Energy loss of fast gluons in a gluonic fluid

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We estimate the energy loss of a fast gluon propagating in a gluonic fluid due to the radiative process: gg $\to$ ggg by relaxing some of the approximations used in previous works.

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The energy loss of high energy partons propagating through quark gluon plasma (QGP) is a field of high contemporary interest. Experimentally the energy dissipation has been measured through the suppression of the transverse momentum (p_T) distribution of hadrons produced in Au+Au relative to the binary scaled p+p collisions at the same centre of mass energy. The nature of the suppression may be used as a tool for diagnosis of QGP formation in nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC). The two most common mechanisms for the energy loss are elastic and in-elastic or radiative processes. Among the various in-elastic processes involving quarks and gluons, the process $\mathrm{g+g \to g+g+g}$ plays the leading role. The momentum spectrum of the radiated gluon can be used to estimate the energy loss of fast partons propagating in QGP. The momentum distribution of the radiated gluons from the above process, considered by Gunion and Bertsch (GB) [1] long ago, has drawn some attention recently [2, 3]. In the present work, we will estimate the energy dissipation due to the processes $\mathrm{g+g \to g+g+g}$ by relaxing some of the approximations considered earlier [1–3]. The results obtained here may be extended for other partonic processes such as $q+q \to q+q+\mathrm{g}$, $q+\mathrm{g} \to q+\mathrm{g}+\mathrm{g}$ etc.

We consider the process, $\mathrm{g}(k_1)+\mathrm{g}(k_2) \to \mathrm{g}(k_3)+\mathrm{g}(k_4)+\mathrm{g}(k_5)$, the square of the invariant amplitude for this reaction can be written as [4]:

$$
\begin{aligned}
|M_{\mathrm{gg\to ggg}}|^2 &= \frac{1}{2}g^6\frac{N_c^3}{N_c^2-1}\frac{\mathcal{N}}{\mathcal{D}} \\
&\times [(12345)+(12354)+(12435) \\
&+ (12453)+(12534)+(12543) \\
&+ (13245)+(13254)+(13425) \\
&+ (13524)+(14235)+(14325)], \qquad (1)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{N} &= (k_1.k_2)^4+(k_1.k_3)^4+(k_1.k_4)^4 \\
&+ (k_1.k_5)^4+(k_2.k_3)^4+(k_2.k_4)^4+(k_2.k_5)^4 \\
&+ (k_3.k_4)^4+(k_3.k_5)^4+(k_4.k_5)^4; \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D} &= (k_1.k_2)(k_1.k_3)(k_1.k_4)(k_1.k_5)(k_2.k_3) \\
&\times (k_2.k_4)(k_2.k_5)(k_3.k_4)(k_3.k_5)(k_4.k_5), \qquad (3)
\end{aligned}
$$

and

$$(ijklm) = (k_i.k_j)(k_j.k_k)(k_k.k_l)(k_l.k_m)(k_m.k_i). \qquad (4)$$

$N_c(=3)$ is the number of colors, $g=\sqrt{4\pi\alpha_s}$ is the colour charge and α_s is the strong coupling.

The quantity, $|M_{\mathrm{gg\to ggg}}|^2$ after simplification can be written as (see appendix):

$$
\begin{aligned}
|M|^2{}_{\mathrm{gg\to ggg}} &= 12g^2|M_{\mathrm{gg\to gg}}|^2_{GB}\frac{1}{k_\perp^2} \\
&\times [(1+\frac{t}{2s}+\frac{5t^2}{2s^2}-\frac{t^3}{s^3}) \\
&- (\frac{3}{2\sqrt{s}}+\frac{4t}{s\sqrt{s}}-\frac{3t^2}{2s^2\sqrt{s}})k_\perp \\
&+ (\frac{5}{2s}+\frac{t}{2s^2}+\frac{5t^2}{s^3})k_\perp^2)], \qquad (5)
\end{aligned}
$$

where $|M_{\mathrm{gg\to gg}}|^2_{GB}=(9/2)g^4s^2/t^2$, $s=(k_1+k_2)^2, t=(k_1-k_3)^2$, $u=(k_1-k_4)^2$, $k_\perp$ is transverse momentum of the radiated gluon. Although the term $O(k_\perp^{-2})$ is the most dominant one for the soft gluon emission, the other terms in Eq. 5 make non-negligible contributions to gluonic energy loss and hence has crucial importance for the phenomenology of heavy ion collisions at ultra-relativistic energies. In consequence of this, the spectrum of the radiated gluon, derived by using the ratio of the amplitude square of the radiative process, gg $\to$ ggg to that of the elastic process, gg $\to$ gg, becomes,

