)	0.99 (-34.68)
3	0.99 (-23.47)	0.99 (-24.16)	0.93 (-27.78)	0.00 (-30.66)
4	0.15 (-22.25)	0.94 (-23.50)	0.02 (-25.27)	0.89 (-29.29)
5	0.89 (-22.07)	0.94 (-22.82)	0.90 (-25.19)	0.05 (-29.25)
6	0.93 (-20.83)	0.95 (-22.64)	0.81 (-22.04)	0.90 (-26.67)
7	0.76 (-19.66)	0.01 (-22.07)	0.95 (-20.21)	0.00 (-26.18)

where $\psi_k^{\alpha tnn}$ is the k-th orthonormal state obtained by diagonalizing the Hamiltonian matrix only within the basis states of $\alpha+t+n+n$ configurations (from 1 to 150 on the horizontal axis of Figs. 1-4). It can be confirmed that states affected by the three triton states have small values; the second $1/2^-$ and third $3/2^-$ states have the values less than 1 %, and the 4th $1/2^-$ and 5th $3/2^-$ states have 0.02 and 0.05, respectively. The second and 4th $1/2^+$ states have mixed structure and the values are 0.86 and 0.15. Since the wave functions of $\alpha+t+n+n$ and three triton configurations are non-orthogonal, the sum of the values of Table II and Table III for each state does not become unity.

Recently, it has been proposed that the strong enhancement of isoscalar monopole (E0) transitions can

TABLE IV: The calculated energy (MeV) and squared isoscalar E0 transition strength from the ground state (fm^4) for the $3/2^-$ states.

energy (MeV)	squared E0 (fm^4)
-40.59	—
-34.68	0.03
-30.66	0.01
-29.29	40.51
-29.25	2.11
-26.67	21.68
-26.18	0.01

be a measure of the existence of cluster structure [20]. The isoscalar E0 operator has a form of $\sum_i r_i^2$, where the sum over i is for all the nucleons. From the experimental side, the presence of cluster states in ^{13}C has been suggested by measuring the isoscalar E0 transitions from the ground $1/2^-$ state induced by the ^{13}C$(\alpha, \alpha')^{13}$C reaction [21]. Also from the theoretical side, the relation between the monopole transition strength and the cluster structure has been widely discussed [22–27]. The squared isoscalar E0 transition strength from the ground $3/2^-$ state to low-lying $3/2^-$ states is listed in Table IV. Although the strength to the 4th $3/2^-$ state shows an enhanced value of ~ 40 fm^4, the strengths to the third and 5th $3/2^-$ states, which have large components of three tritons, are very suppressed. This is considered to be due to the small rms radius of the third and 5th $3/2^-$ state, and the other reason comes from the fact that the wave functions of the ground and those $3/2^-$ states have completely different structure.

IV. CONCLUSION

We focused on the non-α cluster structure in light neutron-rich nuclei; three triton structure in ^{9}Li. This is an analogy to the case of three α state in ^{12}C (Hoyle state). For this purpose, we introduced both three triton wave functions and $\alpha + t + n + n$ wave functions for the low-lying states of ^{9}Li, and coupling effect has been taken into account.

The calculated second $1/2^-$ and third $3/2^-$ states have dominantly the components of three triton clusters, and the states are well below the threshold energy of three tritons. The calculated rms radii are quite small, and the E0 transition strength from the ground state to the third $3/2^-$ state is also small. Therefore, this nature should be called "three triton correlation" rather than "three triton cluster"". The situation is the same for other three triton states close to the threshold.

On the other hand, the positive parity states correspond to higher nordal states, and the states have slightly larger rms radii. However, the mixing (or coupling) effect of two components, $\alpha + t + n + n$ and $t + t + t$ is important in these states.

Nevertheless, three triton correlation is important and states with dominantly the three triton components appear in the low-lying states of ^{9}Li.

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