$$
\begin{aligned}
\frac{dn_g}{d^2k_\perp d\eta} &= \left[\frac{dn_g}{d^2k_\perp d\eta}\right]_{GB}[(1+\frac{t}{2s}+\frac{5t^2}{2s^2}-\frac{t^3}{s^3}) \\
&- (\frac{3}{2\sqrt{s}}+\frac{4t}{s\sqrt{s}}-\frac{3t^2}{2s^2\sqrt{s}})k_\perp \\
&+ (\frac{5}{2s}+\frac{t}{2s^2}+\frac{5t^2}{s^3})k_\perp^2)], \qquad (6)
\end{aligned}
$$

where η is the rapidity of the radiated gluon, the subscript GB has been used to indicate the gluon spectrum obtained using the approximation considered in [1] (see also [5]) which is given by,

$$\left[\frac{dn_g}{d^2k_\perp d\eta}\right]_{GB}=\frac{C_A\alpha_s}{\pi^2}\frac{q_\perp^2}{k_\perp^2[(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+m_D^2]}, \qquad (7)$$

$m_D=\sqrt{\frac{2\pi}{3}\alpha_s(T)(C_A+\frac{N_F}{2})}T$, is the thermal mass of the gluon [6], N_F is the number of flavors contributing in the gluon self-energy loop, $C_A=3$ is the Casimir invariant for the SU(3) adjoint representation, α_s is the temperature dependent strong coupling [7] and $q_\perp$ is the transverse momentum transfer. The thermal mass in the denominator of Eq. 7 has been introduce to shield the infrared divergence arising from the massless intermediary gluon exchange.

The energy loss of a gluon passing through the QGP medium can now be calculated using the gluon spectrum of Eq. 6. The energy loss per collision can now be estimated as:

$$\epsilon = \int d^2k_\perp d\eta \frac{dn_g}{d^2k_\perp d\eta} k_0 \theta(\Lambda^{-1} - \tau_F) \times \theta(E - k_\perp cosh\eta), \quad (8)$$

where $k_0 = k_\perp cosh\eta$ is the energy of the radiated gluon and τ_F is the formation time of the gluon. The first θ-function in Eq. 8, involving Λ^{-1} or interaction time, is introduced for the LPM effect. The effect is actually due to a characteristic destructive interference phenomenon caused by finite formation time of the radiated gluon with 4-momentum $k_5 = (k_0, k_\perp, k_z)$ defined by $\tau_F \sim 1/\Delta E$, where ΔE is the energy lost by the particle in a single collision. In effect, τ_F is the minimum time needed to resolve the transverse wave-packet of the quanta with $\Delta x_\perp \sim 1/k_\perp$ from its high energy parent($E >> k_0$). In a nutshell, the radiated gluon must have a formation time greater than the mean free time. τ_F in Eq. 8 is the formation time, given by: $\tau_F = cosh\eta/k_\perp$. It is the time within which another interaction, if occurs, results in the suppression of the emission of the gluon. Destructive interference among the radiation amplitudes associated with multiple scattering is expected when the formation time is larger than the mean free path, λ. When $\tau_F >> \lambda$ the scattering centers cannot resolve the emitted quanta and the incoherent contribution of each scattering breaks down. This effect is called Landau-Pomeranchuk-Migdal (LPM) suppression. This suppression imposes some restriction on the phase space of the radiated gluon. The second θ-function sets the upper limit for the energy of the radiated gluon.

To proceed further, we replace $q_\perp^2$ by its average value evaluated as follows:

$$\langle q_\perp^2 \rangle = \frac{1}{\sigma_{el}} \int_{m_D^2}^{\frac{s}{4}} dq_\perp^2 \frac{d\sigma_{el}}{dq_\perp^2} q_\perp^2, \quad (9)$$

where

$$\sigma_{el} = \int_{m_D^2}^{\frac{s}{4}} dq_\perp^2 \frac{d\sigma_{el}}{dq_\perp^2}. \quad (10)$$

For dominant small angle scattering ($t \to 0$),

$$\frac{d\sigma_{el}}{dq_\perp^2} = C_i \frac{2\pi\alpha_s^2}{q_\perp^4}. \quad (11)$$

C_i is 9/4, 1 and 4/9 for gg, qg and qq scattering. $\langle q_\perp^2 \rangle$ is then obtained as,

$$\langle q_\perp^2 \rangle = \frac{s m_D^2}{s - 4m_D^2} \ln(\frac{s}{4m_D^2}). \quad (12)$$

For $\sqrt{s} \to \infty$, i.e. in the high-energy limit one can make the replacement $t \sim -q_\perp^2$ [8]. In contrast to the previous works [2, 3] where the value of s was taken as $s \sim 18T^2$ we put $s \sim 6ET$ in Eq. 12, allowing the possibility for the incident gluon to remain out of thermal equilibrium ($E \neq 3T$). With all the above ingredients we are now ready to evaluate energy loss (dE/dx) of a fast gluon in the QGP as follows:

$$-\frac{dE}{dx} = \epsilon \cdot \Lambda. \quad (13)$$

The interaction rate, Λ has been evaluated by using the procedure similar to [9].

We have displayed the results of the ratio, $|M|^2_{\rm gg\to ggg}/|M_{\rm gg\to gg}|^2_{GB}$ for the approximations used in Refs. [2, 3] in Fig. 1 and compared the results with the present calculation. We observe that for a 10 GeV incident gluon the ratios obtained in the present calculation differ by 5-10% from that of Ref. [3] for the range of temperature considered here. The corresponding difference is about 2-8% when we consider the matrix element of Ref. [2].

The energy loss of a fast gluon moving in a gluonic plasma has been estimated by using Eqs. 8 and 13. The result is displayed in Fig. 2. We find that the magnitude of energy loss with the current gluon spectrum (Eq. 6) is larger than the one obtained with spectrum of Ref. [1] for gluon energy above 20 GeV. This will have crucial consequences on the heavy ion phenomenology at RHIC and LHC collision energies.

In the evaluation of the energy loss, dE/dx, the gluon spectrum is required as an input. In Fig. 3 we compare the temperature variation of dE/dx (normalized by the dE/dx for GB approximation) with those obtained by using the gluon spectra of Refs. [2, 3]. The importance of the terms $\mathcal{O}(k_\perp^{-1})$ and $\mathcal{O}(k_\perp^0)$ is evident from the results displayed in the figure for $E = 10$ GeV.

In summary, we perform a comparative study of the radiative energy loss of a high energy gluon propagating through a thermal gluonic medium by taking the momentum spectrum of the radiated gluons from Eq. 6 and Refs. [2, 3]. We find that the previously neglected terms $\mathcal{O}(k_\perp^{-1})$ and $\mathcal{O}(k_\perp^0)$ may play crucial role in energy loss of gluons in a gluonic plasma.

Appendix

In this appendix we derive Eq. 5 for the square of the invariant amplitude for the radiative process, gg $\to$ ggg upto order $\mathcal{O}({\rm k_\perp}^0)$ and $\mathcal{O}(\frac{\rm t^3}{\rm s^3})$. Consider the reaction:

$$\rm g(k_1) + g(k_2) \to g(k_3) + g(k_4) + g(k_5), \quad (14)$$

where k_5 is the four-momentum of the radiated gluon. The Mandelstam variables for the above process are defined as:

$$s = (k_1 + k_2)^2, \quad t = (k_1 - k_3)^2$$

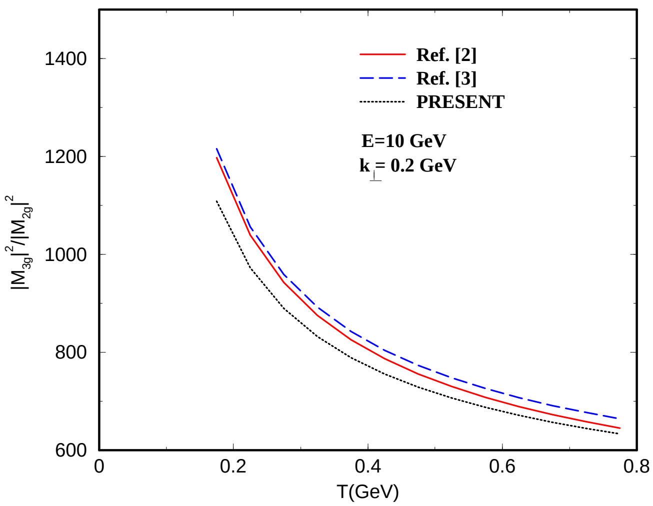

FIG. 1: (color online) Variation of the ratios of radiative to collisional matrix element square with temperature.

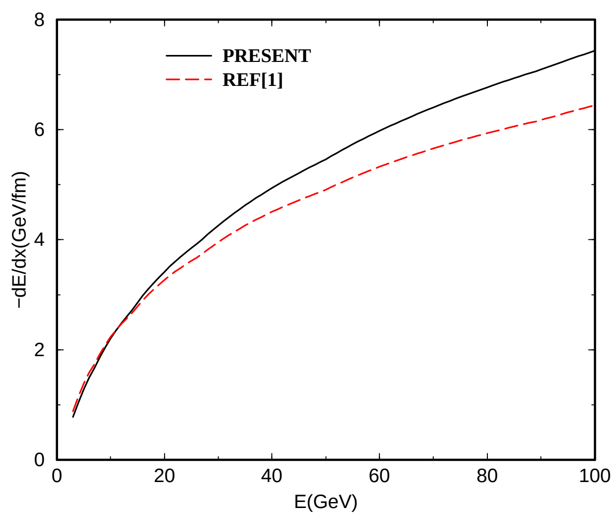

FIG. 2: (color online) Energy loss of a gluon propagating through a thermal gluonic medium at temperature, $T = 300$ MeV.

$$u = (k_1 - k_4)^2, \quad s' = (k_3 + k_4)^2$$
$$t' = (k_2 - k_4)^2, \quad u' = (k_2 - k_3)^2. \qquad (15)$$

Gluons being massless we can write:

$$k_1.k_2 = \frac{s}{2}, \quad k_1.k_3 = -\frac{t}{2}$$
$$k_1.k_4 = -\frac{u}{2}, \quad k_3.k_4 = \frac{s'}{2}$$
$$k_2.k_4 = -\frac{t'}{2}, \quad k_2.k_3 = -\frac{u'}{2}. \qquad (16)$$

We also have the relations:

$$k_1.k_5 = \frac{s+t+u}{2}, \quad k_2.k_5 = \frac{s+t'+u'}{2}$$

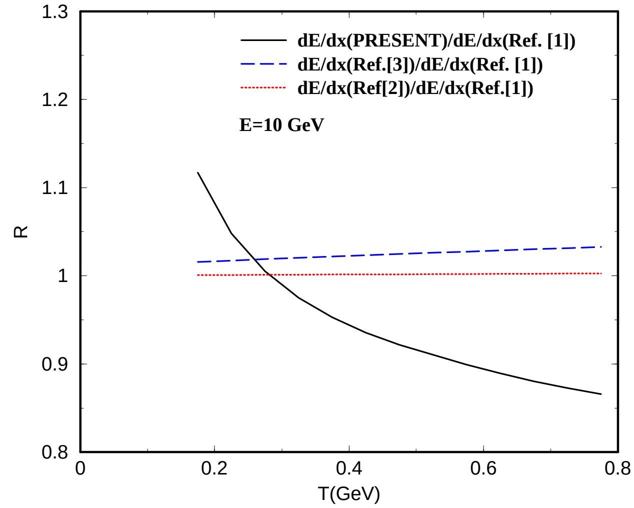

FIG. 3: (color online) Variation of dE/dx (normalized by the dE/dx for GB approximation) with T. Solid line indicates the result for the present work. Dashed (dotted) line depicts the energy loss obtained from the gluon spectrum of Ref. [3] ([2]).

$$k_3.k_5 = \frac{s+t'+u}{2}, \quad k_4.k_5 = \frac{s+t+u'}{2}. \qquad (17)$$

For soft gluon emission:

$$s + t + u + s' + t' + u' = 0. \qquad (18)$$

The matrix element square of the radiative process gg → ggg is given by [4]:

$$|M_{\mathrm{gg}\to\mathrm{ggg}}|^2 = \frac{1}{2}g^6\frac{N_c^3}{N_c^2-1}\frac{\mathcal{N}}{\mathcal{D}}$$
$$\times\ [(12345) + (12354) + (12435)$$
$$+\ (12453) + (12534) + (12543)$$
$$+\ (13245) + (13254) + (13425)$$
$$+\ (13524) + (14235) + (14325)], \qquad (19)$$

where N_c is the number of colors, $g = \sqrt{4\pi\alpha_s}$ is the strong coupling,

$$\mathcal{N} = (k_1.k_2)^4 + (k_1.k_3)^4 + (k_1.k_4)^4$$
$$+\ (k_1.k_5)^4 + (k_2.k_3)^4 + (k_2.k_4)^4 + (k_2.k_5)^4$$
$$+\ (k_3.k_4)^4 + (k_3.k_5)^4 + (k_4.k_5)^4, \qquad (20)$$

$$\mathcal{D} = (k_1.k_2)(k_1.k_3)(k_1.k_4)(k_1.k_5)(k_2.k_3)$$
$$\times\ (k_2.k_4)(k_2.k_5)(k_3.k_4)(k_3.k_5)(k_4.k_5), \qquad (21)$$

and

$$(ijklm) = (k_i.k_j)(k_j.k_k)(k_k.k_l)(k_l.k_m)(k_m.k_i). \qquad (22)$$

Simplifying Eq. 19 we get,

$$
\begin{aligned}
|M_{\rm gg\to ggg}|^2 &= 16g^6\frac{N_c^3}{N_c^2-1}\mathcal{N}\\
&\times [\frac{1}{s'(s+u+t)(s+u'+t')}[\frac{1}{tt'}+\frac{1}{uu'}]\\
&+ \frac{1}{s(s+u+t)(s+u'+t)}[\frac{1}{tt'}+\frac{1}{uu'}]\\
&- \frac{1}{t'(s+u+t)(s+u+t')}[\frac{1}{uu'}+\frac{1}{ss'}]\\
&- \frac{1}{u'(s+u+t)(s+u'+t)}[\frac{1}{tt'}+\frac{1}{ss'}]\\
&- \frac{1}{u(s+u'+t')(s+u+t')}[\frac{1}{tt'}+\frac{1}{ss'}]\\
&- \frac{1}{t(s+u'+t')(s+u'+t)}[\frac{1}{uu'}\\
&+ \frac{1}{ss'}]; \qquad (23)
\end{aligned}
$$

and $\mathcal{N}$ can now be written as:

$$
\begin{aligned}
\mathcal{N} &= \frac{1}{16}[s^4+t^4+u^4+s'^4+t'^4+u'^4\\
&+ (s+t+u)^4+(s+t'+u')^4+(s+t'+u)^4\\
&+ (s+t+u')^4]. \qquad (24)
\end{aligned}
$$

For a soft gluon emission ($k_5\to 0$), $s\to s'$, $t\to t'$, $u\to u'$. We can express the transverse momentum of the emitted gluon as:

$$
\begin{aligned}
k_\perp^2 &= 4(k_1.k_5)(k_2.k_5)/s\\
&= (s+t+u)(s+t'+u')\\
&= (s+t+u)^2/s\\
&. \qquad (25)
\end{aligned}
$$

Using Eqs. 23, 24 and 25, the square of the matrix element can be written as:

$$
\begin{aligned}
|M|^2{}_{\rm gg\to ggg} &= \frac{27}{2}g^6(s^4+t^4+u^4+2s^2k_\perp^4)\frac{1}{sk_\perp^2}\\
&\times [\frac{1}{s}(\frac{1}{t^2}+\frac{1}{u^2})\\
&- \frac{1}{t}(\frac{1}{s^2}+\frac{1}{u^2})\\
&- \frac{1}{u}(\frac{1}{t^2}+\frac{1}{s^2})]\\
&= g^2(\frac{27}{2}g^4s^4)(1+\frac{t^4}{s^4}+\frac{u^4}{s^4}+2\frac{k_\perp^4}{s^2})\\
&\times \frac{1}{s^2k_\perp^2t^2}[1+\frac{t^2}{u^2}-\frac{t}{s}-\frac{st}{u^2}-\frac{s}{u}-\frac{t^2}{us}]\\
&= g^2(\frac{9}{2}g^4\frac{s^2}{t^2})(3+3\frac{t^4}{s^4}+3\frac{u^4}{s^4}+6\frac{k_\perp^4}{s^2})\frac{1}{k_\perp^2}\\
&\times [1+\frac{t^2}{u^2}-\frac{t}{s}-\frac{st}{u^2}-\frac{s}{u}-\frac{t^2}{us}]\\
&= g^2(\frac{9}{2}g^4\frac{s^2}{t^2})(3(1+\frac{u^4}{s^4})+3\frac{t^4}{s^4}+6\frac{k_\perp^4}{s^2})\\
&\times \frac{1}{k_\perp^2}[1-\frac{s}{u}-(1+\frac{s^2}{u^2})\frac{t}{s}+(\frac{s^2}{u^2}-\frac{s}{u})\frac{t^2}{s^2}]\\
&= g^2|M_{\rm gg\to gg}|^2_{GB}(3(1+\frac{u^4}{s^4})+3\frac{t^4}{s^4}+6\frac{k_\perp^4}{s^2})\\
&\times \frac{1}{k_\perp^2}[(1-\frac{s}{u})-(1+\frac{s^2}{u^2})\frac{t}{s}\\
&+ (\frac{s^2}{u^2}-\frac{s}{u})\frac{t^2}{s^2}], \qquad (26)
\end{aligned}
$$

where the subscript GB stands for the approximation used by Gunion and Bertsch [1]. For the elastic process,

$$
|M_{\rm gg\to gg}|^2_{GB} = \frac{9}{2}g^4\frac{s^2}{t^2}. \qquad (27)
$$

On simplifying Eq. 26 we obtain,

$$
\begin{aligned}
|M|^2{}_{\rm gg\to ggg} &= g^2|M_{\rm gg\to gg}|^2_{GB}\\
&\times \frac{1}{k_\perp^2}[(3-3\frac{s}{u}+3\frac{u^4}{s^4}-3\frac{u^3}{s^3})-(3\frac{t}{s}+3\frac{st}{u^2}\\
&+ 3\frac{u^4t}{s^5}+3\frac{u^2t}{s^3})\\
&+ (3\frac{t^2}{u^2}-3\frac{t^2}{us}+3\frac{u^2t^2}{s^4}-3\frac{u^3t^2}{s^5})\\
&+ (3\frac{t^4}{s^4}-3\frac{t^4}{us^3})\\
&- (3\frac{t^5}{s^5}+3\frac{t^5}{u^2s^3})+(3\frac{t^6}{u^2s^4}-3\frac{t^6}{us^5})\\
&+ (6\frac{k_\perp^4}{s^2}-6\frac{k_\perp^4}{us})-(6\frac{k_\perp^4t}{s^3}+6\frac{k_\perp^4t}{u^2s})\\
&+ (6\frac{k_\perp^4t^2}{u^2s^2}-6\frac{k_\perp^4t^2}{us^3})]. \qquad (28)
\end{aligned}
$$

In the proposed kinematic limit we set terms which are linear in $k_\perp$ to zero and keep terms $\mathcal{O}(k_\perp^0), \mathcal{O}(k_\perp^{-1})$ and $\mathcal{O}(k_\perp^{-2})$ in $|M|^2{}_{\rm gg\to ggg}$. We also neglect terms $\mathcal{O}(\frac{t^4}{s^4})$ and higher order in the matrix element. To proceed further one needs to express the Mandelstam variable, u in terms of s, t and $k_\perp$ by using the following relation:

$$
\begin{aligned}
{k_\perp}^2 &= \frac{(s+t+u)^2}{s}\\
\Rightarrow u &= \sqrt{s}k_\perp - s - t\\
\Rightarrow \frac{1}{u} &= \frac{1}{(\sqrt{s}k_\perp - s - t)}\\
\Rightarrow \frac{1}{u} &= -\frac{1}{s}[1-(\frac{k_\perp}{\sqrt{s}}-\frac{t}{s})]^{-1}\\
\Rightarrow \frac{1}{u} &\approx -\frac{1}{s}[1+(\frac{k_\perp}{\sqrt{s}}-\frac{t}{s})
\end{aligned}
$$

$$
\begin{aligned}
&+ \ (\frac{k_\perp}{\sqrt{s}} - \frac{t}{s})^2 \\
&+ \ (\frac{k_\perp}{\sqrt{s}} - \frac{t}{s})^3 \\
&+ \ (\frac{k_\perp}{\sqrt{s}} - \frac{t}{s})^4 \\
&+ \ (\frac{k_\perp}{\sqrt{s}} - \frac{t}{s})^5 + ... \qquad (29)
\end{aligned}
$$

The binomial expansion of $[1-(\frac{k_\perp}{\sqrt{s}}-\frac{t}{s})]^{-1}$ converges if $(\frac{k_\perp}{\sqrt{s}}-\frac{t}{s})<1$. For the kinematic limit mentioned above *i.e.* for $k_\perp \to 0$ and keeping terms upto $\mathcal{O}(\frac{t^3}{s^3})$, the inequality $(\frac{k_\perp}{\sqrt{s}}-\frac{t}{s})<1$ is satisfied. We have checked that terms beyond $(\frac{k_\perp}{\sqrt{s}}-\frac{t}{s})^5$ in the expression of $\frac{1}{u}$ are not required for the kinematic limit under consideration. With all these we get,

$$
\begin{aligned}
\frac{1}{u} \ &= \ -\frac{1}{s}[(1-\frac{t}{s}+\frac{t^2}{s^2}-\frac{t^3}{s^3}) \\
&+ \ (\frac{1}{\sqrt{s}}-\frac{2t}{s\sqrt{s}}+\frac{3t^2}{s^2\sqrt{s}})k_\perp \\
&+ \ (\frac{1}{s}-\frac{3t}{s^2}+\frac{6t^2}{s^3})k_\perp^2]. \qquad (30)
\end{aligned}
$$

Similarly $1/u^2$ can be written as

$$
\begin{aligned}
\frac{1}{u^2} \ &= \ \frac{1}{s^2}[(1-\frac{2t}{s}+\frac{3t^2}{s^2}-\frac{4t^3}{s^3}) \\
&+ \ (\frac{2}{\sqrt{s}}-\frac{6t}{s\sqrt{s}}+\frac{12t^2}{s^2\sqrt{s}})k_\perp \\
&+ \ (\frac{3}{s}-\frac{12t}{s^2}+\frac{30t^2}{s^3})k_\perp^2]. \qquad (31)
\end{aligned}
$$

For the assumed kinematic conditions u^4/s^4 can be expressed as follows:

$$
\begin{aligned}
\frac{u^4}{s^4} \ &= \ [(1+\frac{4t}{s}+\frac{6t^2}{s^2}+\frac{4t^3}{s^3}) \\
&- \ (\frac{4}{\sqrt{s}}+\frac{12t}{s\sqrt{s}}+\frac{12t^2}{s^2\sqrt{s}})k_\perp \\
&+ \ (\frac{6}{s}+\frac{12t}{s^2}+\frac{6t^2}{s^3})k_\perp^2]. \qquad (32)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\frac{u^3}{s^3} \ &= \ -[(1+\frac{3t}{s}+\frac{3t^2}{s^2}+\frac{t^3}{s^3}) \\
&- \ (\frac{3}{\sqrt{s}}+\frac{6t}{s\sqrt{s}}+\frac{3t^2}{s^2\sqrt{s}})k_\perp \\
&+ \ (\frac{3}{s}+\frac{3t^2}{s^2})k_\perp^2]; \qquad (33)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{u^2}{s^2} \ &= \ [(1+\frac{2t}{s}+\frac{t^2}{s^2})-(\frac{2}{\sqrt{s}}+\frac{2t}{s\sqrt{s}})k_\perp \\
&+ \ \frac{1}{s}k_\perp^2]. \qquad (34)
\end{aligned}
$$

Putting Eqs. 30 to 34) in 28 we get,

$$
\begin{aligned}
|M|^2{}_{\mathrm{gg}\to\mathrm{ggg}} \ &= \ 12g^2|M_{\mathrm{gg}\to\mathrm{gg}}|^2_{GB}\frac{1}{k_\perp^2} \\
&\times \ [(1+\frac{t}{2s}+\frac{5t^2}{2s^2}-\frac{t^3}{s^3}) \\
&- \ (\frac{3}{2\sqrt{s}}+\frac{4t}{s\sqrt{s}}-\frac{3t^2}{2s^2\sqrt{s}})k_\perp \\
&+ \ (\frac{5}{2s}+\frac{t}{2s^2}+\frac{5t^2}{s^3})k_\perp^2)]. \qquad (35)
\end{aligned}
$$

The terms $\mathcal{O}(k_\perp^{-1})$ and $\mathcal{O}(k_\perp^0)$ contributes to the energy loss of the gluons in a gluonic plasma and hence are important for heavy ion phenomenology at RHIC and LHC energies. These terms were absent in the previous work [3] (also in [2])

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